On Valency Based Topological Properties of Zigzag-edge Coronoid Graph

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Abstract
The Chemical graph theory is related with the chemical structure of different compounds. By applying some mathematical tackles, a chemical graph is rehabilitated into a real constant. This constant identity has the property that it elaborate the characteristics of the molecule. These constants is called topological invariants. In this editorial, we find some topological invariants via M-polynomial for the molecular structure of Zigzag-edge Coronoid graph.

Keywords: M-Polynomial, Zigzag-edge, Coronoid, topological indices

Introduction

Topological indices in theoretical chemistry has a great interest. The topological indices help us to understand the different sorts of the chemical substance. So topological index has a key role that show the chemical structure to a mathematical number and is used to explain a molecule under testing.

Topological indices are calculated from their definition; however, these are also calculated by using their M-polynomial. For a graph H, the M-polynomial defined in 2015[6] as:

$$M(H,x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(H) x^i y^j$$

Where $\delta = \min\{d_a | a \in V(H)\}$, $\Delta = \max\{d_a | a \in V(H)\}$ and $m_{ij}(H)$ is the counting of edges $ab \in$ E(H) such that $\{d_a, d_b\} = \{i,j\}$. Table 2 shows some well known degree based topological invariants in terms of via M-Polynomial. M-polynomial of many graphs are introduced [1, 3, 5, 9-17, 28-31, 35, 39] . For more information about the topological indices, the reader can look at the articles [2, 8, 20-27, 36-38, 40]. In this article we calculate M-polynomials and topological invariants of $ZC(g,h,q)$, The zigzag-edge coronoid $ZC(g,h,q)$, shown in Figure 1, is obtained by fusing 3 linear polyacenes of length g,h and q respectively.

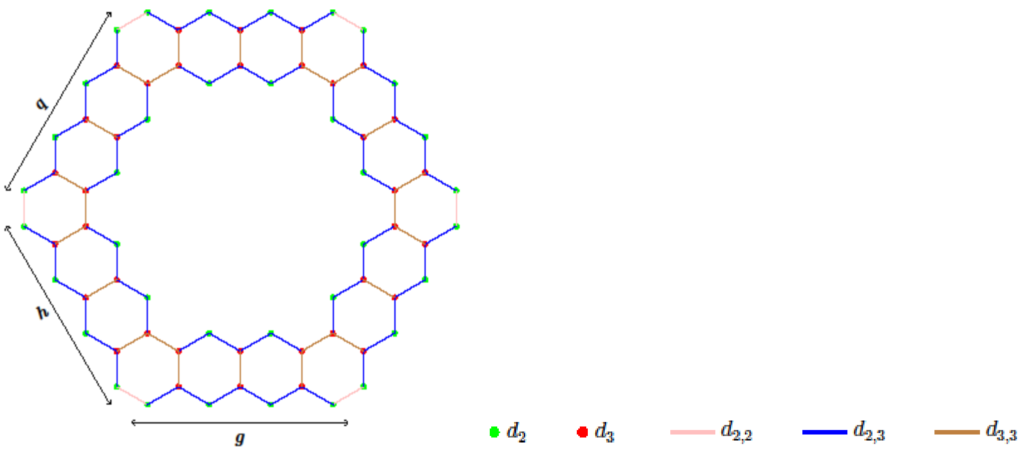

Figure 1: Zigzag-edge Coronoid $ZC(g,h,q)$

Table 1: partition of edges of $ZC(g,h,q)$

(d_a, d_b)	Number of edges
(2,2)	6
(2,3)	$8(g+h+q)-36$
(3,3)	$2(g+h+q)$
Total edges	$10(g+h+q)-30$

Table 2: Degree-based Topological indices

Atom-bond connectivity index[7]	$ABC[ZC(g,h,q)]$	=	$\sum_{uv \in E(ZC(g,h,q))} \sqrt{\frac{d_u + d_v - 2}{d_u . d_v}}$
Geometric arithmetic index[4]	$GA[ZC(g,h,q)]$	=	$\sum_{uv \in E(ZC(g,h,q))} \frac{2\sqrt{d_u . d_v}}{d_u + d_v}$
First K Banhtti index[14]	$B_1[ZC(g,h,q)]$	=	$\sum_{uv \in E(ZC(g,h,q))} (d_u + d_{uv})$

Second K Banhtti index[14]	$B_2[ZC(g,h,q)]$	=	$\sum_{uv\in E(ZC(g,h,q))} (d_u.d_{uv})$
First K hyper Banhatti index[32]	$HB_1[ZC(g,h,q)]$	=	$\sum_{uv\in E(ZC(g,h,q))} (d_u+d_{uv})^2$
Second K hyper Banhatti index[32]	$HB_2[ZC(g,h,q)]$	=	$\sum_{uv\in E(ZC(g,h,q))} (d_u.d_{uv})^2$
Modified first K Banhatti index[33]	${}^mB_1[ZC(g,h,q)]$	=	$\sum_{uv\in E(ZC(g,h,q))} \frac{1}{d_u+d_{uv}}$
Modified second K Banhatti index[33]	${}^mB_2[ZC(g,h,q)]$	=	$\sum_{uv\in E(ZC(g,h,q))} \frac{1}{d_u.d_{uv}}$
Harmonic K Banhatti index[33]	$H_b[ZC(g,h,q)]$	=	$\sum_{uv\in E(ZC(g,h,q))} \frac{2}{d_u+d_{uv}}$

Table 3: Degree-based Topological indices via M-polynomial

Topological invariants	Derivation from M(G;x,y)		
Atom-bond connectivity index	$ABC[ZC(g,h,q)]$		$D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y\frac{1}{2}[f(x,y)]_{x=1}$
Geometric arithmetic index	$GA[ZC(g,h,q)]$		$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[f(x,y)]_{x=1}$
First K Banhatti index	$B_1[ZC(g,h,q)]$		(D_x+D_y) $+2D_xQ_{-2}J)[f(x,y)]_{x=y=1}$
Second K Banhatti index	$B_2[ZC(g,h,q)]$		$D_xQ_{-2}J(D_x+D_y)[f(x,y)]_{x=1}$
First K hyper Banhatti index	$HB_1[ZC(g,h,q)]$		$(D_x^2+D_y^2+2D_x^2Q_{-2}J$ $+2D_xQ_{-2}J(D_x$ $+D_y))[f(x,y)]_{x=y=1}$
Second K hyper Banhatti index	$HB_2[ZC(g,h,q)]$		$D_x^2Q_{-2}J(D_x^2+D_y^2)[f(x,y)]_{x=1}$
Modified first K Banhatti index	${}^mB_1[ZC(g,h,q)]$		$S_xQ_{-2}J(L_x+L_y)[f(x,y)]_{x=1}$
Modified second K Banhatti index	${}^mB_2[ZC(g,h,q)]$		$S_xQ_{-2}J(S_x+S_y)[f(x,y)]_{x=1}$
Harmonic K Banhatti index	$H_b[ZC(g,h,q)]$		$2S_xQ_{-2}J(L_x+L_y)[f(x,y)]_{x=1}$

Where the operator used are defined as

$D_xf(x,y)=x\frac{\partial(f(x,y))}{\partial x}$, $D_yf(x,y)=y\frac{\partial(f(x,y))}{\partial y}$, $L_xf(x,y)=f(x^2,y)$, $L_yf(x,y)=f(x,y^2)$,

$S_xf(x,y)=\int_0^x\frac{f(t,y)}{t}dt$, $S_yf(x,y)=\int_0^y\frac{f(x,t)}{t}dt$, $Jf(x,y)=f(x,x)$, $Q_\alpha f(x,y)=x^\alpha f(x,y)$,

$$D_x^{\frac{1}{2}}f(x,y)=\sqrt{x\frac{\partial(f(x,y))}{\partial x}}\cdot\sqrt{f(x,y)},\qquad D_y^{\frac{1}{2}}f(x,y)=\sqrt{y\frac{\partial(f(x,y))}{\partial y}}\cdot\sqrt{f(x,y)},$$

$$S_x^{\frac{1}{2}} f(x,y) = \sqrt{\int_0^x \frac{f(t,y)}{t} dt} \cdot \sqrt{f(x,y)}, \qquad S_y^{\frac{1}{2}} f(x,y) = \sqrt{\int_0^y \frac{f(x,t)}{t} dt} \cdot \sqrt{f(x,y)}.$$

M-Polynomial of Zigzag-edge Coronoid Graph

Theorem 1 *If Zigzag-edge coronoid is denoted by* $ZC(g,h,q)$ *then for* $g,h,q \geq 3$*, its M-polynomial is*

$$M[ZC(g,h,q);x,y] = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$$

Proof. Let ZC(g,h,q) be a Zigzag-edge coronoid then we have

$$E_{2,2}(ZC(g,h,q)) = \{e = uv \in ZC(g,h,q): d_u = 2, d_v = 2\}$$
$$\Rightarrow |E_{2,2}ZC(g,h,q)| = 6$$
$$E_{2,3}(ZC(g,h,q)) = \{e = uv \in ZC(g,h,q): d_u = 2, d_v = 3\}$$
$$\Rightarrow |E_{2,3}ZC(g,h,q)| = (8(g+h+q)-36)$$
$$E_{3,3}ZC(g,h,q)) = \{e = uv \in ZC(g,h,q): d_u = 3, d_v = 3\}$$
$$\Rightarrow |E_{3,3}ZC(g,h,q)| = 2(g+h+q)$$

By applying the definition of M-polynomial we have

$$M(ZC(g,h,q);x,y) = \sum_{\delta \leq i \leq j \leq \Delta} m_{ij}(ZC(g,h,q))x^i y^j$$
$$M(ZC(g,h,q);x,y) = \sum_{2 \leq i \leq j \leq 3} m_{ij}(ZC(g,h,q))x^i y^j$$
$$M(ZC(g,h,q);x,y) = \sum_{2\leq 2} m_{22}(ZC(g,h,q))x^2y^2 + \sum_{2 \leq 3} m_{23}(ZC(g,h,q))x^2y^3 + \sum_{3 \leq 3} m_{33}(ZC(g,h,q))x^3y^3$$
$$M(ZC(g,h,q);x,y) = |E_{2,2}|x^2y^2 + |E_{2,3}|x^2y^3 + |E_{3,3}|x^3y^3$$
$$M(ZC(g,h,q);x,y) = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$$

Topological Invariants of Zigzag-edge coronoid

Theorem 2 *Let* $ZC(x,y,z)$ *be a Zigzag-edge coronoid*

$$M[ZC(g,h,q);x,y] = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$$

- $ABC[ZC(g,h,q)] = (\frac{4\sqrt{2}+24}{3\sqrt{2}})(g+h+q) - \frac{30}{\sqrt{2}}$
- $GA[ZC(g,h,q)] = (\frac{16\sqrt{6}}{5} + 2)(g+h+q) + (6 - \frac{72\sqrt{6}}{5})$
- $B_1[ZC(g,h,q)] = 116(g+h+q) - 348$
- $B_2[ZC(g,h,q)] = 168(g+h+q) - 492$
- $HB_1[ZC(g,h,q)] = 684(g+h+q) - 2004$
- $HB_2[ZC(g,h,q)] = 1512(g+h+q) - 4020$
- ${}^mB_1[ZC(g,h,q)] = \frac{368}{105}(g+h+q) - \frac{51}{5}$
- ${}^mB_2[ZC(g,h,q)] = \frac{23}{9}(g+h+q) - 7$

• $H_b[ZC(g,h,q)] = \frac{736}{105}(g+h+q) - \frac{102}{5}$

Proof. Let $M[ZC(g,h,q);x,y] = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$

• **The atom-bond connectivity index**

$$f(x,y) = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$$
$$S_y^{\frac{1}{2}}f(x,y) = \frac{6}{\sqrt{2}}x^2y^2 + \frac{1}{\sqrt{3}}(8(g+h+q)-36)x^2y^3 + \frac{2}{\sqrt{3}}(g+h+q)x^3y^3$$
$$S_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y) = 3x^2y^2 + \frac{1}{\sqrt{6}}(8(g+h+q)-36)x^2y^3 + \frac{2}{3}(g+h+q)x^3y^3$$
$$JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y) = 3x^4 + \frac{1}{\sqrt{6}}(8(g+h+q)-36)x^5 + \frac{2}{3}(g+h+q)x^6$$
$$Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y) = 3x^2 + \frac{1}{\sqrt{6}}(8(g+h+q)-36)x^3 + \frac{2}{3}(g+h+q)x^4$$
$$D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}f(x,y) = 3\sqrt{2}x^2 + \frac{1}{\sqrt{2}}(8(g+h+q)-36)x^3 + \frac{4}{3}(g+h+q)x^4$$
$$ABC[ZC(g,h,q)] = D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}[f(x,y)]_{x=1}$$
$$= (\frac{4\sqrt{2}+24)}{3\sqrt{2}})(g+h+q) - \frac{30}{\sqrt{2}}.$$

• **The geometric arithmetic index**

$$f(x,y) = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$$
$$D_y^{\frac{1}{2}}f(x,y) = 6\sqrt{2}x^2y^2 + \sqrt{3}(8(g+h+q)-36)x^2y^3 + 2\sqrt{3}(g+h+q)x^3y^3$$
$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y) = 12x^2y^2 + \sqrt{6}(8(g+h+q)-36)x^2y^3 + 6(g+h+q)x^3y^3$$
$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y) = 12x^4 + \sqrt{6}(8(g+h+q)-36)x^5 + 6(g+h+q)x^6$$
$$S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y) = 3x^4 + \frac{\sqrt{6}}{5}(8(g+h+q)-36)x^5 + (g+h+q)x^6$$
$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}f(x,y) = 6x^4 + \frac{2\sqrt{6}}{5}(8(g+h+q)-36)x^5 + 2(g+h+q)x^6$$
$$GA[ZC(g,h,q)] = 2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[f(x,y)]_{x=1}$$
$$= (\frac{16\sqrt{6}}{5} + 2)(g+h+q) + (6 - \frac{72\sqrt{6}}{5}).$$

• **The first K Banhatti index**

$$f(x,y) = 6x^2y^2 + (8(g+h+q)-36)x^2y^3 + 2(g+h+q)x^3y^3$$
$$D_xf(x,y) = 12x^2y^2 + 2(8(g+h+q)-36)x^2y^3 + 6(g+h+q)x^3y^3$$
$$D_yf(x,y) = 12x^2y^2 + 3(8(g+h+q)-36)x^2y^3 + 6(g+h+q)x^3y^3$$
$$(D_x+D_y)f(x,y) = 24x^2y^2 + 5(8(g+h+q)-36)x^2y^3 + 12(g+h+q)x^3y^3$$

$(D_x + D_y)f(x,y)_{x=y=1} = 52(g+h+q) - 156$

$Jf(x,y) = 6x^4 + (8(g+h+q) - 36)x^5 + 2(g+h+q)x^6$

$Q_{-2}Jf(x,y) = 6x^2 + (8(g+h+q) - 36)x^3 + 2(g+h+q)x^4$

$D_xQ_{-2}Jf(x,y) = 12x^2 + 3(8(g+h+q) - 36)x^3 + 8(g+h+q)x^4$

$2D_xQ_{-2}Jf(x,y) = 24x^2 + 6(8(g+h+q) - 36)x^3 + 16(g+h+q)x^4$

$2D_xQ_{-2}Jf(x,y)_{x=1} = 64(g+h+q) - 192$

$B_1[ZC(g,h,q)] = (D_x + D_y + 2D_xQ_{-2}J)[f(x,y)]_{x=y=1}$

$= 116(g+h+q) - 348.$

• **The second K Banhatti index**

$f(x,y) = 6x^2y^2 + (8(g+h+q) - 36)x^2y^3 + 2(g+h+q)x^3y^3$

$D_xf(x,y) = 12x^2y^2 + 2(8(g+h+q) - 36)x^2y^3 + 6(g+h+q)x^3y^3$

$D_yf(x,y) = 12x^2y^2 + 3(8(g+h+q) - 36)x^2y^3 + 6(g+h+q)x^3y^3$

$(D_x + D_y)f(x,y) = 24x^2y^2 + 5(8(g+h+q) - 36)x^2y^3 + 12(g+h+q)x^3y^3$

$J(D_x + D_y)f(x,y) = 24x^4 + 5(8(g+h+q) - 36)x^5 + 12(g+h+q)x^6$

$Q_{-2}J(D_x + D_y)f(x,y) = 24x^2 + 5(8(g+h+q) - 36)x^3 + 12(g+h+q)x^4$

$D_xQ_{-2}J(D_x + D_y)f(x,y) = 48x^2 + 15(8(g+h+q) - 36)x^3 + 48(g+h+q)x^4$

$B_2[ZC(g,h,q)] = D_xQ_{-2}J(D_x + D_y)[f(x,y)]_{x=1}$

$= 168(g+h+q) - 492.$

• **The first K hyper Banhatti index**

$f(x,y) = 6x^2y^2 + (8(g+h+q) - 36)x^2y^3 + 2(g+h+q)x^3y^3$

$D_x^2f(x,y) = 24x^2y^2 + 4(8(g+h+q) - 36)x^2y^3 + 18(g+h+q)x^3y^3$

$D_y^2f(x,y) = 24x^2y^2 + 9(8(g+h+q) - 36)x^2y^3 + 18(g+h+q)x^3y^3$

$(D_x^2 + D_y^2)f(x,y) = 48x^2y^2 + 13(8(g+h+q) - 36)x^2y^3 + 36(g+h+q)x^3y^3$

$(D_x^2 + D_y^2)f(x,y)_{x=y=1} = 140(g+h+q) - 420$

$Jf(x,y) = 6x^4 + (8(g+h+q) - 36)x^5 + 2(g+h+q)x^6$

$Q_{-2}Jf(x,y) = 6x^2 + (8(g+h+q) - 36)x^3 + 2(g+h+q)x^4$

$D_x^2Q_{-2}Jf(x,y) = 24x^2 + 9(8(g+h+q) - 36)x^3 + 32(g+h+q)x^4$

$2D_x^2Q_{-2}Jf(x,y) = 48x^2 + 18(8(g+h+q) - 36)x^3 + 64(g+h+q)x^4$

$2D_x^2Q_{-2}J[f(x,y)]_{x=1} = 208(g+h+q) - 600$

$D_xf(x,y) = 12x^2y^2 + 2(8(g+h+q) - 36)x^2y^3 + 6(g+h+q)x^3y^3$

$D_yf(x,y) = 12x^2y^2 + 3(8(g+h+q) - 36)x^2y^3 + 6(g+h+q)x^3y^3$

$(D_x + D_y)f(x,y) = 24x^2y^2 + 5(8(g+h+q) - 36)x^2y^3 + 12(g+h+q)^3y^3$

$J(D_x + D_y)f(x,y) = 24x^4 + 5(8(g+h+q) - 36)x^5 + 12(g+h+q)x^6$

$Q_{-2}J(D_x + D_y)f(x,y) = 24x^2 + 5(8(g+h+q) - 36)x^3 + 12(g+h+q)x^4$

$D_xQ_{-2}J(D_x + D_y)f(x,y) = 48x^2 + 15(8(g+h+q) - 36)x^3 + 48(g+h+q)x^4$

$2D_xQ_{-2}J(D_x + D_y)f(x,y) = 96x^2 + 30(8(g+h+q) - 36)x^3 + 96(g+h+q)x^4$

$2D_xQ_{-2}J(D_x + D_y)[f(x,y)]_{x=1} = 336(g+h+q) - 984$

$HB_1[ZC(g,h,q)] = (D_x^2 + D_y^2 + 2D_x^2Q_{-2}J + 2D_xQ_{-2}J(D_x + D_y))[f(x,y)]_{x=y=1}$

$= 684(g+h+q) - 2004.$

• **The second K hyper Banhatti index**

$f(x,y) = 6x^2y^2 + (8(g+h+q) - 36)x^2y^3 + 2(g+h+q)x^3y^3$

$D_x^2f(x,y) = 24x^2y^2 + 4(8(g+h+q) - 36)x^2y^3 + 18(g+h+q)x^3y^3$

$D_y^2f(x,y) = 24x^2y^2 + 9(8(g+h+q) - 36)x^2y^3 + 18(g+h+q)x^3y^3$

$(D_x^2 + D_y^2)f(x,y) = 48x^2y^2 + 13(8(g+h+q) - 36)x^2y^3 + 36(g+h+q)x^3y^3$

$J(D_x^2 + D_y^2)f(x,y) = 48x^4 + 13(8(g+h+q) - 36)x^5 + 36(g+h+q)x^6$

$Q_{-2}J(D_x^2 + D_y^2)f(x,y) = 48x^2 + 13(8(g+h+q) - 36)x^3 + 36(g+h+q)x^4$

$D_x^2Q_{-2}J(D_x^2 + D_y^2)f(x,y) = 192x^2 + 117(8(g+h+q) - 36)x^3 + 576(g+h+q)x^4$

$HB_2[ZC(g,h,q)] = D_x^2Q_{-2}J(D_x^2 + D_y^2)[f(x,y)]_{x=1}$

$= 1512(g+h+q) - 4020.$

• **Modified first K Banhatti index**

$f(x,y) = 6x^2y^2 + (8(g+h+q) - 36)x^2y^3 + 2(g+h+q)x^3y^3$

$L_xf(x,y) = 6x^4y^2 + (8(g+h+q) - 36)x^4y^3 + 2(g+h+q)x^6y^3$

$L_yf(x,y) = 6x^2y^4 + (8(g+h+q) - 36)x^2y^6 + 2(g+h+q)x^3y^6$

$(L_x + L_y)f(x,y) = 6x^4y^2 + (8(g+h+q) - 36)x^4y^3 + 2(g+h+q)x^6y^3$

$+6x^2y^4 + (8(g+h+q) - 36)x^2y^6 + 2(g+h+q)x^3y^6$

$J(L_x + L_y)f(x,y) = 12x^6 + (8(g+h+q) - 36)(x^7 + x^8) + 4(g+h+q)x^9$

$Q_{-2}J(L_x + L_y)f(x,y) = 12x^4 + (8(g+h+q) - 36)(x^5 + x^6) + 4(g+h+q)x^7$

$S_xQ_{-2}J(L_x + L_y)f(x,y) = 3x^4 + (8(g+h+q) - 36)(\frac{1}{5}x^5 + \frac{1}{6}x^6) + \frac{4}{7}(g+h+q)x^7$

${}^{m}B_1[ZC(g,h,q)] = S_xQ_{-2}J(L_x + L_y)[f(x,y)]_{x=1}$

$= \frac{368}{105}(g+h+q) - \frac{51}{5}.$

• **Modified second K Banhatti index**

$f(x,y) = 6x^2y^2 + (8(g+h+q) - 36)x^2y^3 + 2(g+h+q)x^3y^3$

$S_x f(x,y) = 3x^2y^2 + \frac{1}{2}(8(g+h+q) - 36)x^2y^3 + \frac{2}{3}(g+h+q)x^3y^3$

$S_y f(x,y) = 3x^2y^2 + \frac{1}{3}(8(g+h+q) - 36)x^2y^3 + \frac{2}{3}(g+h+q)x^3y^3$

$(S_x + S_y)f(x,y) = 6x^2y^2 + \frac{5}{6}(8(g+h+q) - 36)x^2y^3 + \frac{4}{3}(g+h+q)x^3y^3$

$J(S_x + S_y)f(x,y) = 6x^4 + \frac{5}{6}(8(g+h+q) - 36)x^5 + \frac{4}{3}(g+h+q)x^6$

$Q_{-2}J(S_x + S_y)f(x,y) = 6x^2 + \frac{5}{6}(8(g+h+q) - 36)x^3 + \frac{4}{3}(g+h+q)x^4$

$S_x Q_{-2}J(S_x + S_y)f(x,y) = 3x^2 + \frac{5}{18}(8(g+h+q) - 36)x^3 + \frac{1}{3}(g+h+q)x^4$

${}^{m}B_2[ZC(g,h,q)] = S_x Q_{-2}J(S_x + S_y)[f(x,y)]_{x=1}$

$= \frac{23}{9}(g+h+q) - 7.$

• **Harmonic K Banhatti index**

$f(x,y) = 6x^2y^2 + (8(g+h+q) - 36)x^2y^3 + 2(g+h+q)x^3y^3$

$L_x f(x,y) = 6x^4y^2 + (8(g+h+q) - 36)x^4y^3 + 2(g+h+q)x^6y^3$

$L_y f(x,y) = 6x^2y^4 + (8(g+h+q) - 36)x^2y^6 + 2(g+h+q)x^3y^6$

$(L_x + L_y)f(x,y) = (6x^4y^2 + (8(g+h+q) - 36)x^4y^3 + 2(g+h+q)x^6y^3)$

$+(6x^2y^4 + (8(g+h+q) - 36)x^2y^6 + 2(g+h+q)x^3y^6)$

$J(L_x + L_y)f(x,y) = 12x^6 + (8(g+h+q) - 36)(x^7 + x^8) + 4(g+h+q)x^9$

$Q_{-2}J(L_x + L_y)f(x,y) = 12x^4 + (8(g+h+q) - 36)(x^5 + x^6) + 4(g+h+q)x^7$

$S_x Q_{-2}J(L_x + L_y)f(x,y) = 3x^4 + (8(g+h+q) - 36)(\frac{1}{5}x^5 + \frac{1}{6}x^6) + \frac{4}{7}(g+h+q)x^7$

$2S_x Q_{-2}J(L_x + L_y)f(x,y) = 6x^4 + (8(g+h+q) - 36)(\frac{2}{5}x^5 + \frac{1}{3}x^6) + \frac{8}{7}(g+h+q)x^7$

$H_b[ZC(g,h,q)] = 2S_x Q_{-2}J(L_x + L_y)[f(x,y)]_{x=1}$

$= \frac{736}{105}(g+h+q) - \frac{102}{5}$

Conclusion

In the existing paper, we have closed form of M-polynomial for the graph Zigzag-edge Coronoid and then we computed many degree-based topological invariants, which supports to shrink the number of experiments. These topological invariants can give insight information for biological, chemical and physical features of $ZC(g,h,q)$.

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