The Connected Distance - K Domination Number of Some Families of Graphs

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Abstract

distance - k dominating set $D \subseteq V$ of a graph G = (V, E) is a connected distance - k dominating set if the induced sub-graph $\langle D \rangle$ is connected. The connected distance -k domination number $\gamma_{c\leq k}(G)$ of G is the minimum cardinality of a minimal connected distance - k dominating set of G.The connected distance - k domination transition number of a graph G is defined as $\tau_{c\leq k}(G) = \gamma_{c\leq k}(G) - \gamma_{\leq k}(G)$and is denoted as $\tau_{c\leq k}(G)$.In this paper, we defined the notion of connected distance - k domination and connected distance - k domination transition number in graphs. We got many bounds on connected distance - k dominationnumber and connected distance - k domination transition number. Exact values of these new parameters are obtained for some standard graphs and also their relationship with other domination parameters were obtained. Nordhaus - Gaddum type results were also obtained for these new parameters.

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1. INTRODUCTION

In this paper, we considered only simple, finite, connected and undirected graphs. Let n and m denote the order and size of the graph G. We used the terminology of [2].Let $\Delta(G)\,(\,\delta(G)\,)$ denote the maximum (minimum) degree. The least (greatest) integer greater (less) than or equal to x is$\lceil x \rceil (\lfloor x \rfloor)$. The independence number β_0(G) is the maximum cardinality among the independent set of vertices of G. A vertex v of G is called a support if it is adjacent to a pendent vertex. Any vertex of degree greater than one is called an internal vertex.

A non-empty subset $D \subseteq V$ of vertices in a graph G = (V, E) is called a dominating set if every vertex in V-D is adjacent to at least one vertex of D.The domination number of G is the minimumcardinality of a minimal dominating set and it is denoted by γ (G). The upper domination number $\Gamma(G)$ of G is the maximum cardinality of a dominating set. A recent survey of γ (G) can be found in [3]. The term "domination" was first used by Ore [6].

Various types of domination parameters have been defined and studied by severalauthors and more than 80 models of domination were listed in the Appendixof Haynes et al. [4]. Sampathkumar and Walikar [7] introduced the conceptof connected domination in graphs. A dominating set $D \subseteq V$of G is called aconnected dominating set if the induced sub-graph$\langle D \rangle$is connected.Theminimum cardinality of a minimal connected dominating set of G is called the connecteddomination number of G and is denoted by$\gamma_c(G)$. Cockayne et al.[2] introduced the concept of total domination in graphs. A dominating set$D \subseteq V$of G is called a total dominating set of G if $\langle D \rangle$has no isolated vertices.The minimum cardinality of a total dominating set of G is called the totaldomination number of G and is denoted by $\gamma_t(G)$.

A non-empty set $D \subseteq V$ of vertices in a graph G = (V,E) is a distance -k dominating set if every vertex in V-D is within distance -k of atleast one vertex in D. The distance -k domination number $\gamma_{\leq k}(G)$ of G equals the minimum cardinality of a minimal distance -k dominating set in G [2].

Definition 1.1

A distance - k dominating set$D \subseteq V$ in a graph G= (V, E) is a connected distance -k dominating set, if the induced sub-graph $\langle D \rangle$is connected. The connected distance - k domination number $\gamma_{c\leq k}(G)$ of G is the minimum cardinality of minimalconnected distance - k dominating set. The upper connected distance - k domination number $\Gamma_{c\leq k}(G)$ of G is the maximum cardinality of aconnected distance - k dominating set.

Example1.2

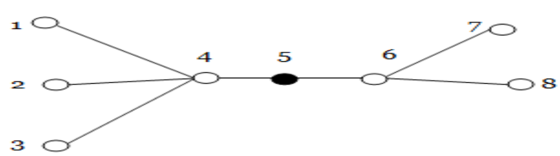


Figure.1

Here $\gamma(G) = 2, \gamma_c(G) = 3$, when $k = 2, \gamma_{\leq 2}(G) = 1, \gamma_{c\leq 2}(G) = 1, \Gamma_{c\leq 2}(G)$ =9.

2. Standard graphs and their exact values of $\gamma_{c\leq k}(G)$for k = 2

The connected distance - 2 domination numbers $\gamma_{c\leq 2}(G)$ of some standard graphs is given below.

2.1. **Observation** $\gamma_{c\leq k}(G)$ **for k = 2**

1. For any Path P_n,

$$\gamma_{c\leq 2}(P_n) = \begin{Bmatrix} n-4, n\geq 5 \\ 1, n<5 \end{Bmatrix}$$

2. For any Cycle C_n,

$$\gamma_{c\leq 2}(C_n) = \begin{Bmatrix} n-4, n\geq 5 \\ 1, n<5 \end{Bmatrix}$$

3. For a Lollipop graph $I_{m,n}$ for all m

$$\gamma_{c\leq 2}(I_{m,n}) = \begin{Bmatrix} n-2, n\geq 3 \\ 1, n<3 \end{Bmatrix}$$

4. For any Grid graph P_{ixj} for i= 2,3 and $j \geq 3$

$$\gamma_{c\leq 2}(P_{ixj}) = j-2$$

2.2. **Observation**

1. For any Wheel graph W_n, for n$\geq$ 3

$$\gamma_{c\leq k}(W_n) = 1$$

2. For any Friendship graph F_n, for n$\geq$ 2

$$\gamma_{c\leq k}(F_n) = 1$$

3. For any Complete graph K_n, for n$\geq$ 3

$$\gamma_{c\leq k}(K_n) = 1$$

4. For any Star graph $K_{1,m}$, for m $\geq$1

$$\gamma_{c\leq k}(K_{1,m}) = 1$$

5. For any Complete bipartite graph $K_{n,m}$, for m,n $\geq$ 1,

$$\gamma_{c\leq k}(K_{n,m}) = 1$$

6. For any Book graph B_n , for n $\geq$3

$$\gamma_{c\leq k}(B_n)=1$$

7. For any Helm graph H_n , for n $\geq$3

$$\gamma_{c\leq k}(H_n)=1$$

8. For any n-Barbell graph, for n $\geq$3

$$\gamma_{c\leq k}(n-barbell)=1$$

9. For any Ladder Rung graph L_n,

$$\gamma_{c\leq k}(L_n)=0$$

10. For any graph G,

$$\gamma_{c\leq k}(K_n+G)=1$$

3. Exact value of $\boldsymbol{\gamma_{c\leq k}}(\mathbf{G})$ with other graph theoretical parameters

Theorem 3.1

For any star graph $\gamma(K_{1,n}) = \gamma_c(K_{1,n}) = \gamma_{\leq k}(K_{1,n}) = \gamma_{c\leq k}(K_{1,n}) = 1$.

Proof

By the definition of the star graph, one vetex is adjacent to all other vertices. Therefore the domination number is one. The maximum distance in the star graph is one so we get$\gamma_{\leq k}(K_{1,n}) = 1$. From [7], singleton sets are connected. Hence $\gamma(K_{1,n}) = \gamma_c(K_{1,n}) = \gamma_{\leq k}(K_{1,n}) = \gamma_{c\leq k}(K_{1,n}) = 1$.

Theorem 3.2

If Gis a Friendship graph, or a Wheel graph, or a Complete graph then it hold the following equality$\gamma(G) = \gamma_c(G) = \gamma_{\leq k}(G) = \gamma_{c\leq k}(G) = 1$

Theorem3.3

If G is a book graph, or a Helm graph, or a Complete bipartite graph then$\gamma_{\leq k}(G) = \gamma_{c\leq k}(G) = 1$

Theorem3.4

If G is a Complete bipartite graph or aBook graph then it satisfies the equality$\gamma_{c\leq k}(G) = \gamma_c(G) - 1$.

Theorem 3.5

If G is a Book graph or a Complete bipartite graph then $\gamma_{c\leq 2}(G) = \gamma(G) - 1$.

Theorem3.6

For any Helm graph H_n, $\gamma_{c\leq k}(H_n) = \gamma_c(H_n) - (n-1)$, where n is a suffix number of H_n.

Theorem3.7

In Helm graph H_n, $\gamma_{c\leq k}(H_n) = \gamma(H_n) - (n-1)$.

Proposition 3.8

A graph G is a Wheel graph or a Star graph or a complete graph with n ≥2, then $\left(\frac{n}{\Delta(G)+1}\right) = \gamma_{c\leq k}(G)$.

Proposition 3.9

A graph G with n ≥2, then $\left\lfloor\frac{n}{\Delta(G)+1}\right\rfloor = \gamma_{c\leq k}(G)$ where G is a Path or Cycle or a Book graph or a Helm graph or a complete bipartite graph.

4. Bounds on the connected distance - 2 domination number

Proposition4.1

For any connected graph G, $\gamma_{\leq k}(G) \leq \gamma_{c\leq k}(G)$.

Proof

Every connected distance - k dominating set of G is a distance - k dominating set of G, we have $\gamma_{\leq k}(G) \leq \gamma_{c\leq k}(G)$.

Proposition 4.2

For any connected graph G, $\gamma_{c\leq k}(G) \leq \gamma_c(G)$.

Proof

Every connected dominating set of G is a connected distance - k dominating set of G, we have $\gamma_{c\leq k}(G) \leq \gamma_c(G)$.

Proposition 4.3

For any connected graph G, $\gamma_{c\leq k}(G) \leq \gamma(G)$, if and only if G is not a Path or a Cycle.

Proposition 4.4

Let G be a connected graph and H be a connected spanning sub-graph of G. Then every connected distance – k dominating set of H is also a connected distance – k dominating set of G, then $\gamma_{c\leq 2}(G) \leq \gamma_{c\leq 2}(H)$.

Hedetniemi and Laskar gave an inequality for connected domination number namely, $\gamma_c(G) \leq n - \Delta(G)$ in[2].

Theorem 4.5

For any connected graph G, $\gamma_{c\leq k}(G) \leq n - \Delta(G)$.

Proof

By the reference [4], we have$\gamma_c(G) \leq n - \Delta(G)$ and by the proposition 4.2 $\gamma_{c\leq k}(G) \leq \gamma_c(G)$.

Hence we get , $\gamma_{c\leq k}(G) \leq n - \Delta(G)$.

Theorem 4.6

For any connected graph G, $\gamma_{c\leq k}(G) \leq 2\beta_0 - 1, and \gamma_{c\leq k}(G) \leq 3\gamma(G) - 2$.

Proof

By the reference [2], we have$\gamma_c(G) \leq 2\beta_0 - 1, and \gamma_c(G) \leq 3\gamma(G) - 2$, and by the proposition 4.2 $\gamma_{c\leq k}(G) \leq \gamma_c(G)$.

Hence we get ,$\gamma_{c\leq k}(G) \leq 2\beta_0 - 1, and \gamma_{c\leq k}(G) \leq 3\gamma(G) - 2$.

Corollary 4.7

For any connected graph G, $\gamma_{c\leq k}(G) \leq 3\gamma_{\leq k}(G) - 2$.

Bo and Liu showed [2] that the irredundance number satisfies the inquality $\gamma_c(G) \leq 3\ ir(G) - 2$.

Theorem 4.6

For any connected graph G, $\gamma_{c\leq k}(G) \leq 3\ ir(G) - 1$.

Proof

By the reference [2], we have$\gamma_c(G) \leq 3\ ir(G) - 2$, and by the proposition 4.2 $\gamma_{c\leq k}(G) \leq \gamma_c(G)$.

Hence we get $\gamma_{c\leq k}(G) \leq 3\ ir(G) - 1$.

Theorem 4.7

For any connected graph G of order n,

(i). $\gamma_{c\leq k}(G) + \gamma_{\leq k}(G) \leq n$

(ii). $\gamma_{c\leq k}(G) + \gamma(G) \leq n$

Theorem 4.8

If G is a connected graph of order $n \geq 5$, then

(i). $\gamma_{c\leq 2}(G) = n - 4$ if and only if G is either a $P_n$or a C_n.

(ii).$1 \leq \gamma_{c\leq k}(G) \leq n - 4$.

Norduaus – Gaddum Type results

For any graph G and $\overline{G}$ without isolated vertices,

(i). $2 \leq \gamma_{c\leq k}(G) + \gamma_{c\leq k}(\overline{G}) \leq 2(n-4)$

(ii). $1 \leq \gamma_{c\leq k}(G).\gamma_{c\leq k}(\overline{G}) \leq (n-4)^2$

5. Graphs with equality between connected – k domination and distance – kdomination numbers of a graph G

Theorem 5.1

Let G be any connected graph and let D be the minimum connected distance – 2 dominating set of G. If$\gamma_{\leq 2}(G) = \gamma_{c\leq 2}(G)$, then $\Delta(\langle D\rangle) < \Delta(G)$ where $\Delta(G)$ denotes the maximum degree of a vertex in G [1].

Proof

Suppose $\Delta(\langle D\rangle) = \Delta(G)$. Let $v \in D$ be such that $deg\langle D\rangle = \Delta(G)$. Then D- $\{v\}$ is a distance - 2 dominating set of cardinality$\gamma_{\leq 2}(G) - 1$, which is a contradiction. Hence we prove the result.

Theorem 5.2

For a tree T_n of order$n \geq 3$, $\gamma_{\leq k}(T_n) = \gamma_{c\leq k}(T_n)$ (for k = 2) if and only if every internal vertex of T_n has a support within distance two [1].

Proof

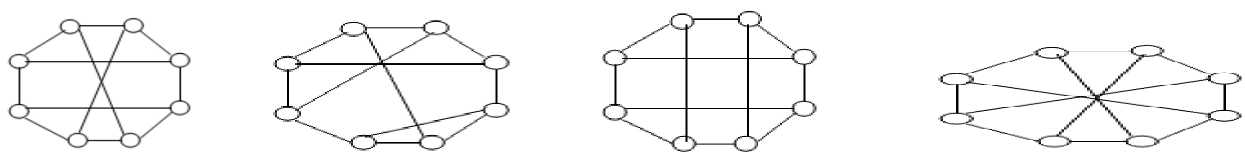
Let T_n be a tree of order$n \geq 3$. Let D be the set of all internal vertices which are within distance two from the support of tree is the unique minimum connected distance – 2 dominating set in tree.

Let $\gamma_{\leq 2}(T_n) = \gamma_{c\leq 2}(T_n)$. If there exists an internal vertex v which is not have a support within distance two, then D-{v } is a distance – 2 dominating set of cordinality $\gamma_{\leq 2}(T_n) - 1$, which is a contradiction.

Conversely, let every internal vertex of T_n has a support within distance two. Then $\gamma_{\leq 2}(T_n) \geq |D|$. Hence we get $\gamma_{\leq k}(T_n) = \gamma_{c\leq k}(T_n)$for k = 2.

Theorem 5.3

Let G be a connected cubic graph of order $n \leq 8$. If$\gamma_{\leq 2}(G) = \gamma_{c\leq 2}(G)$, then G is isomorphic to $K_4, \bar{C}_6, K_{3,3}, G_1 or G_2 or G_3 or G_4$ where $G_1,\ G_2, G_3\ and G_4$ are given in Fig 2.



G1 G2 G3 G4

Figure 2.

Proof

If $G = K_4, \bar{C}_6, K_{3,3}$, we get $\gamma_{\leq 2}(G) = \gamma_{c\leq 2}(G) = 1$. When $G = G_1 or G_2 or G_3 or G_4$, then $\gamma_{\leq 2}(G) = \gamma_{c\leq 2}(G) = 2$.

Theorem 5.4

For any path graph P_n,$\gamma_{\leq k}(C(P_n)) = 1 = \gamma_{c\leq k}(C(P_n))$.

Proof

Let P_n be any path of length n-1 with vertices $v_1, v_2, v_3, \dots v_n$. On the process of centralization of P_n , let u_i be the vertex of subdivision of the edges $v_i v_{i+1}$ for $1 \leq i \leq n$. Also let $v_i u_i = e_i$ and $u_i v_{i+1} = e'_i$ for $1 \leq i \leq n-1$.

In $C(P_n)$, the vertex v_i is adjacent to all vertices except the verticesv_{i+1} and v_{i-1} $(1 \leq i \leq n-1)$ but it is of distance two to the vertex v_i and the remaining verticesu_i are all within the distance two the vertex v_i. Hence we have $\gamma_{\leq 2}(C(P_n)) = 1 = \gamma_{c\leq 2}(C(P_n))$. It is true for the distance k.

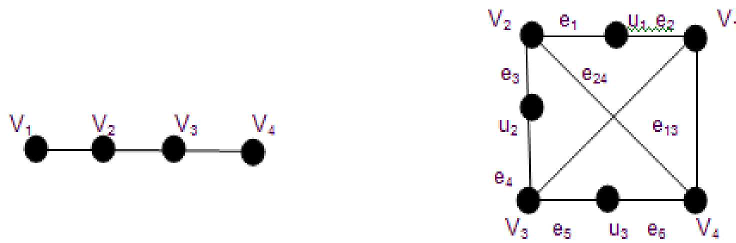


Figure 3: Path graph with 4 vertices and its central graph.

Theorem 5.5.

For any cycle graph C_n, the distance - k domination number of the central graph, $\gamma_{\leq k}(C(C_n)) = 1 = \gamma_{c\leq k}(C(C_n))$.

Proof

Let $V(C_n) = \{v_1, v_2, v_3, \dots v_n\}$ and $E(C_n) = \{e_1, e_2, e_3, \dots e_n\}$ where$e_n = v_n v_1$and $e_i = v_i v_{i+1}$, $(1 \leq i \leq n-1)$.

Apply the definition of the central graph to the cycle graph, has the vertex set $V(C_n) \cup \{u_i / 1 \leq i \leq n\}$ where u_i is a vertex of subdivision of the edge $v_i v_{i+1}$, $(1 \leq i \leq n-1)$ and u_n is a vertex of subdivision of the edges $v_n v_1$.

In $C(C_n)$, let any vertex $v_i$which is adjacent to all vertices except the verticesv_{i+1} and $v_{i-1}$$(1 \leq i \leq n-1)$ but the adjacent vertices are all exactly atthe distance two to the vertex v_i and also all other vertices u_i are within the distance two the vertex v_i. Hence we have $\gamma_{\leq 2}(C(P_n)) = 1 = \gamma_{c\leq 2}(C(C_n))$.

Ie, $\gamma_{\leq k}(C(C_n)) = 1 = \gamma_{c\leq k}(C(C_n))$.

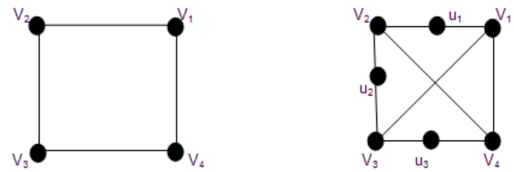


Figure 4.: The graph of C_4 and its $C(C_4)$.

Theorem 5.6.

For any star graph $K_{1,n}$, the distance - k domination numberof the central graph, $\gamma_{\leq k}[C(K_{1,n})] = 1 = \gamma_{c\leq k}(C(K_{1,n}))$.

Proof

Let $V(K_{1,n}) = \{v, v_1, v_2, v_3, \dots v_n\}$ where $\deg v = n$. Applying the definition of the central graph of star graph, the subdivision vertices are dented by $u_1, u_2, u_3, \dots, u_n$.We denote the new edges as$e_i = v_i u_i$ and $e'_i = v u_i$. In the central graph of a star graph , the central vertex v form a distance – 2 dominating set. Hence we have $\gamma_{\leq 2}[C(K_{1,n})] = 1 = \gamma_{c\leq 2}(C(K_{1,n}))$. So this result is true for all k, ie $\gamma_{\leq k}[C(K_{1,n})] = 1 = \gamma_{c\leq k}(C(K_{1,n}))$.

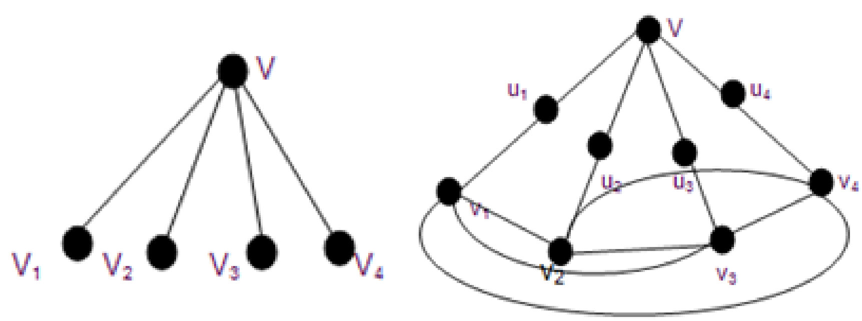


Figure 5: The Star graph with (1,5) vertices and its central graph.

Theorem 5.6

For any cycle graphC_n,the distance - 2 domination numberof the Middle graph, $\gamma_{\leq k}(M(C_n)) = 1 = \gamma_{c\leq k}(M(C_n))$.

Proof

Let $V(C_n) = \{v_1, v_2, v_3, \dots v_n\}$ and $E(C_n) = \{e_1, e_2, e_3, \dots e_n\}$ where$e_n = v_n v_1$and $e_i = v_i v_{i+1}$, $(1 \leq i \leq n-1)$. By the definition of middle graph $M(C_n)$ has the vertex set $V(C_n) \cup E(C_n)$ in which each e_i is adjacent with e_{i+1} for $i = 1,2,3 \dots, n-1$ and e_n is adjacent with v_1. In (C_n) ,$v_1, e_1, v_2, e_2, v_3, e_3, \dots e_{n-1}, v_1$ induces a cycle of length 2n.

In $M(C_n)$, we select any one vertex v_i and the other vertices in $M(C_n)$ are all within the distance two to the vertex v_i . Hence we have $\gamma_{\leq 2}(M(C_n)) = 1 = \gamma_{c\leq 2}(M(C_n))$.

Theorem 5.7

For any star graph $K_{1,n}$, the distance -k domination number of the Middle graph, when k = $2\gamma_{\leq k}\left[M\left(K_{1,n}\right)\right] = 1 = \gamma_{c\leq k}\left[M\left(K_{1,n}\right)\right]$.

Proof

Let $\{v, v_1, v_2, v_3, \dots v_n\}$be the vertices of the $K_{1,n}$and $\{e_1, e_2, e_3, \dots e_n\}$ be the edges of $K_{1,n}$ By the definition of Middle graph, we have $V\left[M\left(K_{1,n}\right)\right] = \{v\} \cup \{e_i/1 \leq i \leq n\} \cup \{ v_i/1 \leq i \leq n\}$ in which the vertices $e_1, e_2, e_3, \dots e_n, v$ induces a clique of order $n+1$.

In $M\left(K_{1,n}\right)$ the vertex v is adjacent to $\{e_i/1 \leq i \leq n\}$ and $\{v_1, v_2, v_3, \dots v_n\}$ is an independent set and also each v_i $(1 \leq i \leq n)$ is exactly at a distance two to the vertex $\{v\}$. Since each e_i is adjacent to v_i. Hence $\{v\}$ will form a distance – 2 dominating set of $M\left(K_{1,n}\right)$, i.e $\gamma_{\leq 2}\left[M\left(K_{1,n}\right)\right] = 1 = \gamma_{c\leq 2}\left[M\left(K_{1,n}\right)\right]$. Therefore we get the general result$\gamma_{\leq k}\left[M\left(K_{1,n}\right)\right] = 1 = \gamma_{c\leq k}\left[M\left(K_{1,n}\right)\right]$.

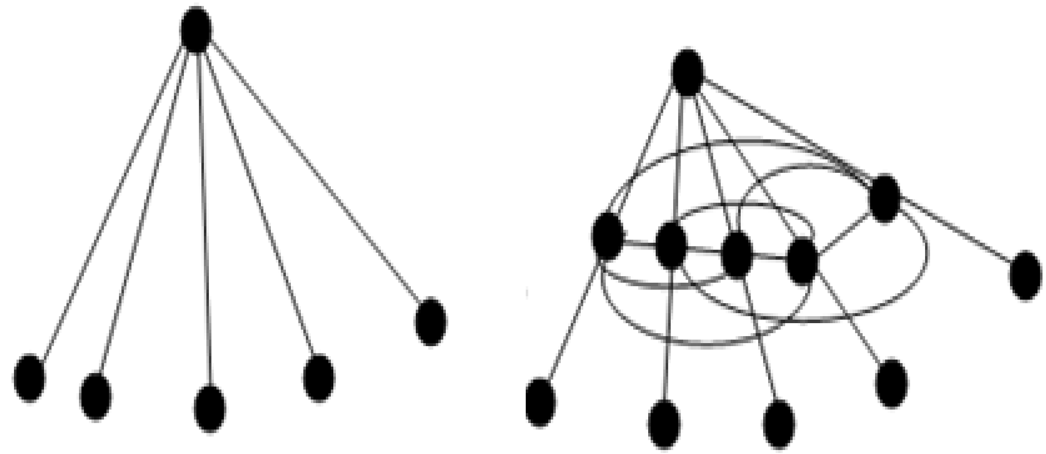


Figure 6: The Star graph with (1,5) vertices and its middle graph.

In [4] Kaspar. S etal introduced the concept of connected domination transition number of a graph G as a difference between the connected domination and domination numbers of G and is denoted as $\tau_c(G)$.

Definition 6.1

The connected distance - k domination transition number of a graph G is defined as the difference between the connected distance - k domination and distance - k domination numbers of G i.e $\tau_{c\leq k}(G) = \gamma_{c\leq k}(G) - \gamma_{\leq k}(G)$and is denoted as $\tau_{c\leq k}(G)$.

In the example 1.2 $\tau_{c\leq k}(G) = 0$, when k =2

7.Exact values of $\tau_{c\leq k}(G)$for some standard graphs

The connected distance - k domination transitionnumber$\tau_{c\leq k}(G)$ of some standard graphs is given below.

7.1. **Observation**

1. For n≥ 3, and k=2, $\tau_{c\leq 2}(P_n) = \tau_{c\leq 2}(C_n)$

2. $\tau_{c\leq k}(K_n) = 0$.

3. $\tau_{c\leq k}(H_n) = 0$.

4. $\tau_{c\leq k}(K_{1,m}) = 0$, $\tau_{c\leq 2}(K_{1,m} + G) = 0$, where G is any graph.

5. $\tau_{c\leq k}(K_{n,m}) = 0$.

6.$\tau_{c\leq k}(B_n) = 0$.

7.$\tau_{c\leq k}(F_n) = 0$.

8.$\tau_{c\leq k}(W_n) = 0$.

Proposition 7.1

For any connected graph G, then $\tau_{c\leq k}(G) \leq \tau_c(G)$.

Proof

Every connected domination transition number of a graph G is a connected distance – k domination transition number of G.Thus we have $\gamma_{\leq k}(G) \leq \gamma_{c\leq k}(G)$.

Proposition 7.2

For any connected graph G, then $\tau_{c\leq k}(G) \leq \left|\Delta(G) - \frac{n\Delta(G)}{1+\Delta(G)}\right|$ when k = 2.

Proof

By the reference [4] , we have $\tau_c(G) \leq \left|\Delta(G) - \frac{n\Delta(G)}{1+\Delta(G)}\right|$ and proposition 7.1,$\tau_{c\leq k}(G) \leq \tau_c(G)$ we get $\tau_{c\leq 2}(G) \leq \left|\Delta(G) - \frac{n\Delta(G)}{1+\Delta(G)}\right|$ $when\ k = 2$.

Proposition 7.3

For any connected graph G, then $\tau_{c\leq 2}(G) \leq n - 3$,when k=2.

Proof

By the reference [4] , we have $\tau_c(G) \leq n - 3$ and proposition 7.1,$\tau_{c\leq k}(G) \leq \tau_c(G)$ we get $\tau_{c\leq 2}(G) \leq n - 3$, when k=2.

Proposition 7.4

For any connected graph G, then $\tau_{c\leq k}(G) \leq n - \gamma(G) - 2$.

Proof

By the reference [4] , we have $\tau_c(G) \leq n - \gamma(G) - 2$ and proposition 7.1,$\tau_{c\leq k}(G) \leq \tau_c(G)$ we get $\tau_{c\leq k}(G) \leq n - \gamma(G) - 2$.

Proposition 7.5

Let G be a connected graph and H be a connected spanning sub-graph of G, then $\tau_{c\leq k}(G) \leq \tau_{c\leq k}(H)$.

8. APPLICATIONS OF MINIMUM CONNECTED DISTANCE– k (for k=2) DOMINATING SET

8.1 School Bus Routing

Nowadays, almost all schools operate school buses for transporting children for to and fro services. Among many points, three important points to be noted are (i) The running time of a bus between school and its terminus (ii) Maximum number of students on a bus at any time and (iii) the maximum distance a student has to walk to board a school bus.Consider a street map of a city shown in fig.3 where each edge represents one city block. Let us assume that the school is located at the vertex starting point and the management committee of the school decides that no student shall walk more than two blocks to board a school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child that uses the school bus and returns to the school. Clearly this bus route forms a minimum connected distance - 2 dominating set.

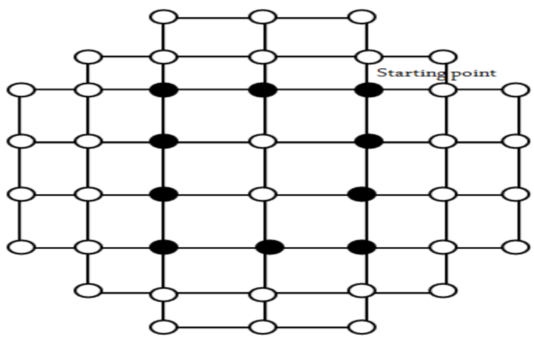


Figure 3.

8.2 Mobile Ad-Hoc Network

The connected Dominating set has been a classic subject studied in graph theory since 1975. In 1990s it has been found to have important applications in communication networks, especially in wireless networks, as a virtual backbone. Now the connected dominating set has become a hot research topic in computer Science.

Connected distance - 2 dominating sets are very useful in the computation of routing for mobile ad hoc networks.In this application, a minimum connected distance - 2 dominating set is used as a backbone for communications, and nodes that are not in this set communicate by passing messages through neighbors that are in the set.

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