

Reliability Comparison of the Shafts when Shear Stress follow the Different Distributions

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Abstract

Torsion refers to twisting of a straight member under the action of a turning moment or torque which tends to produce a rotation about the longitudinal axis. In workshops and factories, a turning force is always applied to transmit energy by rotation. This turning force is applied to the rim of a pulley, keyed to the shaft or at any other suitable point at some distance from the axis of the shaft. This paper deals with the reliability analysis of the shafts subjected to torsion, when shear stress follows the different distributions. The reliability of a shaft subjected to torsion has been compared with Lindley, Weibull, Exponential and Rayleigh distributions.

Keywords: Reliability, Shear stress, Lindley distribution, Weibull distribution, Rayleigh distribution, Exponential distribution, Total torque, Modulus of rigidity, Circular solid shaft, Hollow circular shaft.

1. Introduction

Reliability of a system is the probability that a system perform its intended purpose for a given period of time under stated environment conditions. In some cases system failures occur due to certain type of stresses acting on them.

A shaft of circular section is said to be in pure torsion when it is subject to equal and opposite end couples whose axes are coincide with the axes of the shaft. Since all sections of the shaft are identical and subject to the same torque then the shaft is in pure torsion. From symmetrical considerations, it may be realized that cross-sections of the shaft do not change in their shape as they turn about the longitudinal axis.

Anil Misra et al. [1] discussed with the view of developing methods for reliability based design, the finite difference technique was combined with the Monte-Carlo simulation method to create a probabilistic load–displacement analysis. The Monte-Carlo simulation method was used, in lieu of other closed-form probabilistic techniques, due to the complexity of the load–displacement analysis. Dr. Edward E. Osakue et al. [2] studied fatigue shaft design verification for bending and torsion. Dr. Edward E. Osakue et al. [3] studied probabilistic fatigue design of shaft for bending and torsion. Dr. R. K. Bansal [4] and R. S. Khurmi [9] discussed shear stress produced in a circular shaft subjected to torsion, torque transmitted by a circular solid shaft and a hollow circular shaft. E. Balagurusamy [5] discussed stress dependent hazard models. K. C. Kapur and L. R. Lamberson [6] discussed design of a shaft subjected to torsion, when a shaft is subjected to a torque a shearing stress is produced in the shaft. M. E. Ghitany et al. [7] studied the Lindley distribution and outlined that its mathematical properties are more flexible than those of the exponential distribution. P. Hari Prasad et al. [8] studied reliability analysis of symmetrical columns with eccentric loading from Lindley distribution. T. S. Uma Maheswari

et al. [10] obtained reliability analysis of unsymmetrical columns subjected to eccentric load for stress follows exponential distribution.

2. Statistical model

The probability of failure as function of time can be defined by

$$F(t) = P(t' \leq t), \quad t \geq 0.$$

where t' is a random variable denoting the time to failure.

Reliability function is

$$R(t) = 1 - F(t) = P(t' \geq t), \quad t \geq 0.$$

The failure rate of the almost all components is stress dependent. For such cases, a power function model is defined [9] as given below

$$Z(t) = h(t) \sigma_1^a \sigma_2^b$$

where $Z(t)$ is the failure rate at rated stress conditions, $h(t)$ is hazard function, σ_1 and σ_2 are stress ratios for two different kind of stresses and a, b are positive constants.

3. Lindley, Weibull, Exponential, and Rayleigh Distributions

The Lindley distribution was introduced by D.V. Lindley in the context of Bayesian inference in 1958. The Weibull distribution is a continuous probability distribution. It is named after Swedish mathematician Waloddi Weibull who described it in detail in 1951. The Weibull variate is commonly used as a lifetime distribution in reliability engineering. The exponential distribution is the probability distribution of the time between events in a process in which events occur continuously and independently at a constant average rate. Among the probability distributions, the Rayleigh distribution is one of the most commonly used distributions. The Rayleigh distribution introduced by Lord Rayleigh in 1880 and it has appeared as a special case of the Weibull distribution.

The hazard rate function is a way to model data distribution in survival analysis. This paper considered the hazard rate function of Lindley, Weibull, Exponential, and Rayleigh Distributions.

The hazard rate function $h(t)$	
Lindley distribution	$h(t) = \frac{\lambda^2(1+t)}{1+\lambda(1+t)}$
Weibull distribution	$h(t) = \frac{k}{\lambda^k} t^{k-1}$
Exponential distribution	$h(t) = \frac{1}{\lambda}$
Rayleigh distribution	$h(t) = \frac{t}{\lambda^2}$

4. Torsion of shafts

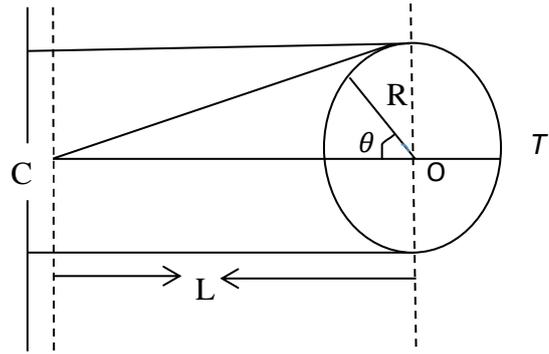
A shaft is in torsion, when equal and opposite torques is applied at the two ends of a shaft. The torque is the product of the force applied (tangentially to the ends of shaft) and radius of the shaft. Due to the application of the torques at the two ends, the shaft is subjected to a twisting moment. This causes the shear stresses and shear strains in the material of the shaft.

4.1 Shear stress produced in a circular shaft subjected to torsion

The maximum shear stress (τ) induced in a shaft subjected to twisting moment [4] is given by

$$\frac{\tau}{R} = \frac{C \times \theta}{L} \Rightarrow \tau = \frac{R \times C \times \theta}{L}$$

where radius of the shaft R , modulus of rigidity C , angle of twisting moment θ and length of the shaft L .



The reliability function for shear stress is

$$R(t) = e^{-\int_0^t Z(t)dt}$$

where $Z(t) = h(t) \times \tau$.

The reliability function $R(t)$	
Lindley distribution	$R(t) = \left[e^{-\frac{\lambda \times t \times R \times C \times \theta}{L}} \right] \left[\frac{1 + \lambda + \lambda t}{1 + \lambda} \right]^{\frac{R \times C \times \theta}{L}}$
Weibull distribution	$R(t) = \exp \left[- \left(\frac{t}{\lambda} \right)^k \times \frac{R \times C \times \theta}{L} \right]$
Exponential distribution	$R(t) = \exp \left[\frac{-t \times R \times C \times \theta}{\lambda \times L} \right]$
Rayleigh distribution	$R(t) = \exp \left[\frac{-t^2 \times R \times C \times \theta}{2\lambda^2 \times L} \right]$

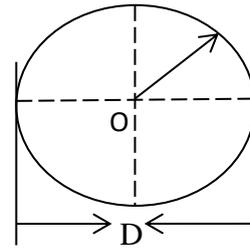
4.2 Torque transmitted by a circular solid shaft

The maximum torque transmitted by a circular solid shaft is obtained from the maximum shear stress induced at the outer surface of the solid shaft.

The torque (T) transmitted by a solid shaft [4] is given by

$$T = \frac{\pi}{16} \times \tau \times D^3 \Rightarrow \tau = \frac{16T}{\pi \times D^3}$$

where maximum shear stress τ and diameter of circular solid shaft D .



The reliability function for the circular solid shaft is

$$R(t) = e^{-\int_0^t Z(t) dt}$$

where $Z(t) = h(t) \times \tau$

The reliability function $R(t)$	
Lindley distribution	$R(t) = \left[e^{-\frac{\lambda \times t \times 16T}{\pi \times D^3}} \right] \left[\frac{1 + \lambda + \lambda t}{1 + \lambda} \right]^{\frac{16T}{\pi \times D^3}}$
Weibull distribution	$R(t) = \exp \left[-\left(\frac{t}{\lambda} \right)^k \times \frac{16T}{\pi D^3} \right]$
Exponential distribution	$R(t) = \exp \left[\frac{-t \times 16T}{\lambda \times \pi D^3} \right]$
Rayleigh distribution	$R(t) = \exp \left[\frac{-t^2 \times 16T}{2\lambda^2 \times \pi D^3} \right]$

4.3 Torque transmitted by a hollow circular shaft

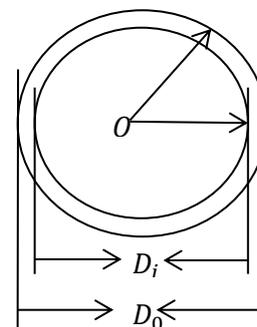
Torque transmitted by a hollow circular shaft is obtained in the same way as for a solid shaft.

The torque (T) transmitted by hollow circular shaft [4] is given by

$$T = \frac{\pi \times \tau}{16} \left[\frac{D_0^4 - D_i^4}{D_0} \right]$$

$$\Rightarrow \tau = \frac{16T \times D_0}{\pi [D_0^4 - D_i^4]}$$

where maximum shear stress τ , external diameter of hollow circular shaft D_0 and internal diameter of hollow circular shaft D_i .



The reliability function for hollow circular shaft is

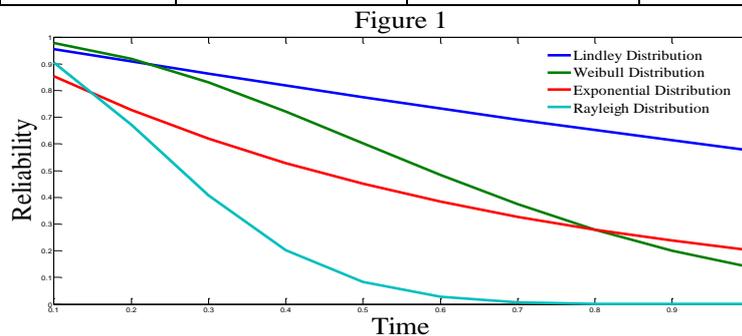
$$R(t) = e^{-\int_0^t Z(t) dt}$$

where $Z(t) = h(t) \times \tau$

The reliability function R(t)	
Lindley distribution	$R(t) = \left[e^{-\frac{\lambda \times t \times 16T \times D_0}{\pi \times [D_0^4 - D_i^4]}} \right] \left[\frac{1 + \lambda + \lambda t}{1 + \lambda} \right]^{\frac{16T \times D_0}{\pi \times [D_0^4 - D_i^4]}}$
Weibull distribution	$R(t) = \exp \left[-\left(\frac{t}{\lambda}\right)^k \times \frac{16T \times D_0}{\pi(D_0^4 - D_i^4)} \right]$
Exponential distribution	$R(t) = \exp \left[\frac{-t \times 16T \times D_0}{\lambda \times \pi(D_0^4 - D_i^4)} \right]$
Rayleigh distribution	$R(t) = \exp \left[\frac{-t^2 \times 16T \times D_0}{2\lambda^2 \times \pi(D_0^4 - D_i^4)} \right]$

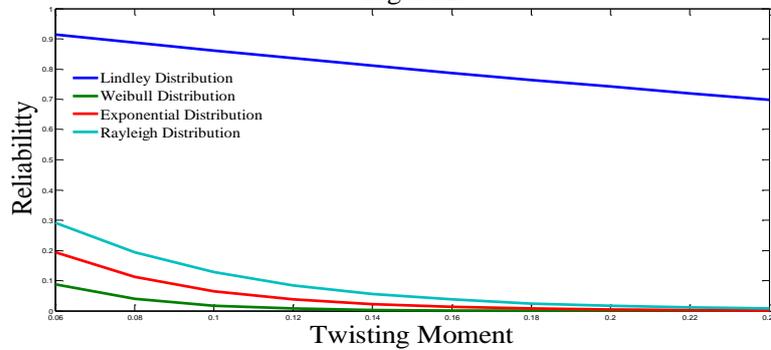
5. Numerical results and discussion for reliability

Table 1				
$\lambda=0.8, R=16 \text{ mm}, C=73.1 \text{ GPa}, \theta=0.0698 \text{ radian}, L=64 \text{ mm}.$				
t	Lindley	Weibull	Exponential	Rayleigh
0.1	0.954489	0.978669741	0.85261313	0.905148863
0.2	0.908947	0.919418557	0.726949149	0.671243419
0.3	0.863738	0.83014411	0.619806389	0.407830866
0.4	0.819168	0.720820864	0.528455065	0.203011277
0.5	0.775487	0.602172759	0.450567727	0.082794265
0.6	0.732897	0.484146672	0.38415996	0.027664343
0.7	0.691559	0.374722548	0.327539826	0.007573219
0.8	0.651596	0.279264756	0.279264756	0.001698559
0.9	0.613101	0.200437429	0.238104798	0.000312119
1.0	0.576137	0.138570467	0.203011277	4.69895E-05



\square	Lindley	Weibull	Exponential	Rayleigh
0.06	0.914235	0.088071992	0.193060822	0.291253138
0.08	0.887313	0.039184751	0.111581501	0.193060822
0.10	0.861185	0.017433973	0.064489684	0.127972805
0.12	0.835826	0.007756676	0.037272481	0.08482839
0.14	0.811213	0.003451079	0.021542016	0.05622957
0.16	0.787325	0.001535445	0.012450431	0.037272481
0.18	0.764141	0.000683146	0.007195856	0.024706535
0.20	0.741640	0.000303943	0.004158919	0.016377039
0.22	0.719801	0.00013523	0.002403691	0.010855727
0.24	0.698605	6.0166E-05	0.001389238	0.007195856

Figure 2



\square	Lindley	Weibull	Exponential	Rayleigh
0.1	0.987582	1.67951E-33	3.69938E-05	1.87291E-18
0.2	0.956535	3.8795E-09	0.006082254	3.69938E-05
0.3	0.913900	0.000159934	0.03332036	0.010722093
0.4	0.864497	0.006940171	0.077988804	0.077988804
0.5	0.811637	0.040436607	0.129904982	0.195389241
0.6	0.757588	0.106118600	0.182538653	0.321788036
0.7	0.703896	0.190577771	0.232742437	0.434725
0.8	0.651596	0.279264756	0.279264756	0.528455065
0.9	0.601364	0.363355056	0.321788036	0.604146376
1.0	0.553621	0.438982402	0.360423338	0.664852279

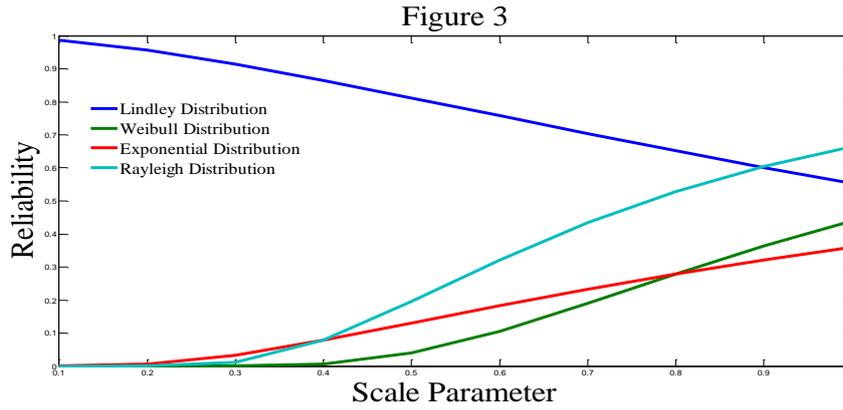


Table 4
 $\lambda=0.8, t=0.9, R=6 \text{ mm}, L=32 \text{ mm}, \theta=0.0698 \text{ radian.}$

C	Lindley	Weibull	Exponential	Rayleigh
4.2	0.979139	0.933084794	0.940034749	0.965813893
16.2	0.921904	0.765563875	0.787793178	0.874443737
28.2	0.868014	0.628118742	0.660207607	0.791717591
40.2	0.817274	0.515349753	0.553284918	0.716817696
52.2	0.769500	0.422826689	0.463678694	0.649003653
64.2	0.724519	0.346914708	0.388584478	0.587605111
76.2	0.682167	0.284631547	0.325652005	0.532015136
88.2	0.642291	0.233530362	0.272911643	0.481684211
100.2	0.604746	0.191603604	0.228712748	0.436114809
112.2	0.569396	0.157204145	0.191672003	0.394856469

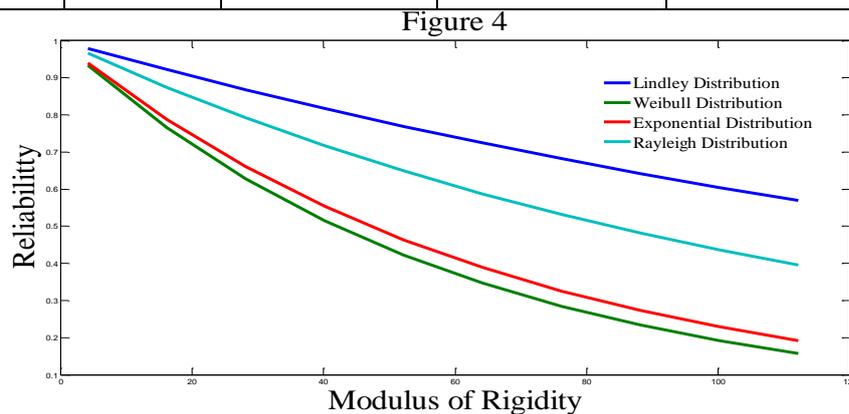


Table 5				
$\lambda=0.8, t=0.6, R= 4 \text{ mm}, C=73.1 \text{ GPa}, \theta=0.0698 \text{ radian.}$				
L	Lindley	Weibull	Exponential	Rayleigh
55	0.913566	0.80976114	0.757060857	0.900894731
50	0.905344	0.79285291	0.736281471	0.89154127
45	0.895396	0.772666005	0.711657397	0.88024104
40	0.883115	0.748153618	0.682032137	0.866316956
35	0.867572	0.717776877	0.64574741	0.848737718
30	0.847272	0.679184408	0.600352678	0.82585237
25	0.819649	0.628615736	0.542110404	0.794845836
20	0.779892	0.559733836	0.465167836	0.750505068
15	0.717871	0.46129146	0.360423338	0.682032137
10	0.608232	0.313301967	0.216381116	0.563257857

Figure 5

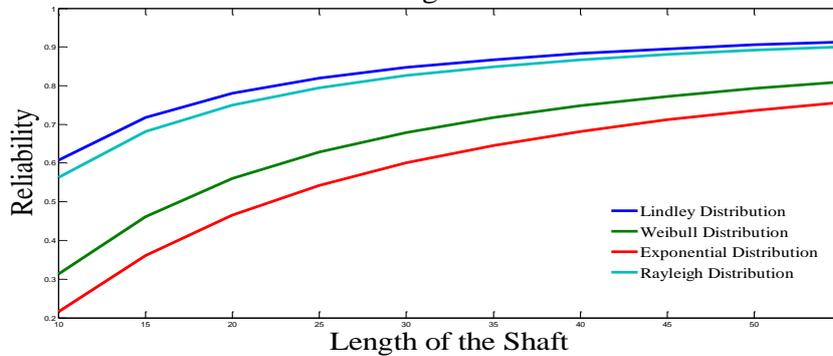


Table 6				
$\lambda=0.2, \pi=3.1415926, t=0.4, D=198 \text{ mm.}$				
T	Lindley	Weibull	Exponential	Rayleigh
98900000	0.366675	1.554E-110	4.34673E-57	4.34673E-57
88900000	0.405824	1.9698E-99	2.17287E-51	2.17287E-51
78900000	0.449152	2.49698E-88	1.08619E-45	1.08619E-45
68900000	0.497108	3.16523E-77	5.42971E-40	5.42971E-40
58900000	0.550183	4.0123E-66	2.71424E-34	2.71424E-34
48900000	0.608924	5.08608E-55	1.35681E-28	1.35681E-28
38900000	0.673938	6.44722E-44	6.78251E-23	6.78251E-23
28900000	0.745893	8.17262E-33	3.39048E-17	3.39048E-17
18900000	0.825531	1.03598E-21	1.69486E-11	1.69486E-11
8900000	0.913671	1.31323E-10	8.47236E-06	8.47236E-06

Figure 6

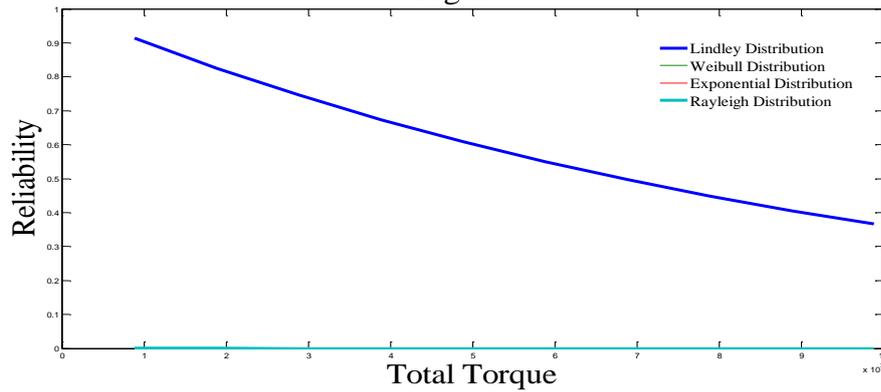


Table 7

$\square\square\square\square\square$, $t=0.8$, $T=982000$, $\pi=3.1415926$, $D_i = 142$ m.

D_0	Lindley	Weibull	Exponential	Rayleigh
240	0.954026	0.20056979	0.438400764	0.438400764
230	0.946583	0.153513793	0.382182624	0.382182624
220	0.937185	0.109202347	0.32088532	0.32088532
210	0.925022	0.069913529	0.255237078	0.255237078
200	0.908761	0.038162868	0.187065524	0.187065524
190	0.886057	0.016089566	0.120078851	0.120078851
180	0.852341	0.004279575	0.060849983	0.060849983
170	0.797379	0.000439784	0.018926206	0.018926206
160	0.692692	3.60317E-06	0.001607273	0.001607273
150	0.423490	1.82778E-13	2.89656E-07	2.89656E-07

Figure 7

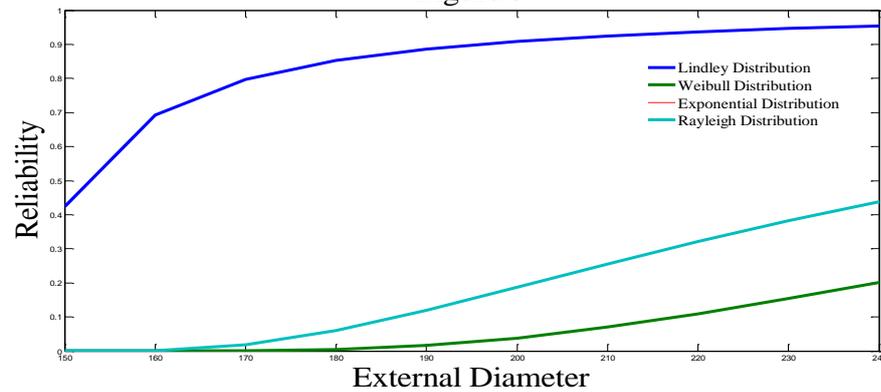
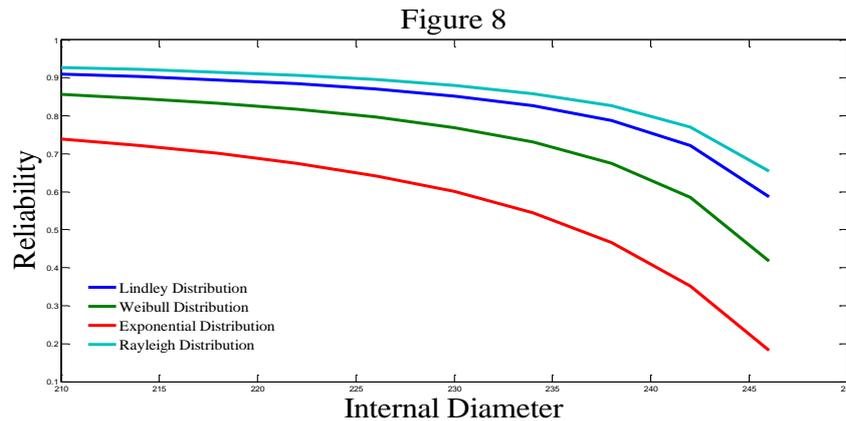


Table 8

$\square\square\square\square\square$, $t=0.4$, $T=982000$, $\pi=3.1415926$, $D_0 = 252$ m.

D_i	Lindley	Weibull	Exponential	Rayleigh
246	0.587534	0.417768345	0.182589922	0.653686025
242	0.721213	0.584861816	0.351685826	0.770085093
238	0.787328	0.675414706	0.465538792	0.826016968
234	0.826547	0.731507998	0.543827213	0.858747002

230	0.852432	0.769482159	0.600182156	0.880178528
226	0.870753	0.796810303	0.642409481	0.895267843
222	0.884371	0.817365639	0.675091594	0.906443367
218	0.894869	0.833349504	0.701050479	0.915034192
214	0.903189	0.846103613	0.722105835	0.921828675
210	0.909929	0.856491662	0.739479147	0.927323964



6. Conclusion

The reliability of a circular shaft subjected to torsion when shear stress follow Lindley, Weibull, Exponential and Rayleigh distributions has been derived. The reliability of torque transmitted by a circular solid shaft and a hollow circular shaft has been derived. It is observed that the reliability of the shaft decreases when time and modulus of rigidity increases and the reliability decreases when external diameter and length of the shaft decreases. Reliability of the shaft does not affect the total torque and twisting moment for the Weibull, Exponential and Rayleigh distributions.

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