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Usig R-Statistical Software to Perform Simulations on Centralized and Decentralized Beta Distributions

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Abstract. The beta distribution is one of the continuous probability distributions of application significance and is widely used to study the behavior of some random variables. The name is derived from the beta name of the mathematical function that appears in its equation. Beta functions are a class of random and controllable variables in the shape of a distribution, which is a continuous probability distribution defined over periods (0 , 1), In addition to the use of R programs showing how to apply these properties and some relationships through simulation, the R program was used to find the distributions resulting from the beta distribution relationship (central and decentralized of the first type) with some distributions that were derived mathematically and through the resulting distribution from the relationship.

Key words and phrases. Beta Distribution . Beta Distribution of First Kind. R-statistical

1. INTRODUCTION

The beta distribution is one of the important statistical distributions and has many applications in various fields, especially in the applications of dependency and production quality control. This distribution is very important in terms of application in various fields. The purpose of this study is to derive the beta distributions of the first and second types (central and decentralized) and to find the statistical properties of the two distributions such as the mean, median, mode, variance, standard deviation and skew coefficient, and to find some relationships for the first and second beta distributions with some distributions. They were only formulas, but they were derived and proven.

In addition to using the R program that shows how to apply these properties and some relationships through simulation, the R program was used to find the distributions resulting from the beta distribution relationship (central and eccentric of the first type) with some distributions that have been mathematically derived and through the resulting distribution from the relationship, was generated Samples of different sizes and finding the correlation coefficient, variance and skew coefficient at different sample sizes and the extent of the effect of different sample sizes when finding each of the correlation coefficient, variance and skew coefficient in terms of increase or decrease in the resulting value.

The problem of this research lies in identifying one of the important probability distributions, which is the central and decentralized beta distribution of the first and second types, which has wide uses, and identifying the theoretical properties and how to use it, as well as acquiring the skill in using the statistical program R, to simulate some theoretical proofs to verify in practice.

previous works:

Shanny Lin, Shaohui Liu, and Hao Zhu

Porto, Portugal — June 27 – July 1, 202

Risk-Aware Learning for Scalable Voltage Optimization in Distribution Grids
Through his research he developed a risk-aware educational framework

To achieve scalable decision rules in the distribution network effort

optimization problem. To learn optimum reactive power.

research aims :

In this research, we seek to achieve the following

1- Identifying the central and decentralized beta distribution of the first and second types.

2- Studying the relationship of the central and decentralized beta distribution to some other distributions

3- Studying the theoretical properties of the central and decentralized beta distribution

4- How to use RV in centralized and decentralized beta distribution applications

The importance of the research The importance of this research is in identifying one of the important probability distributions, which is the central and decentralized beta distribution of the first and second types, which has wide uses and the identification of theoretical properties and how to use it, as well as gaining skill in using the R-statistical program, for the simulation of some theoretical proofs to verify In practical terms.

2. Beta Distribution characteristics[1]

In mathematics, the beta function is of great importance in this distribution, also known as the first genus Euler integral. It is a function given by the following relationship:

$$\beta(n, m) = \int_0^1 t^{n-1}(1-t)^{m-1} \qquad n, m \text{ wave constant}$$

Beta Function Properties: There are many properties of a beta function, including:

The first feature:

$$\beta(n, m) = \beta(m, n)$$

The second feature:

$$\beta(n,m) = \frac{1}{n}$$

The third feature:

$$\beta(n,m) = \frac{n-1}{m} - \frac{n-2}{m+1}\,\beta(m+2, n-2)$$

3. Beta Distribution of First Kind [1]

This distribution is derived from the beta function and is one of the distributions of practical importance in the field of production control, where it is noticed that many relative phenomena are better described by their impact ratio, so that the description is more clear, such as the success rate, the impurities ratio, the invalid cut-off ratio, and so on. Thus, such phenomena describe their random results with a random variable that follows in its changes a probability distribution called the beta distribution

$$f(x) = \begin{Bmatrix} \dfrac{\frac{\alpha+\beta}{2}}{\sqrt{\frac{\alpha}{2}\sqrt{\frac{\beta}{2}}}} x^{\frac{\alpha}{2}-1}(1-x)^{\frac{\beta}{2}-1} & ,0 < x < 1 \quad \alpha,\beta > 0 \\ 0 & , \text{ow} \end{Bmatrix}$$

Therefore, variable x has a beta distribution with parameters $(\frac{\alpha}{2}, \frac{\beta}{2})$ and is symbolized by the symbol $x \sim \beta\,(\frac{\alpha}{2}, \frac{\beta}{2})$.

3.1.Characteristics of a central beta distribution of the first type[2] :

Probability distributions have many shapes and different characteristics, and this is determined by the standard deviation, the arithmetic mean, the mode, and others.

3.1.1.Characteristics

$$F(m) = 0.5$$

$$\frac{\sqrt{\frac{\alpha+\beta}{2}}}{\sqrt{\frac{\alpha}{2}-1}} \sum_{i=0}^{\frac{\beta}{2}} C_i^{\frac{\beta}{2}-1} (-1)^{\frac{\beta}{2}-i-1} \frac{\frac{\alpha+\beta}{2}-i-1}{\frac{\alpha+\beta}{2}-i-1} = 0.5$$

$$m = \frac{\alpha+\beta}{2} - i - \sqrt{\frac{\left(0.5 \times \frac{\alpha+\beta}{2} - i - 1\right)}{AB}}$$

3.1.2. The mode

$$f(x)=\frac{\sqrt{\frac{\alpha+\beta}{2}}}{\sqrt{\frac{\alpha}{2}\sqrt{\frac{\beta}{2}}x^2(1-x)}}\frac{\beta}{2}-1$$

$$\ln f(x)=\ln(c)+(a-1)\ln(x)+(b-1)\ln(1-x)$$

$$c=\frac{\sqrt{\frac{\alpha+\beta}{2}}}{\sqrt{\left(\frac{\alpha}{2}\right)\frac{\beta}{2}}},\ a=\frac{\alpha}{2},\ b=\frac{\beta}{2}$$

$$\frac{\partial \ln f(x)}{\partial x}=\frac{f'(x)}{f(x)}=\text{zero}+\frac{(a-1)}{x}+\frac{(b-1)}{(1-x)}(-1)$$

$$\frac{f'(x)}{f(x)}=\text{zzro}+\frac{(a-1)}{x}+\frac{(b-1)}{(1-x)}(-1)$$

We multiply both sides by the middle

$$f'(x)=f(x)\left[\frac{(a-1)}{x}-\frac{(b-1)}{(1-x)}\right]$$

let $f'(x)=0, f(x)>0$

$$\frac{(a-1)(1-x)-x(b-1)}{x(1-x)}=0$$

$$(a-1)-(a-1)x-x(b-1)=0$$

$$(a-1)-x((a-1)+(b-1))=0$$

$$(a-1)=x(a+b-2x)$$

$$\therefore x=\frac{a-1}{a+b-2}$$

$$x=\frac{\frac{\alpha-1}{2}}{\frac{\alpha+\beta-2}{\alpha+\beta-4}}=\frac{2}{\alpha+2}\quad ,\alpha+\beta>4$$

$$f''(x)=f(x)\left[\frac{-(a-1)}{x^2}-\frac{(b-1)}{(1-x)^2}\right]+f'(x)\left[\frac{(a-1)}{x}-\frac{(b-1)}{(1-x)}\right]$$

$$f''(x)=f(x)\left[\frac{-(a-1)}{x^2}-\frac{(b-1)}{(1-x)^2}\right]+f(x)\left[\frac{(a-1)}{x}-\frac{(b-1)}{(1-x)}\right]^2$$

$$f''(x)=f(x)\left[\left[\frac{-(a-1)}{x^2}-\frac{(b-1)}{(1-x)^2}\right]+\left[\frac{(a-1)}{x}-\frac{(b-1)}{(1-x)}\right]^2\right]$$

$$f''(x) - f(x)\left[\left[\frac{-(a-1)}{\left[\frac{a-1}{a+b-2}\right]^2} - \frac{(b-1)}{\left[1-\left(\frac{a-1}{a+b-2}\right)\right]^2}\right] + \left[\frac{(a-1)}{\left[\frac{a-1}{a+b-2}\right]} - \frac{(b-1)}{\left[1-\left(\frac{a-1}{a+b-2}\right)\right]}\right]^2\right]$$

Thus, $f''(x)$ has an absolute limit at $x = \frac{a-1}{a+b-2}$ in the case of $\alpha > 1, \beta > 1$, where it is noticed that $f''(x) < 0$ is negative

3.1.3. inversion points

$$f''(x) = f(x)\left[\left[\frac{(a-1)}{x} - \frac{(b-1)}{(1-x)}\right]^2 - \left[\frac{(a-1)}{x^2} + \frac{(b-1)}{(1-x)^2}\right]\right] = 0$$

$$\left[\frac{[(1-x)(a-1) - x(b-1)]^2}{x^2(1-x)^2}\right] - \left[\frac{x^2(b-1) + (a-1)(1-x)^2}{x^2(1-x)^2}\right] = 0$$

$$(1-x)^2(a-1)^2 - 2x(1-x)(a-1)(b-1) = X^2(b-1)^2 - X^2(b-1) - (a-1)(1-x)^2 = 0$$

$$(1-2x+X^2)(a-1)^2 - 2x\,(a-1)(b-1)2X^2(a-2)(b-1) + X^2(b-1)^2 + X^2(a-1) - (1-2x+X^2)(b-1) = 0$$

$$(a-1)^2 - 2x\,a-1)^2 + 2X^2(a-1)(b-1) + 2X^2(b-1)^2 - X^2(a-1) - (b-1) + 2x(b-1) - X^2(b-) = 0$$

$$X^2[(a-1)^2 + 2(a-1)(b-1) + (b-1)^2 - (b-1) - (a-1)] + 2x\,[(a-1) - (a-1)(b-1) - (a-1)^2] + [(a-1)^2 - (a-1)] = 0$$

Compensation in common law

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = [(a-1)^2 + 2(a-1)(b-1) + (b-1)^2 - (b-1) - (a-1)]$$

$$b = 2[(a-1) - (a-1) - (b-1) - (a-1)^2]$$

$$c = [(a-1)^2 - (a-1)]$$

After compensation in common law a , b , c we get

$$x_{1,2}=\frac{a-1}{a+b-2}\pm\frac{1}{a+b-2}\sqrt{\frac{(a-1)(b-1)}{a+b-2}}$$

$$x_{1,2}=\frac{\frac{\alpha}{2}-1}{\frac{\alpha+\beta}{2}-2}\pm\frac{1}{\frac{\alpha+\beta}{2}-2}\sqrt{\frac{\left(\frac{\alpha}{2}-1\right)\left(\frac{\beta}{2}-1\right)}{\frac{\alpha+\beta}{2}-3},\frac{\alpha+\beta}{2}>3}$$

3.1.4.Moment generating function of the first type [3].

$$\therefore \mu_x(t)=E(e^{tx})$$

$$\because \mu_x(t)=\int_0^t e^{tx}f(x)dx$$

$$=\frac{1}{\beta\left(\frac{\alpha}{2},\frac{\beta}{2}\right)_0^1}\int_0^t e^{tx}x^{\frac{\alpha}{2}-1}(1-x)^{\frac{b}{2}-1}dx$$

$$=\frac{1}{\beta\left(\frac{\alpha}{2},\frac{\beta}{2}\right)}\int_0^{\frac{\alpha}{2}-1}(1-x)^{\frac{b}{2}}1\left[1+\frac{tx}{1!}+\frac{t^2x^2}{2!}+\cdots\right]dx$$

$$-\frac{1}{\beta\left(\frac{\alpha}{2},\frac{\beta}{2}\right)}\left[\int_0^1 x^{\frac{\alpha}{2}-1}(1-x)^{\frac{b}{2}1}dx+\frac{t}{1!}\int_0^{\frac{\alpha}{2}}(1-x)^{\frac{b}{2}}dx+\frac{t^2}{2!}\cdots\right]$$

$$\therefore \mu_x(t)=\frac{1}{\beta\left(\frac{\alpha}{2},\frac{\beta}{2}\right)}\left[\beta\left(\frac{\alpha}{2},\frac{\beta}{2}\right)+\frac{t}{1!}\beta\left(\frac{\alpha}{2}+1,\frac{\beta}{2}\right)+\frac{t^2}{2!}\beta\left(\frac{\alpha}{2}+2,\frac{\beta}{2}\right)+\cdots\right]$$

we get :

$$\mu_x(t)=1+\frac{\alpha\quad t}{\alpha+\beta}+,\frac{(\alpha)(\alpha+2)\quad t^2}{(\alpha+\beta)((\alpha+\beta+2)\ \ 2!}+\frac{(\alpha)(\alpha+2)\quad t^3}{(\alpha+\beta)((\alpha+\beta+2)(\alpha+\beta+4)\ \ 3!}+\cdots$$

3.2.Beta Distribution of Second Kind [4].

In statistics and probability theory, a beta distribution of the second type known as (beta distribution of the second type or quoted beta distribution) is completely of continuous probability distributions defined by a random variable 1 in terms of two parameters 1.2 and has a probability density function defined as follows:

$$f(x)=\begin{cases}\dfrac{\sqrt{\frac{\alpha+\beta}{2}}}{\sqrt{\frac{\alpha}{2}\sqrt{\frac{\beta}{2}}}}\ \dfrac{x^{\frac{\alpha}{2}-1}}{\frac{\alpha+\beta}{2}} & ,0<x<\infty\quad \alpha,\beta>0\\ 0 & ,\text{ow}\end{cases}$$

Therefore, variable x has a beta distribution with parameters $(\frac{\alpha}{2},\frac{\beta}{2})$ and is symbolized by the symbol $x \sim \beta'\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$.

3.2.2.Characteristics[5]

3.2.3. variance

$$V(X) - E(X^2) - [E(X)]^2 - \frac{(\alpha+\lambda)^2 + 2(\alpha+2\lambda)}{(\beta-2)(\beta-4)} - \left(\frac{(\alpha+\lambda)}{(\beta-2)}\right)^2$$

$$= \frac{(\alpha+\lambda)^2 + 2(\alpha+2\lambda)}{(\beta-2)(\beta-4)} - \frac{(\alpha+\lambda)^2}{(\beta-2)^2}$$

$$= \frac{[(\alpha+\lambda)^2 + 2(\alpha+2\lambda)][\beta-2] - (\alpha+\lambda)^2[\beta-4]}{(\beta-2)^2(\beta-4)}$$

$$= \frac{(\alpha^2 + 2\alpha\lambda + \lambda^2 + 2\alpha + 4\lambda)[\beta-2] - (\alpha^2 + 2\alpha\lambda + \lambda^2)[\beta-4]}{(\beta-2)^2(\beta-4)}$$

$$= \frac{\alpha^2\beta - 2\alpha^2 + 2\alpha\beta\lambda - 4\alpha\lambda + \lambda^2\beta - 2\lambda^2 + 2\alpha\beta - 4\alpha + 4\lambda\beta - 8\lambda - \alpha^2\beta + 4\alpha^2 - 2\alpha\beta\lambda + 8\alpha\lambda - \lambda^2\beta + 4\lambda}{(\beta-2)^2(\beta-4)}$$

$$= \frac{2\alpha^2 + 2\alpha\lambda + 2\lambda^2 + 2\alpha\beta - 4\alpha + 4\lambda\beta - 8\lambda}{(\beta-2)^2(\beta-4)}$$

Thus, the variance takes the following form:

$$v(x) = \frac{2[\alpha^2 + \alpha\lambda + \lambda^2 + \alpha\beta - 2\alpha + 2\lambda\beta - 4\lambda]}{(\beta-2)^2(\beta-4)}$$

3.2.4. distributive function

$f_x(x) = p(X \leq \mu) = \int_0^x f(u)du$

$$= \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}} \sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \int_0^x \frac{u^{\frac{\alpha}{2}+r-1}}{\frac{\alpha+\beta}{2}+r} \, du$$

$$\text{let} : v = \frac{u}{1+u}$$

$$v(1+u) = u$$

$$v + vu = u$$

$$v = u - vu$$

$$v = u(1-v)$$

$$u = \frac{v}{(1-v)}$$

$$\frac{du}{dv} = \frac{1}{(1-v)^2}\,dv$$

$$F_X(x) = \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}} \sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \frac{\left(\frac{v}{(1-v)}\right)^{\frac{\alpha}{2}+r-1}}{\left(1+\frac{v}{(1-v)}\right)^{\frac{\alpha+\beta}{2}+r}} \frac{1}{(1-v)^2}\,dv$$

$$= \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}} \sqrt{\frac{\alpha+\beta}{2}+r}}{r!} \frac{xv^{\frac{\alpha}{2}+r-1}(1-v)^{\frac{-\alpha}{2}}r+1}{\sqrt{\frac{\beta}{2}}+r} \int_0^{\frac{\alpha+\beta}{2}} (1-v)^{-2}dv$$

$$= \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \int_0^x v^{\frac{\alpha}{2}+r-1}\ (1-v)^{\frac{\beta}{2}-1}\ dv$$

$$\Rightarrow \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \int_0^x v^{\frac{\alpha}{2}+r-1} \sum_{i=0}^{\frac{\beta}{2}-1} c_i^{\frac{\beta}{2}-1}(-1)^{\frac{\beta}{2}-i-1}\ v^{\frac{\beta}{2}-i-1}\ dv$$

$$= \sum_{r}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \sum_{i=0}^{2} C_i^{\frac{\beta}{2}-1}(-1)^{\frac{\beta}{2}-i-1} \int_0^x v^{\frac{\alpha+\beta}{2}+r-i-2}\ dv$$

$$= \sum_{r}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \sum_{i=0}^{2} C_i^{\frac{\beta}{2}-1}(-1)^{\frac{\beta}{2}-i-1} \frac{\left[v^{\frac{\alpha+\beta}{2}+r-i-1}\right]^x}{v^{\frac{\alpha+\beta}{2}+r-i-1}}$$

$$\therefore F_X(x) = \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r}\sqrt{\frac{\beta}{2}}} \sum_{i=0}^{2} C_i^{\frac{\beta}{2}-1}(-1)^{\frac{\beta}{2}-i-1} \frac{x^{\frac{\alpha+\beta}{2}+r-i-1}}{\frac{\alpha+\beta}{2}+r-i-1}$$

3.2.5. Median

$F(m) = 0.5$

$$= \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2}+r}}{\sqrt{\frac{\alpha}{2}+r\sqrt{\frac{\beta}{2}}}} \sum_{i=0}^{\frac{\beta}{2}-1} C_{i=0}^{\frac{\beta}{2}-1}(-1)^{\frac{\beta}{2}-i-1} \frac{x^{\frac{\alpha+\beta}{2}+r-i-1}}{\frac{\alpha+\beta}{2}+r-i-1} = 0.5$$

let : $A = \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!}$

$$B = \frac{\sqrt{\frac{\alpha+\beta}{2} + r}}{\sqrt{\frac{\alpha}{2} + r\sqrt{\frac{\beta}{2}}}}$$

$$C = \sum_{i=0}^{\frac{\beta}{2}-1} C_i^{\frac{\beta}{2}-1} (-1)^{\frac{\beta}{2}-i-1}$$

$$m = x^{\frac{\alpha+\beta}{2}+r-i-1} \sqrt{\frac{(0.5 * x^{\frac{\alpha+\beta}{2}+r-i-1})}{ABC}}$$

3.2.6. Mode

$$f(x) = \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!} \frac{\sqrt{\frac{\alpha+\beta}{2} + r}}{\sqrt{\frac{\alpha}{2} + r\sqrt{\frac{\beta}{2}}}\, 2^{\frac{\alpha+\beta}{2}+r}} \frac{x^{\frac{\alpha}{2}+r-1}}{(1+x)^{\frac{\alpha+\beta}{2}+r}}, 0 < x < \infty$$

let : $d = \sum_{r=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^r e^{\frac{-\lambda}{2}}}{r!}, c = \frac{\sqrt{\frac{\alpha+\beta}{2}}}{\sqrt{\frac{\alpha}{2}\sqrt{\frac{\beta}{2}}}}, a = \frac{\alpha}{2} + r, b = \frac{\beta}{2}$

$$\ln f(x) = hn(c) + (a-1)\ln(x) - (a+b)\ln(1+x)$$

$$\frac{f'(x)}{f(x)} = \text{ zero } + \frac{(a-1)}{x} - \frac{(a+b)}{(1+x)}$$

$$f'(x) = f(x)\left[\frac{(a-1)}{x} - \frac{(a+b)}{(1+x)}\right] = 0$$

$$\text{let } f'(x) = 0, \qquad f(x) > 0 \text{ then } \frac{(a-1)}{x} - \frac{(a+b)}{(1+x)} = 0$$

$$\frac{(a-1)}{x} - \frac{(a+b)}{(1+x)} = 0$$

$$\Rightarrow \frac{(a-1)(1+x)(a+b)x}{x(1+x)} = 0$$

$$(a-1)(1+x)(a+b)x = 0$$

$$(a-1) + (a-1)x - ax - bx = 0$$

$$a - 1 + ax - x - ax - bx = 0$$

$$a - 1 - x(1 + b) = 0$$

$$a - 1 = x(b + 1)$$

$$x = \frac{a - 1}{b + 1}$$

$$x = \frac{\frac{\alpha}{2} + r - 1}{\frac{\beta}{2} + 1}$$

$$\Rightarrow x = \frac{(\frac{\alpha}{2} + r - 1) * 2}{\left(\frac{\beta}{2} + 1\right) * 2}$$

$$x = \frac{\alpha + r - 1}{\beta + 1}$$

$$f''(x) = f(x)\left[\frac{-(a-1)}{x^2} - \frac{(\alpha+\beta)}{(1-x)^2}\right] + f'(x)\left[\frac{(a-1)}{x} - \frac{(\alpha+\beta)}{(1-x)}\right]$$

$$f''(x) = f(x)\left[\frac{-(a-1)}{x^2} - \frac{(\alpha+\beta)}{(1-x)^2}\right] + f(x)\left[\frac{(a-1)}{x} - \frac{(\alpha+\beta)}{(1-x)}\right]^2$$

$$f''(x) = f(x)\left[\left[\frac{-(a-1)}{x^2} - \frac{(\alpha+\beta)}{(1-x)^2}\right] + \left[\frac{(a-1)}{x} - \frac{(\alpha+\beta)}{(1-x)}\right]^2\right]$$

$$f''(x) - f(x)\left[\left[\frac{-(a-1)}{\left[\frac{a-1}{a+b-2}\right]^2} - \frac{(\alpha+\beta)}{\left[1-\left(\frac{a-1}{a+b-2}\right)\right]^2}\right] + \left[\frac{(a-1)}{\left[\frac{a-1}{a+b-2}\right]} - \frac{(\alpha+\beta)}{\left[1-\left(\frac{a-1}{a+b-2}\right)\right]}\right]\right]^2 - f(x) < 0$$

Thus, $f''(x)$ has an absolute limit at $x = \frac{a-1}{a+b}$ where it is noticed that $f''(x) < 0$ is negative

3.2.7. inversion points

$$f''(x) = f(x)\left[\left[\frac{(a-1)}{x} - \frac{\alpha+\beta}{(1-x)}\right]^2 - \left[\frac{(a-1)}{x^2} + \frac{\alpha+\beta}{(1-x)^2}\right]\right] = 0$$

$$\left[\frac{[(1+x)(a-1) - x(\alpha+\beta)]^2}{x^2(1-x)^2}\right] - \left[\frac{(a-1)(1+x)^2 + (\alpha+\beta)x^2}{x^2(1-x)^2}\right] = 0$$

$$(a-1)^2(1-x)^2 - 2x(1+x)(a+b)(a-1) + (a+1)^2X^2 - (a-1)(1-x)^2 = 0$$

$$(1-2x+X^2)(a-1)^2-2x\,(a-1)(a+b)-2X^2(a-1)(a+b)+(a+1)^2X^2 - (a-1)\,(1-2x+X^2)+(a+b)X^2=0$$

$$(a-1)^2-2x(\,a-1)^2+(a-1)X^2-2x(a-1)(a+b)-2X^2(a-1)(a+b) + (a+b)^2X^2-(a-1)-2x(a-1)-X^2(a-1)(a+b)X^2=0$$

$$X^2[(a-1)^2+2(a-1)(a+b)+(a+b)^2-(b-1)-(a-1)] + 2x\,[(a-1)-(a-1)(b-1)-(a-1)^2]+[(a-1)^2-(a-1)]=0$$

Compensation in common law

$$x_{1,2}=\frac{-b\ \pm\sqrt{b^2-4ac}}{2a}$$

$$a=[(a-1)^2+2(a-1)(b-1)+(b-1)^2-(b-1)-(a-1)]$$

$$b=2[(a-1)-(a-1)-(b-1)-(a-1)^2]$$

$$c=[(a-1)^2-(a-1)]$$

After compensation in common law a , b , c we get:

$$x_{1,2} = \frac{2[(a-1)(b-1)-(a-1)(b-1)-(a-1)^2]\pm\sqrt{\begin{matrix}4(a-1)^2[2(b-1)+(a+b)^2]-4(a-1)^2-2(a \\ (a-1)](a+b)^2+(a+b)-(a-1)][(a-1)^2\end{matrix}}}{2[(a-1)^2-(a-1)]}$$

$$x_{1,2} = \frac{2[(a-1)(b-1)-(a-1)(b-1)-(a-1)^2]\pm\sqrt{\begin{matrix}4(a-1)^2[2(b-1)+(a+b)^2]-4(a-1)^2-2(a \\ (a-1)](a+b)^2+(a+b)-(a-1)][(a-1)^2\end{matrix}}}{2[(a-1)^2-(a-1)]}$$

4. Practical application using the R . program

R software is considered one of the most important software in statistics and in several fields. Today, it is taught in different subjects in universities, and it uses statistical calculations and graphs. R is one of the languages that has recently emerged and is rapidly increasing in the statistics and bioinformatics sector. This program can be downloaded from the following website:

http://www.r-project.org

4.1.Using the R program on beta distribution applications

The R program can be used in beta applications and distributions, as well as other distributions. But before we start with these applications, we must first know the four functions, and their general form in the R program, then we do some applications on them and summarize the results we get through these applications, and the following table shows the general form of the four functions of a beta distribution of the first type

Table (1,3) shows the general form of the four functions of the beta distribution using the R . program

function form	Her job
dbeta ( )	It is used to calculate the probability density function of a beta distribution at all values of the variable
pbeta ( )	It is used to calculate the distributive function
rbeta ( )	The random distribution function of the beta distribution.
qbeta ( )	It can be used to calculate the hashes of the probability density function of a beta distribution at all probability values

The use of these four functions can be illustrated using the R program for the central beta distribution, through the following commands:

```
dbeta(x, shape1, shape2, ncp = 0, log = FALSE)        #        Density function
pbeta(q, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE)   #   distributive function
qbeta(p, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE)   #   Hash function
rbeta(n, shape1, shape2, ncp = 0)        #        random generation
```

Table (2.3) explains the commands for applying the second type beta distribution using the R . program:

Implementation of a second type beta distribution using the R . program	
Package " VGAM"	The name of the library used
https://www.stat.auckland.ac.nz/~yee/VGAM	Library website on search engine
We express the distributive function of the second type beta distribution in R like this	
dbetapr(x, shape1, shape2, scale = 1, log = FALSE)	
We express the distributive function of the second type beta distribution in R like this	
pbetapr(q, shape1, shape2 , lower.tail = TRUE, scale=1, log.p = FALSE)	
Generate a random sample from the second type beta distribution with a sample size of n and parameter value .... and parameter value	
rbeta(n, df1, df2, ncp1)	
It can be used to calculate the hashes of the probability density function of a beta distribution at all probability values p	

qbetapr(p, shape1, shape2 , lower.tail = TRUE, log.p = FALSE)

Table (3.3) Application of decentralized beta distribution commands of the first type using the R . program

Application of decentralized beta distribution of the first type using the R . program	
Package'sadists'	The name of the library used
https://github.com/shabbychef/sadists/issues	Library website on search engine
We express the probability density function of the eccentric beta distribution of the first type in R like this	
dbeta(x, df1, df2, ncp1, log = FALSE, order .max=6)	
We express the distributive function of the eccentric beta distribution of the first type in R like this	
pbeta(q, df1, df2, ncp1, lower.tail = TRUE, log.p = FALSE, order .max=6)	
Generate a random sample from the first type beta distribution with a sample size of n and parameter value .... and parameter value	
rbeta(n, df1, df2, ncp1)	
It can be used to calculate the hashes of the probability density function of the eccentric beta distribution at all probability values p	
qdnbeta (p, df1, df2, ncp1, lower.tail = TRUE, log.p = FALSE, order .max=6)	

4.2. Plotting a type 1 central beta distribution using the R . program.

There is a command from the R program that can be used to obtain a graph of the density function and the distributive function of a beta distribution according to the parameters to be drawn.

4.2.1. Steps for drawing a beta distribution:

To draw the density function for a beta distribution ($\alpha = 7 , \beta = 3$) Using the R program, we follow the following steps as shown in the following table:

Table (3.4) steps for drawing the probability density function for a beta distribution of the first type using the R . program.

n	The form
1	x<-seq(0,1,by=0.01) Generate values between zero and one
2	y<-dbeta(x,7,3) Calculate the probability distribution values for the beta distribution at the generated values
3	plot(x ,y ,type="l" ,col=2) Plot the observation values with the corresponding density function values
4	legend("topleft",legend=c("alpha=7","beta=3"),fill=c(1,2)) Explanation of the values of the features on the drawing

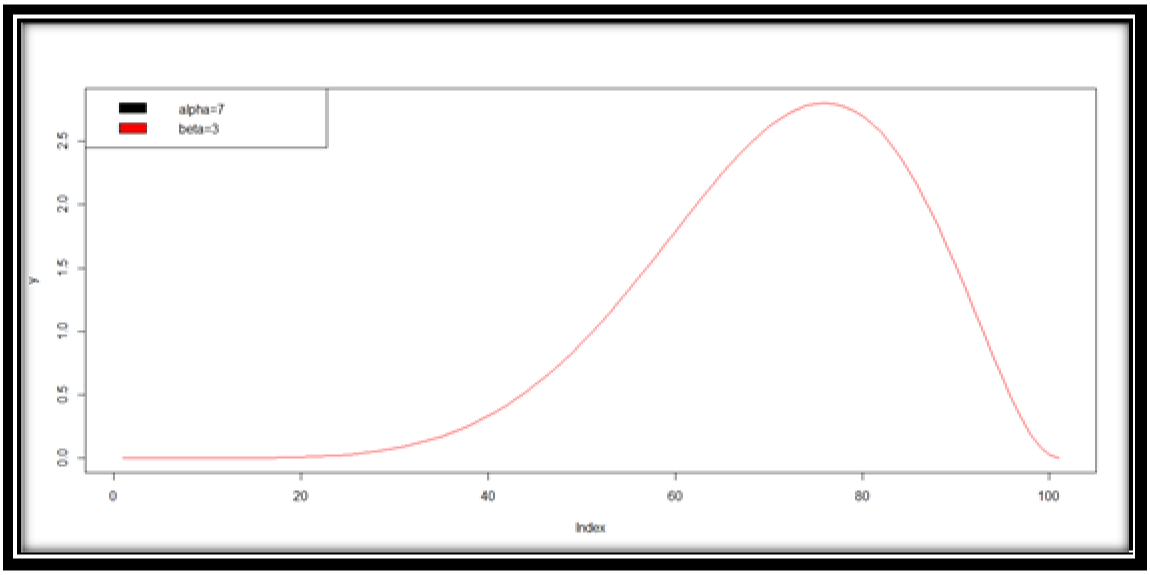

Figure (4.1) shows a graph of the density function of the central beta distribution of the first type

4.3. Plotting a type 1 decentralized beta distribution using the R . program.

There is a command from the R program that can be used to obtain a graph of the density function and the distributive function of the first type decentralized beta distribution according to the steps and general formulas for these commands, as follows:

4.3.1. Steps for decentralized beta distribution:

To draw the density function for a beta distribution ($\alpha = 7$, $\beta = 3$) Using the R program, we follow the following steps as shown in the following table:

Table (4.5) steps for drawing the probability density function for decentralized beta distribution of the first type using the R . program.

n	The form
1	x<-seq(0,1,by=0.01) Generate values between zero and one
2	y<-dbeta(x,7,3) Calculate the probability distribution values for the beta distribution at the generated values
3	plot(x ,y ,type="l" ,col=2) Plot the observation values with the corresponding density function values
4	legend("topleft",legend=c("alpha=7","beta=3"),fill=c(1,2)) Explanation of the values of the features on the drawing

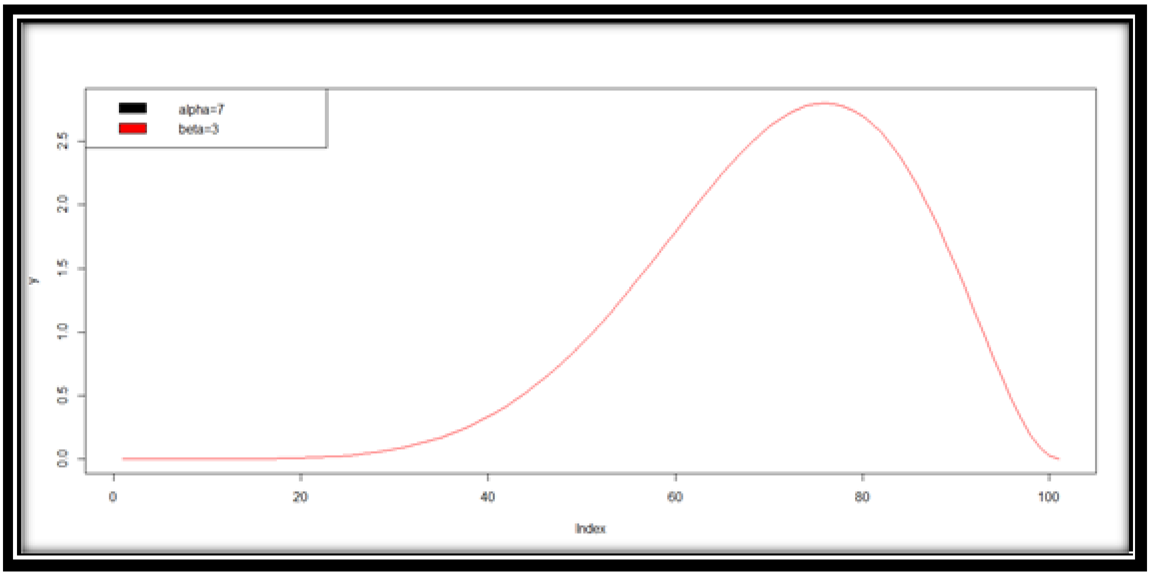

Figure (4.2) shows a graph of the density function of the central decentralized beta distribution of the first type

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