Solving Thermal System Using New Fuzzy Transform

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Abstract
Integral transforms are successful tool for solving mathematical problems in variant fields such as physics, medicine, chemistry and engineering applications. In this research, a new integral transform is introduced which is called “Ro-transform”, basic definitions and theorems have been studied. Moreover, the technique of fuzzy logic introduces for this new integral transform to obtain fuzzy Ro-transform with its theorems, properties and illustrated examples such as a physical problem about heating system. Finally, two integral transforms formulas are derived due to the expansion the kernel function in Ro-transform.

Keywords: Ro-transform, fuzzy Ro- transform, heat system equation.

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1. Introduction

Integral transform is an important method to solve mathematical problems. Appropriate choice of integral transform helps to convert differential equations and integral equations into terms of algebraic equations that can be solve easily [7,6,17]. This kind of integral transform was introduced firstly by Laplace [19], and the Fourier transform was introduced in 1822 by Fourier [19]. In fact, These two integral transforms represented a crucial applications in many fields.
During last two decades, many integral transforms in the class of Laplace transform are introduced such as Sumudu, Elzaki, Hy, Wavelet, sawi and Kamal transforms [2,4,8,11,12,18-25].
In this work, a new integral transform has been introduced, called “Ro-transform” which used to solve linear differential equations, partial differential equations as well. Moreover, brief details about this integral transform has been provided such as, its general definition, the duality property between Laplace transform and Ro-transform besides the related theorems and properties.
Since the novel of fuzzy transforms, mathematically well founded way with many applications, so there is a crucial role for the fuzzy differential equations in various aspects including mathematics, statistics and engineering. Furthermore, one of the most popular theories for modelling many real-world problems, is the fuzzy set theory [21]. Thus, recently, some researchers convert some integral transforms to fuzzy transforms to solve fuzzy differential equations like Allahviranloo and Ahmed [1] as well as fuzzy Sumudu transform and fuzzy Laplace and their application [3,5,10,16], fuzzy Tarig transform [13], etc.
Based on the briefly recapitulated above, we propose fuzzy Ro-transform. general definition, related theorems and introduce fuzzy derivative formulas about the first order and second order derivatives.

An extension for the kernel function in this new integral transform is discussed as well. Thermal system equation is established to examine the results of the proposed technique.

2. Definitions and Theorems

Definition1: Let $f(\varpi)$ be an integrable function defined for $\varpi \geq 0$, let $\Psi(\upsilon) \neq 0$ be positive function and $\Upsilon(\upsilon) \neq 0$ be positive complex function such that $\Psi(\upsilon) = \upsilon^2$ and $\Upsilon(\upsilon) = (i\sqrt[\Omega]{\upsilon})$, $\Omega = 2n+1$, an integral Ro-transform $\Re(\upsilon)$ of $f(\varpi)$ can be defined by the following formula:

$$\Re(\upsilon) = R\{f(\varpi),\upsilon\} = \upsilon^2 \int_0^\infty f(\varpi) e^{-i\sqrt[\Omega]{\upsilon}\varpi} d\varpi, \quad \text{n} \geq 1 \quad . (1)$$

Definition2. (The inverse of Ro-Transform): The Ro-transform inverse of $\Re(\upsilon)$, denoted by $R^{-1}\left[\Re(\upsilon)\right]$ is the piecewise continuous function $f(\varpi)$ on $[0,\infty)$ which satisfies: $R\left|f(\varpi)\right| = \Re(\upsilon)$.

3.Ro-Transform for Some Elementary Functions:

Assume that for any function $f(\varpi)$, the integral in equation (1) exists. So Ro-transform will be:

Function $f(\varpi) = R^{-1}\{\Re(\upsilon)\}$	$\Re(\upsilon) = R\{f(\varpi),\upsilon\}$
$f(\varpi) = 1$	$R\{f(\varpi),\upsilon\} = \dfrac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}}$, $\Omega = 2n+1$
$f(\varpi) = \varpi$	$R\{f(\varpi),\upsilon\} = \dfrac{\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)^2}$, $\Omega = 2n+1$
$f(\varpi) = \varpi^n$	$R\{f(\varpi),\upsilon\} = \dfrac{n!\,\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)^{n+1}}$,n ≥ 1.
$f(\varpi) = \sin a\varpi$	$R\{f(\varpi),\upsilon\} = \dfrac{a\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)^2 + a^2}$
$f(\varpi) = \cos a\varpi$	$R\{f(\varpi),\upsilon\} = \dfrac{\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)}{\left(i\sqrt[\Omega]{\upsilon}\right)^2 + a^2}$
$f(\varpi) = \sinh a\varpi$	$R\{f(\varpi),\upsilon\} = \dfrac{\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)^2 - 1}$, $i\sqrt[\Omega]{\upsilon} > 1$
$f(\varpi) = \cosh a\varpi$	$R\{f(\varpi),\upsilon\} = \dfrac{\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)}{\left(i\sqrt[\Omega]{\upsilon}\right)^2 - 1}$, $i\sqrt[\Omega]{\upsilon} > 1$

$f(\varpi) = e^{\varpi}$ $\qquad R\{f(\varpi), \upsilon\} = \dfrac{\upsilon^2}{i\sqrt[\Omega]{\upsilon} - 1}, i\sqrt[\Omega]{\upsilon} > 1$

Following are proofs for some Ro-transform of some functions:

Proof:

If $f(\varpi) = \varpi$, then

$R(\varpi) = \upsilon^2 \int_0^\infty \varpi e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi$, integrating by parts to get:

$$R\left(\varpi\right) = \upsilon^2 \left[\frac{1}{i\sqrt[\Omega]{\upsilon}} \varpi e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} \Big|_0^\infty + \frac{1}{i\sqrt[\Omega]{\upsilon}} \int_0^\infty \varpi e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi \right].$$

$$= \frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}}.$$

1. If $f(\varpi) = \sinh\varpi$, then

$R(\sinh\varpi) = \upsilon^2 \int_0^\infty \sinh\varpi e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi$, since $\sinh\varpi = \dfrac{e^{\varpi} - e^{-\varpi}}{2}$ Then:

$$= \frac{\upsilon^2}{2} \left[\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi - \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi \right] = \frac{\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)^2 - 1}, i\sqrt[\Omega]{\upsilon} > 1.$$

3. If $f(\varpi) = e^{\varpi}$, then

$$R(e^{\varpi}) = \upsilon^2 \int_0^\infty e^{\varpi} e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi \Rightarrow \upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi \Rightarrow \frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon} - 1}, \quad i\sqrt[\Omega]{\upsilon} > 1.$$

Theorem1:

Let $f\left(\varpi\right)$ is a differentiable function with respect to ϖ, υ^2 is a real positive function and $i\sqrt[\Omega]{\upsilon}$ is complex positive function, then:

1. $R\{f'(\varpi), \upsilon\} = \left(i\sqrt[\Omega]{\upsilon}\right) R[f\left(\varpi\right)] - \upsilon^2 f(0)$,
2. $R\{f''(\varpi), \upsilon\} = \left(i\sqrt[\Omega]{\upsilon}\right)^2 R[f\left(\varpi\right)] - \upsilon^2 \left(i\sqrt[\Omega]{\upsilon}\right) f(0) - \upsilon^2 f'(0)$,
3. $R\{f^{(n)}(\varpi), \upsilon\} = \left(i\sqrt[\Omega]{\upsilon}\right)^n R[f\left(\varpi\right)] - \upsilon^2 \sum_{k=0}^{n-1} \left(i\sqrt[\Omega]{\upsilon}\right)^{n-1-k} f^{(k)}(0)$.

Proof:

1. Since $R\{f'(\varpi), \upsilon\} = \upsilon^2 \int_0^\infty f'\left(\varpi\right) e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi$, integrating by parts:

$$\upsilon^2 \int_0^\infty f'\left(\varpi\right) e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} d\varpi = \left(i\sqrt[\Omega]{\upsilon}\right) R[f(\varpi) - \upsilon\left(i\sqrt[\Omega]{\upsilon}\right) f\left(0\right),$$ thus

$$R\{f'(\varpi), \upsilon\} = \left(i\sqrt[\Omega]{\upsilon}\right) R[f\left(\varpi\right)] - \upsilon^2 f(0)$$

2. $R\{f''(\varpi),\upsilon\}=\upsilon^2\int_0^\infty f''(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi$, integrating by parts twice:

$\left(i\sqrt[\Omega]{\upsilon}\right)^2 R[f(\varpi)]-\upsilon\left(i\sqrt[\Omega]{\upsilon}\right)f(0)-\upsilon^2 f'(0)$.

3. For this n^th derivative can be prove using mathematical induction.

Theorem2: Let $f_1(\varpi)$ and $f_2(\varpi)$ are integrable functions with Ro-transforms $\Re_1(\upsilon)$ and $\Re_2(\upsilon)$ respectively, i.e. $R\{f_1(\varpi)\}=\Re_1(\upsilon)$ and $R\{f_2(\varpi)\}=\Re_2(\upsilon)$.Then it can be shown that $R\{(f_1*f_2)(\varpi)\}=\frac{1}{\upsilon^2}\Re_1(\upsilon)\Re_2(\upsilon)$.

Proof:

$$R(f_1*f_2)=\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}(f_1*f_2)d\varpi \Rightarrow \upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}\left[\int_0^\infty f_1(\varpi)f_2(\varpi-\tau)d\tau\right]d\varpi$$

$$\Rightarrow \upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}f_1(\varpi)d\tau\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}f_2(\varpi-\tau)d\varpi \Rightarrow \frac{1}{\upsilon^2}\Re_1(\upsilon)\Re_2(\upsilon).$$

Following theorem shows the relation between Ro-transform and Laplace transform.

Theorem3: **(Duality between Laplace Transform and Ro-Transform)** Let $f(\varpi)$ an integrable function, if $\Re(\upsilon)$is Ro-transform and F is Laplace transform of $f(\varpi)$ then:

$\Re(\upsilon)=\upsilon^2 F\left(i\sqrt[\Omega]{\upsilon}\right)$.

Proof: From Definition 1:

$$\Re(\upsilon)=R[f(\varpi);\upsilon]=\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}f(\varpi)d\varpi$$

Since Laplace transform is denoted by:

$F(p)=\ell[f(t);p]=\int_0^\infty e^{-pt}f(t)dt$. Then the duality relation will be:

$\Re(\upsilon)=\upsilon^2 F\left(\sqrt[\Omega]{\upsilon}i\right)$.

Theorem4. (Linearity property) Let $f_1(\varpi), f_2(\varpi),..., f_n(\varpi)$ are integrable function have $R_1\left[f_1(\varpi)\right], R_2\left[f_2(\varpi)\right],..., R_n\left[f_n(\varpi)\right]$ and $b_1, b_2,..., b_n$ are constants, then linearity property is defined by:

$$R[b_1 f_1(\varpi)+b_2 f_2(\varpi)+...+b_n f_n(\varpi)]=b_1 R\left[f_1(\varpi)\right]+b_2 R\left[f_2(\varpi)\right]+..+b_n R\left[f_n(\varpi)\right]$$

Proof:

$$R[b_1 f_1(\varpi)+b_2 f_2(\varpi)+\ldots+b_n f_n(\varpi)]$$

$$=\upsilon^2\int_0^\infty [b_1 f_1(\varpi)+b_2 f_2(\varpi)+\ldots+b_n f_n(\varpi)]e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi$$

$$=\upsilon^2\int_0^\infty b_1 f_1(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi+\upsilon^2\int_0^\infty b_2 f_2(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi+\ldots+\upsilon^2\int_0^\infty b_n f_n(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi$$

$$=b_1\upsilon^2\int_0^\infty f_1(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi+b_2\upsilon^2\int_0^\infty f_2(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi+\ldots+b_n\upsilon^2\int_0^\infty f_n(\varpi)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}d\varpi$$

$$=b_1R\left[f_1(\varpi)\right]+b_2R\left[f_2(\varpi)\right]+..+b_nR\left[f_n(\varpi)\right].$$

3. Illustrative Examples: The following examples explain how Ro-transform solves initial value problems that described by ordinary differential equations.

Example 1: Consider the following ordinary differential equation:

$\wp'+2\wp=0\ ;\ \wp(0)=1$

Applying Ro-transform for both sides of the original equation, then $R[\wp']+2R[\wp]=0\ \Rightarrow\left(i\sqrt[\Omega]{\upsilon}\right)R[\wp]-\upsilon^2\wp(0)+2R[\wp]=0$

Using initial condition, to obtain:

$$R[\wp]=\frac{\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)+2}\ .$$

Applying the inverse of Ro-transform for the last equation: $\wp=e^{-2\varpi}$

Example 2: Consider the ordinary differential equation:

$\wp''+\wp=0\ ;\ \wp(0)=0,\wp'(0)=1$

Applying Ro-transform for both sides of the original equation, to get:

$R[\wp'']+R[\wp]=0\ \Rightarrow\ \left(i\sqrt[\Omega]{\upsilon}\right)^2R[\wp]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)\wp(0)-\upsilon^2\wp'(0)+R[\wp]=0$

Using initial conditions, to obtain:

$$R[\wp]=\frac{\upsilon^2}{\left(i\sqrt[\Omega]{\upsilon}\right)^2+1}\ .$$

Applying the inverse of Ro-transform for the last equation, to get: $\wp=\sin\varpi$.

4. Fuzzy Ro-Transform.

Fuzzy transform is method which belongs to fuzzy approximation models. It’s fundamentals are studied from different points of view.

This section contains, fundamental definitions and basic concepts related to the present topic of this paper are given for completeness purpose about fuzzy Ro-transform.

Definition 3 [14]. A fuzzy membership ς in parametric form is a pair $\left(\underline{\varsigma},\bar{\varsigma}\right)$ of functions $\underline{\varsigma}\left(\vartheta\right),\bar{\varsigma}\left(\vartheta\right)$, $\vartheta\in[0,1]$, which fulfill the following requirements:

1. $\underline{\varsigma}\left(\vartheta\right)$ is a bounded increasing left continuous function in $(0,1]$,and right continuous at 0 .
2. $\bar{\varsigma}\left(\vartheta\right)$ is a bounded decreasing left continuous function in $(0,1]$,and right continuous at 0 .
3. $\underline{\varsigma}\left(\vartheta\right)\leq\bar{\varsigma}\left(\vartheta\right)$, $\vartheta\in[0,1]$.

For arbitrary $\varsigma=\left[\underline{\varsigma}\left(\vartheta\right),\bar{\varsigma}\left(\vartheta\right)\right]$ and $\nu=\underline{\nu}\left(\vartheta\right),\bar{\nu}\left(\vartheta\right)$, $\varphi>0$ we define the addition $\varsigma\oplus\nu$, subtraction $\varsigma\ominus\nu$ and scalar multiplication by $\varphi>0$ as in

(a) Addition: $\varsigma\oplus\nu=\underline{\varsigma}\left(\vartheta\right)+\underline{\nu}\left(\vartheta\right),\bar{\varsigma}\left(\vartheta\right)+\bar{\nu}\left(\vartheta\right)$.

(b) Subtraction: $\varsigma\ominus\nu=\underline{\varsigma}\left(\vartheta\right)-\bar{\nu}\left(\vartheta\right),\bar{\varsigma}\left(\vartheta\right)-\underline{\nu}\left(\vartheta\right)$.

(c) Scalar multiplication: $\psi\odot\varsigma=\begin{Bmatrix}\left(\psi\underline{\varsigma},\psi\bar{\varsigma}\right) & \psi\geq 0\\ \left(\psi\underline{\varsigma},\psi\underline{\varsigma}\right) & \psi<0\end{Bmatrix}$.

Definition4. [15]. Presume that $\sigma,\nu\in E$ (E: the set of all fuzzy numbers). If there exists $\kappa\in\mathrm{E}$ such that $\sigma+\nu=\kappa$ then κ is named the Hukuhara difference of σ and ν and is identify by $\sigma\ominus\nu$. $\ominus$ always stands for Hukuhara difference.

Definition 5. [9]. Let $f\left(\varpi\right):\left(a,b\right)\rightarrow\mathrm{E}$ continuous fuzzy-valued function for $\varpi_0\in\left(a,b\right)$, f is strongly generalized differential at ϖ_0. If there exists an element $f'\left(\varpi_0\right)\in\mathrm{E}$ such that:

1. For all $\hbar>0$ sufficiently small $\exists f\left(\varpi_0+\hbar\right)\ominus f\left(\varpi_0\right),\exists f\left(\varpi_0\right)\ominus f\left(\varpi_0-\hbar\right)$ and the limit is

$$f'\left(\varpi_0\right)=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0+\mathrm{h}\right)\ominus f\left(\varpi_0\right)}{\mathrm{h}}=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0\right)\ominus f\left(\varpi_0-\mathrm{h}\right)}{\mathrm{h}}\quad .$$

Or

2. For all $\hbar>0$ sufficiently small $\exists f\left(\varpi_0\right)\ominus f\left(\varpi_0+\hbar\right),\exists f\left(\varpi_0-\hbar\right)\ominus f\left(\varpi_0\right)$ and the limit is

$$f'\left(\varpi_0\right)=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0\right)\ominus f\left(\varpi_0+\mathrm{h}\right)}{-\mathrm{h}}=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0-\mathrm{h}\right)\ominus f\left(\varpi_0\right)}{-\mathrm{h}}.$$

Or

3. For all $\hbar>0$ sufficiently small $\exists f\left(\varpi_0+\hbar\right)\ominus f\left(\varpi_0\right),\exists f\left(\varpi_0-\hbar\right)\ominus f\left(\varpi_0\right)$ and the limit is

$$f'\left(\varpi_0\right)=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0+\mathrm{h}\right)\ominus f\left(\varpi_0\right)}{\mathrm{h}}=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0-\mathrm{h}\right)\ominus f\left(\varpi_0\right)}{-\mathrm{h}}\quad .$$

Or

4. For all $\hbar>0$ sufficiently small $\exists f\left(\varpi_0\right)\ominus f\left(\varpi_0+\hbar\right),\exists f\left(\varpi_0-\hbar\right)\ominus f\left(\varpi_0\right)$ and the limit is

$$f'\left(\varpi_0\right)=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0\right)\ominus f\left(\varpi_0+\mathrm{h}\right)}{-\mathrm{h}}=\lim_{\mathrm{h}\to 0^+}\frac{f\left(\varpi_0-\mathrm{h}\right)\ominus f\left(\varpi_0\right)}{\mathrm{h}}$$

Theorem 5. [9] Assume that $f: R \to \mathrm{E}$ be a function and indicate $f(\varpi) = \left(\underline{f}(\varpi;\vartheta), \overline{f}(\varpi;\vartheta)\right)$ for each $\vartheta \in [0,1]$ Then:

1- If f is the first form, then $\underline{f}(\varpi;\vartheta)$ and $\overline{f}(\varpi;\vartheta)$ are differentiable functions and $f'(\varpi) = \underline{f}(\varpi;\vartheta), \overline{f}(\varpi;\vartheta)$.

2- If f is the second form, then $\underline{f}(\varpi;\vartheta)$ and $\overline{f}(\varpi;\vartheta)$ are differentiable functions and $f'(\varpi) = \overline{f}(\varpi;\vartheta), \underline{f}(\varpi;\vartheta)$.

Theorem 6. [20] Let $f: R \to \mathrm{E}$ and it is represented by $\left[\underline{f}(\varpi;\vartheta), \overline{f}(\varpi;\vartheta)\right]$. For any fixed $\vartheta \in (0,1]$ assume that $\underline{f}(\varpi;\vartheta)$ and $\overline{f}(\varpi;\vartheta)$ are Riemann-integrable functions on [a, b] for every $b \ge a$, and assume there are two positive $\underline{M}_\vartheta$ and $\overline{M}_\vartheta$ such that $\int_a^b \left|\underline{f}(\varpi;\vartheta)\right| d\varpi \le \underline{M}_\vartheta$ and $\int_a^b \left|\overline{f}(\varpi;\vartheta)\right| d\varpi \le \overline{M_\vartheta}$ for every $b \ge a$.Then, $f(\varpi)$ is improper fuzzy Riemann-integrable on $[a,\infty)$. Furthermore, we have: $\int_a^\infty f(\varpi)\, d\varpi = \left[\int_a^\infty \underline{f}(\varpi;\vartheta)\, d\varpi, \int_a^\infty \overline{f}(\varpi;\vartheta)\, d\varpi\right]$.

Definition6. Let $f(\varpi)$ be continuous fuzzy-valued function. Suppose that $\upsilon^2 e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}$ e $f(\varpi)d\varpi$ is an improper fuzzy Riemann-integrable on $[0,\infty)$, then $\upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} f(\varpi)d\varpi$, is called fuzzy Ro-transform and it denoted as:

$$\tilde{\Re}(\upsilon) = R\left[f(\varpi)\right] = \upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} f(\varpi)d\varpi, \qquad \mathrm{n} \ge 1$$

Since From Theorem 6:

$$\upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} f(\varpi)d\varpi = \left(\upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} \underline{f}(\varpi;\vartheta)d\varpi, \upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} \overline{f}(\varpi;\vartheta)d\varpi\right)$$

Using the definition of classical Ro-transform:

$$\gamma\left[\underline{f}(\varpi;\vartheta)\right] = \upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} \underline{f}(\varpi;\vartheta)d\varpi \text{ and } \gamma\left[\overline{f}(\varpi;\vartheta)\right] = \upsilon^2 \int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi} \overline{f}(\varpi;\vartheta)d\varpi$$

So:

$$R\left[f(\varpi;\vartheta)\right] = \gamma\left[\underline{f}(\varpi;\vartheta)\right], \gamma\left[\overline{f}(\varpi;\vartheta)\right].$$

Theorem7. Duality Between Fuzzy Laplace –Ro-Transforms.

If $\tilde{\Re}(\upsilon)$ is fuzzy Ro-transform and $F(\varsigma)$ is fuzzy Laplace transform of $f(\varpi)$ then: $\tilde{\Re}(\upsilon) = \upsilon^2 F\left(i\sqrt[\Omega]{\upsilon}\right)$.

Proof: Let $f(\varpi)\in E$, then:

$$\tilde{\Re}(\upsilon)=\left[\upsilon^2\int_0^\infty \underline{f}(\varpi;\vartheta)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}\,d\varpi\ ,\ \upsilon^2\int_0^\infty \overline{f}(\varpi;\vartheta)e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}\,d\varpi\right]$$

Since fuzzy Laplace transform denoted by:

$$F(\varsigma)=\left[\int_0^\infty \underline{f}(t;\vartheta)e^{-\varsigma t}\,dt\ ,\ \int_0^\infty \overline{f}(t;\vartheta)e^{-\varsigma t}\,dt\right]$$. Then:

$$\tilde{\Re}(\upsilon)=\upsilon^2F\left(i\sqrt[\Omega]{\upsilon}\right).$$

Theorem 8. Let $f_1(\varpi), f_2(\varpi),\dots, f_n(\varpi)$ be continuous fuzzy-valued functions and let $\eta_1,\eta_2,\dots,\eta_n$ are arbitrary constants, then:

$$R\left[\left(\eta_1\ \mathrm{e}\ \ f_1(\varpi)\right)\oplus\left(\eta_2\ \mathrm{e}\ \ f_2(\varpi)\right)\oplus\dots\oplus\left(\eta_n\ \mathrm{e}\ \ f_n(\varpi)\right)\right]=\left(\eta_1\ \mathrm{e}\ \ R\left[f_1(\varpi)\right]\right)\oplus\left(\eta_2\ \mathrm{e}\ \ R\left[f_2(\varpi)\right]\right)\oplus$$
$$\dots\oplus\left(\eta_n\ \mathrm{e}\ \ R\left[f_n(\varpi)\right]\right).$$

Proof:

$$R\left[\left(\eta_1\ \mathrm{e}\ \ f_1(\varpi)\right)\oplus\left(\eta_2\ \mathrm{e}\ \ f_2(\varpi)\right)\oplus\dots\oplus\left(\eta_n\ \mathrm{e}\ \ f_n(\varpi)\right)\right]$$

$$=\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}\left[\left(\eta_1\ \mathrm{e}\ \ f_1(\varpi)\right)\oplus\left(\eta_2\ \mathrm{e}\ \ f_2(\varpi)\right)\oplus\dots\oplus\left(\eta_n\ \mathrm{e}\ \ f_n(\varpi)\right)\right]d\varpi$$

$$=\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}\left[\left(\eta_1\underline{f_1}(\varpi),\left(\eta_1\overline{f}_1(\varpi)\right)\right)+\left(\eta_2\underline{f_2}(\varpi),\left(\eta_2\overline{f}_2(\varpi)\right)\right)+\dots+\left(\eta_n\underline{f_n}(\varpi),\left(\eta_n\overline{f}_n(\varpi)\right)\right)\right]d\varpi$$

$$=\eta_1\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}f_1(\varpi)d\varpi\oplus\eta_2\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}f_2(\varpi)d\varpi\oplus\dots\oplus\eta_n\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}f_n(\varpi)d\varpi$$

$$=\left(\eta_1\ \mathrm{e}\ \ R\left[f_1(\varpi)\right]\right)\oplus\left(\eta_2\ \mathrm{e}\ \ R\left[f_2(\varpi)\right]\right)\oplus\dots\oplus\left(\eta_n\ \mathrm{e}\ \ R\left[f_n(\varpi)\right]\right).$$

Theorem 9. Let $f'(\varpi)$ be continuous fuzzy-valued function and $f(\varpi)$ the primitive of $f'(\varpi)$ on $[0,\infty)$, then:

1. $R\left[f'(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)R\left[f(\varpi)\right]\ominus\upsilon^2f(0)$,where f is the **1st** form differentiable.
2. $R\left[f'(\varpi)\right]=-\upsilon^2f(0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R\left[f(\varpi)\right]$, where f is the **2nd** form differentiable.

Proof: Since $f'(\varpi)$ is continuous fuzzy-valued function, then there are two cases as following:

Case 1. If f is the **1st** form differentiable, for any arbitrary $\vartheta\in[0,1]$,

$$R\left[f'(\varpi)\right]=\gamma\left[\underline{f}'(\varpi,\vartheta)\right],\gamma\left[\overline{f}'(\varpi,\vartheta)\right]$$

From Theorem 1/1:

$$\gamma\left[\underline{f}'(\varpi,\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)\gamma\left[\underline{f}(\varpi,\vartheta)\right]-\upsilon^{2}\underline{f}(0,\vartheta),$$

$$\gamma\left[\overline{f}'(\varpi,\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)\gamma\left[\overline{f}(\varpi,\vartheta)\right]-\upsilon^{2}\overline{f}(0,\vartheta)$$

By Theorem 2:

$$R\left[f'(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)R\left[f(\varpi)\right]\ominus\upsilon^{2}f(0).$$

Case 2. If f is the **2nd** form differentiable, for any arbitrary $\vartheta\in[0,1]$,

$$R\left[f(\varpi)\right]=\gamma\left[\overline{f}(\varpi,\vartheta)\right],\gamma\left[\underline{f}(\varpi,\vartheta)\right]$$

By same way can be get:

$$R\left[f'(\varpi)\right]=-\upsilon^{2}f(0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R\left[f(\varpi)\right].$$

Theorem10. Assume that, $f(\varpi)$, $f'(\varpi)$ are continuous fuzzy-valued functions on $[0,\infty)$, fuzzy derivative of fuzzy Ro-transform about second order will be:

1. If f,f' are the **1st** form differentiable then:

$$R\left[f''(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^{2}R\left[f(\varpi)\right]\ominus\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)f(0)\ominus\upsilon^{2}f'(0).$$

2. If f is the **1st** form differentiable and f' is the **2nd** form differentiable then:

$$R\left[f''(\varpi)\right]=-\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)f(0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^{2}R\left[f(\varpi)\right]\ominus\upsilon^{2}f'(0).$$

3. If f is the **2nd** form differentiable and f' is the **1st** form differentiable then:

$$R\left[f''(\varpi)\right]=-\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)f(0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^{2}R\left[f(\varpi)\right]-\upsilon^{2}f'(0).$$

4. If f,f' are the **2nd** form differentiable then:

$$R\left[f''(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^{2}R\left[f(\varpi)\right]\ominus\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)f(0)-\upsilon^{2}f'(0).$$

Proof: Since $f''(\varpi)$ is continuous fuzzy-valued function and we have four cases such as following:

1. f, f' are the **1st** form differentiable and for any arbitrary $\vartheta\in[0,1]$, then:

$$R\left[f''(\varpi)\right]=\gamma\left[\underline{f}''(\varpi,\vartheta)\right],\gamma\left[\overline{f}''(\varpi,\vartheta)\right]$$

Form Theorem ½:

$$\gamma\left[\underline{f}''(\varpi,\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^{2}\gamma\left[\underline{f}(\varpi,\vartheta)\right]-\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)\underline{f}(0,\vartheta)-\upsilon^{2}\underline{f'(0,\vartheta)}$$

$$\gamma\left[\overline{f}''(\varpi,\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^{2}\gamma\left[\overline{f}(\varpi,\vartheta)\right]-\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)\overline{f}(0,\vartheta)-\upsilon^{2}\overline{f'(0,\vartheta)}$$

By Theorem 2:

$$R\left[f''(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^{2}R\left[f(\varpi)\right]\ominus\upsilon^{2}\left(i\sqrt[\Omega]{\upsilon}\right)f(0)\ominus\upsilon^{2}f'(0).$$

2. f is the **1st** form differentiable and f' **2nd** form differentiable and for any arbitrary $\vartheta \in [0,1]$, then:

$$R\left[f''(\varpi)\right] = \gamma\left[\overline{f}''(\varpi,\vartheta)\right], \gamma\left[\underline{f}''(\varpi,\vartheta)\right]$$

Form Theorem 1/2:

$$\gamma\left[\underline{f}''(\varpi,\vartheta)\right] = \left(i\sqrt[\Omega]{\upsilon}\right)^2 \gamma\left[\underline{f}(\varpi,\vartheta)\right] - \upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)\underline{f}(0,\vartheta) - \upsilon^2\,\underline{f'(0,\vartheta)}$$

$$\gamma\left[\overline{f}''(\varpi,\vartheta)\right] = \left(i\sqrt[\Omega]{\upsilon}\right)^2 \gamma\left[\overline{f}(\varpi,\vartheta)\right] - \upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)\overline{f}(0,\vartheta) - \upsilon^2\,\overline{f'(0,\vartheta)}$$

By Theorem 2:

$$R\left[f''(\varpi)\right] = -\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right) f(0) \ominus \left(i\sqrt[\Omega]{\upsilon}\right)^2 R\left[f(\varpi)\right] \ominus \upsilon^2 f'(0).$$

3. f is the **2nd** form differentiable and f' **1st** form differentiable, and for any arbitrary $\vartheta \in [0,1]$, then:

$$R\left[f''(\varpi)\right] = \gamma\left[\overline{f}''(\varpi,\vartheta)\right], \gamma\left[\underline{f}''(\varpi,\vartheta)\right]$$

Then:

$$R\left[f''(\varpi)\right] = -\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right) f(0) \ominus \left(-i\sqrt[\Omega]{\upsilon}\right)^2 R\left[f(\varpi)\right] - \upsilon^2 f'(0).$$

4. f , f' are the **2nd** form differentiable and for any arbitrary $\vartheta \in [0,1]$, then:

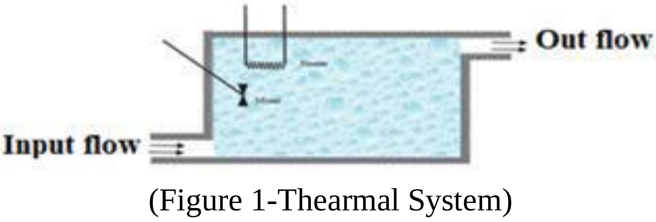
$$R\left[f''(\varpi)\right] = \gamma\left[\underline{f}''(\varpi,\vartheta)\right], \gamma\left[\overline{f}''(\varpi,\vartheta)\right]$$

Then:

$$R\left[f''(\varpi)\right] = \left(i\sqrt[\Omega]{\upsilon}\right)^2 R\left[f(\varpi)\right] \ominus \upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right) f(0) - \upsilon^2 f'(0)$$

Application

Example [13]**:** The following real world example is illustrated the usage of fuzzy Ro-transform to solve the thermal system in the following figure.



(Figure 1-Thearmal System)

The thermal capacitance $\xi = 2$, the temperature ϖ =0.5 thus the model will be:

$$\Upsilon'(\mathrm{h}) = -\frac{1}{\varpi\xi}\Upsilon(\mathrm{h}), \quad \Upsilon(0) = [\underline{\Upsilon}(0;\vartheta), \overline{\Upsilon}(\vartheta;0) \ .$$

There are two cases:

1. If $\Upsilon(\mathrm{h})$ is the 1st form differentiable, then:

Using fuzzy Ro-transform for both sides of original equation.

$$R[\Upsilon'(\mathrm{h})] = -R[\Upsilon(\mathrm{h})]$$

Last equation becomes:

$$\left(i\sqrt[\Omega]{\upsilon}\right)\gamma[\underline{\Upsilon}(\mathrm{h};\vartheta)]-\upsilon^{2}\underline{\Upsilon}(0;\vartheta)=-\gamma[\underline{\Upsilon}(\mathrm{h};\vartheta)],\ \left(i\sqrt[\Omega]{\upsilon}\right)\gamma[\overline{\Upsilon}(\mathrm{h};\vartheta)]-\upsilon^{2}\overline{\Upsilon}(0;\vartheta)=-\gamma[\overline{\Upsilon}(\mathrm{h};\vartheta)]$$

By solve above equations and using the inverse of fuzzy Ro-transform, then:
using the inverse of fuzzy Ro-transform for the last equation:

$$\underline{\Upsilon}(\mathrm{h};\vartheta)=e^{-\varpi}\underline{\Upsilon}(0;\vartheta),\ \overline{\Upsilon}(\mathrm{h};\vartheta)=e^{-\varpi}\overline{\Upsilon}(0;\vartheta)$$

2. If $\Upsilon(\mathrm{h})$ is the 2nd form, then:

Using fuzzy Ro-transform for both sides of original equation.

$$R[\Upsilon'(\mathrm{h})]=-R[\Upsilon(\mathrm{h})]$$

Last equation becomes:

$$\left(i\sqrt[\Omega]{\upsilon}\right)\gamma[\overline{\Upsilon}(\mathrm{h};\vartheta)]-\upsilon\left(i\sqrt[\Omega]{\upsilon}\right)\overline{\Upsilon}(0;\vartheta)=-\gamma[\underline{\Upsilon}(\mathrm{h};\vartheta)],$$

$$\left(i\sqrt[\Omega]{\upsilon}\right)\gamma[\underline{\Upsilon}(\mathrm{h};\vartheta)]-\upsilon\left(i\sqrt[\Omega]{\upsilon}\right)\underline{\Upsilon}(0;\vartheta)=-\gamma[\overline{\Upsilon}(\mathrm{h};\vartheta)]$$

By solve above equations and using the inverse of fuzzy Ro-transform, then:

$$\underline{\Upsilon}(\mathrm{h};\vartheta)=\cosh \mathrm{h}\,\overline{\Upsilon}(0;\vartheta)-\sinh \mathrm{h}\,\underline{\Upsilon}(0;\vartheta),$$

$$\overline{\Upsilon}(\mathrm{h};\vartheta)=\cosh \mathrm{h}\,\underline{\Upsilon}(0;\vartheta)-\sinh \mathrm{h}\,\overline{\Upsilon}(0;\vartheta)$$

Ro-Transform Kernel Extension:

In this section the Ro-transform's kernel has been extended to be $n\in[\infty,-\infty]$, such as the following cases:

First Case: If $n=0$, then:

$$\Re(\upsilon)=R\{f(\varpi),\upsilon\}=\upsilon^{2}\int_{0}^{\infty}f(\varpi)e^{-i\upsilon\varpi}d\varpi .$$

For this case, Ro-transform of simple functions will be:

Function $f(\varpi)=R^{-1}\{\Re(\upsilon)\}$	$\Re(\upsilon)=R\{f(\varpi),\upsilon\}$.
$f(\varpi)=1$	$R\{f(\varpi),\upsilon\}=\upsilon i^{-1}$.
$f(\varpi)=\varpi$	$R\{f(\varpi),\upsilon\}=-1$.
$f(\varpi)=\varpi^{n}$	$R\{f(\varpi),\upsilon\}=\dfrac{n!\,\upsilon^{2}}{(iv)^{n+1}}$.
$f(\varpi)=\sin a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{a\upsilon^{2}}{(i\upsilon)^{2}+a^{2}}$.
$f(\varpi)=\cos a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{i\upsilon^{3}}{(i\upsilon)^{2}+a^{2}}$.
$f(\varpi)=\sinh a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{\upsilon^{2}}{(i\upsilon)^{2}-1}$.

$f(\varpi)=\cosh a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{i\upsilon^3}{(i\upsilon)^2-1}$.
$f(\varpi)=e^{\varpi}$	$R\{f(\varpi),\upsilon\}=\dfrac{\upsilon^2}{(i\upsilon)-1}$.

Note: In this case if has been replaced kernel $e^{-\left(i\sqrt[\Omega]{\upsilon}\right)\varpi}$ by $e^{-\left(i^n\sqrt[\Omega]{\upsilon}\right)\varpi}$, n=0,1,2,..., to get:

$\Re(\upsilon)=R\{f(\varpi),\upsilon\}=\upsilon^2\int_0^\infty f(\varpi)e^{-i\upsilon\varpi}d\varpi;$ it is Mohand transform or Laplace transform multiplied by a constant.

Second Case: If $n\le -1$,

$$\Re(\upsilon)=R\{f(\varpi),\upsilon\}=\upsilon^2\int_0^\infty f(\varpi)e^{-\left(\frac{1}{i\sqrt[\Omega]{\upsilon}}\right)\varpi}d\varpi;$$

So for this case Ro-transform of simple functions will be:

Function $f(\varpi)=R^{-1}\{\Re(\upsilon)\}$	$\Re(\upsilon)=R\{f(\varpi),\upsilon\}$
$f(\varpi)=1$	$R\{f(\varpi),\upsilon\}=\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)$, $\Omega=2n+1$
$f(\varpi)=\varpi$	$R\{f(\varpi),\upsilon\}=\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)^2$, $\Omega=2n+1$
$f(\varpi)=\varpi^n$	$R\{f(\varpi),\upsilon\}=n!\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)^{n+1}$.
$f(\varpi)=\sin a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{a\upsilon^2}{\left(1/i\sqrt[\Omega]{\upsilon}\right)^2+a^2}$
$f(\varpi)=\cos a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{\upsilon^2/i\sqrt[\Omega]{\upsilon}}{\left(1/i\sqrt[\Omega]{\upsilon}\right)^2+a^2}$
$f(\varpi)=\sinh a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{\upsilon^2}{\left(1/i\sqrt[\Omega]{\upsilon}\right)^2-1},\dfrac{1}{i\sqrt[\Omega]{\upsilon}}>1$
$f(\varpi)=\cosh a\varpi$	$R\{f(\varpi),\upsilon\}=\dfrac{\upsilon^2/i\sqrt[\Omega]{\upsilon}}{\left(1/i\sqrt[\Omega]{\upsilon}\right)^2-1}$ $,\dfrac{1}{i\sqrt[\Omega]{\upsilon}}>1$
$f(\varpi)=e^{\varpi}$	$R\{f(\varpi),\upsilon\}=\dfrac{\upsilon^2}{\left(1/i\sqrt[\Omega]{\upsilon}\right)-1},\dfrac{1}{i\sqrt[\Omega]{\upsilon}}>1$

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