A New M-General Model in Constrained Optimization

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Abstract
The idea of this study, we introduce two technique, first technique general n-ordered for objective function in conic model according to expansion of Taylor , the second technique, anew secant equation so is modification of the scaled BFGS method. A wonderful characteristic of the proposed method is that it possesses a globally convergent despite the absence of a convexity postulate on the goal function. And The numerical results proved the efficiency of the new technology compared with the classical method.
Keywords: conic model , BFGS method , globally convergent

1. Introduction

Davidon [7] was the first to propose using conic models for optimization. Sorensen [6] soon after developed an algorithm for updating collinear scalings, the superlinear rate of convergence he was able to achieve. There is a strong connection between this work and those by Grandinetti [16] and Ariyawansa [15]. A solution to the issue of developing algorithms that minimise a conic objective function in a finite number of steps was then addressed . Gourgeon and Nocedal [13], and others considered this problem and developed conjugate gradient analogues to solve it.

We describe a few basic properties of conic functions in constrained optimization problems and how they are defined.

$$\min Q(x) \qquad \dots (1)$$

s.t

$e_i \leq 0$ for i=1,2,……,p

$c_j = 0$ for j=1,2,……,m

$Q : R^n \to R$ Smooth function e_i is inequality constrained and c_j equality constrained [14].

As mentioned in the previous research [1], it is possible to construct an unconstrained objective function as follows

$$\Phi(x,\sigma) = Q(x) + \sigma \mathrm{E}(x) \qquad \dots (2)$$

Where $\mu \to 0$ and [7] defines E (x):

$$\mathrm{E}(x) = \sum_{l=1}^{m} \frac{1}{e_l(x)} \quad \dots (3)$$

we derivative the functions we get :

$$\nabla\Phi(x,\sigma) = \nabla(Q(x)) + \sigma \sum_{l=1}^{m} \left(\frac{-1}{e_l^2(x)}\right) \nabla e_l(x) \quad \dots (5)$$

Now we'll look at the second half, which is an unconstrained optimization method that will help us apprehend in problem (2), where Φ: $R^n \to R$ a continuous real-valued and accessible derivation function. It's an iterative process.[1]

we will write any $x_0 \in R^n$ as $x = x_0 + s$. The conic function is defined in light of this vantage position.

$$Q(x) = Q(x_0 + s) = Q_0 + \frac{g_0 s}{1-a^T s} + \frac{1}{2}\frac{s^T \mathrm{A} s}{(1-a^T s)^2} \quad (6)$$

We can consider that $g(x_{k+1}) = \nabla\Phi(x_{k+1}, \sigma_{k+1})$ (7)

Such that $g_0, a \in \mathbf{R}^n$ and A is an $n \times n$ a matrix with a positive definiteness and symmetry. The horizon vector is called a and the domain of Q is called D, i.e., $D = \{x: 1 - a^T s \neq 0\}$. Since the term $s/(1 - a^T s)$ it is evident that by letting it appear in the second term on the right-hand side of (6), and twice in the third term

$$w = \frac{s}{1-a^T s} \quad (8)$$

the conic function becomes a quadratic in the variable w,

$$Q(x) = Q(x_0 + s) = Q_0 + g_0^T w + \frac{1}{2} w^T \mathrm{A} w \equiv h(w) \quad (9)$$

We can say that s by using the term :

$$s = \frac{w}{1+a^T w} \quad (10)$$

To simplify the formulas we define

$$\gamma(x) = 1 - a^T s = \frac{1}{1+a^T w} \quad (11)$$

so that $w = s/\gamma$. We call $H = \{x: 1 - a^T s = 0\}$ It's important to remember that if $\gamma(x)\gamma(y) < 0$, then x and y are on opposite sides of H the single hyperplane, and vice versa. We'll need to figure out how to relate the derivative of Q to the derivative of h. Since

$$Q(x) = Q(x_0 + s) = Q\left(x_0 + \frac{w}{1+a^T w}\right) = h(w) \quad (12)$$

it follows from the chain rule that

$$h'(w) = \gamma(x)(I - as^T)g(x) \quad (13)$$

so that gradient of Q is denoted by g [18]. As an example function for minimizing, the conic (7) is most beneficial if it has only one minimizer. The conditions [13] guarantee this.

$$A > 0 \text{ and } a^T \mathrm{A}^{-1} g_0 \neq 1 \quad (14)$$

It will be called a normal conic function if it holds. With Broyden [10], Fletcher [11], Goldfarb (12) and Shanno (13), we have a well-known quasi-Newtonian BFGS approach. The BFGS approach is quick and efficient, and it is now utilised to solve unconstrained and constrained optimization problems in a variety of optimization tools. For small and medium-sized unconstrained optimization problems, the BFGS approach proven to be one of the most efficient quasi-Newton methods. Dennis and Moré [20, 1] provided an outstanding exposition of the theoretical features of this method's characteristics and convergence. where approximation to the hessian of function positive definite and symmetric

So, the search for BFGS direction is achieved as a solution of the linear algebraic system.

$$d_k = -B^{-1}{}_k\, g_k \qquad (15)$$

Where g_k is the gradient . In (14) the matrix B_k is the BFGS approximation to the Hessian $\nabla^2 Q(x_k)$ of Q at x_k, being updated by the classical formula:

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k}, \qquad (16)$$

$k = 0,1,\ldots$, where $s_k = x_{k+1} - x_k$ and$y_k = g_{k+1} - g_k$. An important property of the BFGS updating formula (16), which we call standard$BFGS$, is that B_{k+1} inherits the positive definiteness of B_k if $y_k^T s_k > 0$. The condition $y_k^T s_k > 0$ holds

The Line searches are frequently use to ensure the global convergence of nonlinear constrained optimization . The so-called strong Wolfe line search is use here, in which the step length α_k in (6) is determined in such a way that [23,22]

$$Q_{k+1} - Q_k \le \delta \alpha_k g_k^T d_k \qquad (17)$$

$$|g_{k+1}^T d_k| \le -\sigma g_k^T d_k \qquad (18)$$

where the positive constants σ and δ satisfy $0 < \sigma < \delta < 1$. We note that the condition $y_k^T s_k > 0$ is also guaranteed to hold if the stepsize α_k is determined by the exact line search: $\min\{Q(x_k + \alpha d_k), \alpha > 0\}$. Since B_k is positive definite, the search direction d_k generated by (15) is a descent direction of Q at x_k, no matter whether the Hessian is positive definite or not.

Then we'll go through the next two qualities, which will be crucial in our subsequent study. If d_k holds for each search direction, we say the descent condition holds.

$$g_k^T d_k < 0 \quad \forall\, k \geqslant 0 \qquad (19)$$

adding, the sufficient descent condition holds if $\exists\, c > 0$ constant so all direction d_k, obtain

$$g_k^T d_k \leqslant -c\|g_k\|^2 \text{ for all } k \geqslant 0. \qquad (20)$$

2. The Generalization conic model

We have derived a general formula for the conic model, where we expanded the conic model and expanded the space to degree n, and this reduces the cutting error, which makes us obtain more accurate results. This model has several cases where for the time being, we assume that f is sufficiently smooth. We were able to identify a unique scalar for ξ. using the n-order Taylor expansion for the objective conic function around the iterate:

The general form of the conic model is:

$$Q_{k+1} = Q_k + \frac{g^T s}{(1-a^T s)} + \frac{1}{2!}\frac{s^T G s}{(1-a^T s)^2} + \frac{1}{3!}\frac{s^T (Cs) s}{(1-a^T s)^3} + \frac{1}{4!}\frac{s^T (F(s)s) s}{(1-a^T s)^4} + \frac{1}{5!}\frac{s^T ((U(s)s)s) s}{(1-a^T s)^5} + \ldots \ldots \frac{1}{(n-1)!}\frac{s^T (V^{n-1}) s}{(1-a^T s)^{n-1}} \qquad (21)$$

Where $s = x_{k+1} - x, T_{k+1} \in \mathbb{R}^{n\times n\times n}, V_{k+1} \in \mathbb{R}^{n\times n\times n\times n}$ and $U_{k+1} \in \mathbb{R}^{n\times n\times n\times n\times n}$ are symmetric and

$$s^T(C_{k+1} s)s = \sum_{i,j,l=1}^{n} \frac{\partial^3 Q(x_{k+1})}{\partial x^i\, \partial x^j\, \partial x^l} s^i s^j s^l$$

$$s^T((F_{k+1} s)s)s = \sum_{i.j,l,m=1}^{n} \frac{\partial^4 Q(x_{k+1})}{\partial x^i\, \partial x^j\, \partial x^l\, \partial x^m} s^i s^j s^l s^m$$

$$s^T(((U_{k+1} s)s)s)\, s = \sum_{i.j,l,m,n=1}^{n} \frac{\partial^5 Q(x_{k+1})}{\partial x^i\, \partial x^j\, \partial x^l\, \partial x^m\, \partial x^n} s^i s^j s^l s^m s^n$$

Now determined the derivative of eq . (21) then multiplying with s we get :

$$s^T g_{k+1} = s^T g_k + \frac{s^T G s}{(1-a^T s)} + \frac{1}{2!}\frac{s^T (C(s)s)s}{(1-a^T s)^2} + \frac{1}{3!}\frac{s^T (F(s)s)s}{(1-a^T s)^3} + \frac{1}{4!}\frac{s^T U(s)sssS}{(1-a^T s)^4} + \ldots \qquad (22)$$

next mathematical operations and abbreviations we get:

$$s^T G s = (1-a^T s)s^T g_{k+1} - (1-a^T s)s^T g_k - \frac{1}{2!}\frac{s^T (Cs)ss}{(1-a^T s)} - \frac{1}{3!}\frac{s^T (F(s)sss)s}{(1-a^T s)^2} \ldots \qquad (23)$$

$$s^T G s = 2(1-a^T s)^2 (Q_{k+1} - Q_k) - 2(1-a^T s) g_K^T s - \frac{1}{3}\frac{s^T (Cs)s}{(1-a^T s)} - \frac{1}{12(1-a^T s)^2} s^T F(s)s)s + \ldots . \qquad (24)$$

Now by Multiply 4 by ε

$$\varepsilon s^T G s = 2\varepsilon(1-a^T s)^2 (Q_{k+1} - Q_k) - 2\varepsilon(1-a^T s) g_K^T s - \frac{s}{3}\frac{s^T (Cs)s}{(1-a^T s)} - \frac{s}{12(1-a^T s)^2} s^T F(s)s)s + \cdots \qquad (25)$$

And Multiply 3 by $(\varepsilon - 1)$

$$(\varepsilon-1)s^T G s = (\varepsilon-1)(1-a^T s)s^T g_{k+1-} (\varepsilon-1)(1-a^T s)s^T g_k - \frac{(s-1)}{2!}\frac{s^T (Cs)ss}{(1-a^T s)} - \frac{(s-1)}{3!}\frac{s^T (F(s)sss)s}{(1-a^T s)^2} + \ldots . \qquad (26)$$

Then subtracting 26 from 25 we obtained:

$s^TGs = 2\varepsilon(1 - a^Ts)^2(Q_{k+1} - Q_{k+1}) + (-\varepsilon - 1)(1 - a^Ts)g_k^Ts - (\varepsilon - 1)(1 - a^Ts)g_{k+1}^Ts + (\frac{(s-3)}{6(1-a^Ts)})\, s^T(Cs)ss - (\frac{(s-2)}{12(1-a^Ts)^2})s^T(F(s)sss + \cdots$ (27)

The formula when $\varepsilon = n$

$S^TGS = 2n(1 - a^Ts)^2(Q_{k+1} - Q_{k+1}) - (n + 1)(1 - a^Ts)g_k^Ts - (n - 1)(1 - a^Ts)gs + (\frac{(n-3)}{6(1-a^Ts)})\, s^T(Cs)ss * (\frac{(n-2)}{12(1-a^Ts)^2})s^T(F(s)sss + \cdots$ (28)

This is suggested general formula for conic model

Locking that we can be return to all model as

for n=1 the linear model we don’t have derivative

$Q_{k+1} = Q_k$ (29)

for n=2 the quadratic we have first derivative g

$Q_{k+1} = Q + \frac{g_k^Ts}{(1-a^Ts)}$ (30)

$$s^TGs = 2(2)\lambda^2(Q_{k+1} - Q_{k+1}) - (3)\lambda g_k^Ts - (1)\lambda g_{k+1}^Ts$$

$$s^TGs = 4\lambda^2(Q_{k+1} - Q_{k+1}) - 2\,\lambda(g_k + g_{k+1})^Ts + \lambda y_k^Ts$$

and n=3 conic model we have first and second derivative g ,G

$Q_{k+1} = Q_k + \frac{g_k^Ts}{(1-a^Ts)} + \frac{1}{2!}\frac{s^TGs}{(1-a^Ts)^2}$ (31)

Then the general formula for conic model:

$s^TG_ks = 2n\lambda(1 - a^Ts)^2(Q_{k+1} - Q_{k+1}) - (n + 1)(1 - a^Ts)g_k^Ts - (n - 1)(1 - a^Ts)g_{k+1}^Ts + (\frac{(n-3)}{6(1-a^Ts)})\, s^T(Cs)ss * (\frac{(n-2)}{12(1-a^Ts)^2})s^T(F(s)sss$ (32)

3. Generalzation secant equation in constrained optimization:

Second, we develop a novel class of modified secant equations to achieve high order accuracy in approximating the goal function's second-order curvature. Then, we propose a novel SCALCG algorithm modification.

As a next step, we investigated the modified secant equation proposed by (32) B_k, The new approximation of G_k , should be taken into consideration.

$s^TB_ks = 2n\lambda^2(Q_{k+1} - Q_{k+1}) - n\,\lambda(g_k + g_{k+1})^Ts + \lambda y_k^Ts$ (33)

Let $\theta = 2\lambda(Q_{k+1} - Q_{k+1}) - (g_k + g_{k+1})^Ts$ (34)

$s^TB_ks = n\,\lambda\theta + \lambda y_k^Ts = \hat{S}y$ (35)

The suggested of new quasi-Newton equation:

$B_k s = \hat{y}$ (36)

Where

$\hat{y} = \lambda y_k^T + \frac{n\lambda\theta}{s^T u} u$ (37)

with $s^T u \neq 0$ and u ∈ $\Re^n$.

The vectors s_k, y_k, g_k, g_{k+1} For as long as the inside product $s^T u \neq 0$ is still usable you have a few alternatives for substituting the vector u since the choice $u = s^T$ invariance aspect of the QN method is not satisfied, we choose to adopt a different approach $u = y_k$ [13].

The standing by $u = y_k$ This means the Q.N. equation's can be reduced to the next formula.

$\hat{y} = \lambda y_k^T + \frac{n\lambda\theta}{s^T y_k} y_k$ (38)

Let $\varrho = 1 + \frac{n\theta}{s^T y_k}$

$B_k s = \hat{y} = \lambda\varrho y_k$ (39)

It is now possible to adapted quasi-Newton updating formulas when y_k is changed by $\hat{y}$. As a result the inverse BFGS formula given By

$H_{k+1} = (I - p_k s_k y_k^T) H_k (I - p_k y_k s_k^T) + p_k s_k s_k^T$ (40)

Then

$$\hat{H}_{k+1} = (I - p_k s_k \hat{y}_k^T)\hat{H}_k(I - p_k \hat{y}_k s_k^T) + p_k s_k s_k^T$$

Where $\hat{y} = \lambda\varrho y_k$

$\hat{H}_{k+1} = (I - p_k s_k \lambda\varrho y_k^T)\hat{H}_k(I - p_k \lambda\varrho y_k s_k^T) + p_k s_k s_k^T$ (41)

$\hat{H}_{k+1} = (I - p_k s_k y_k^T)\hat{H}_k(I - p_k \lambda s_k^T) + \frac{p_k}{\lambda\varrho} s_k s_k^T$ (42)

Where $\varrho = 1 + \frac{n\theta}{s^T y_k}$, $p = \frac{1}{s^T y_k}$

4. Properties of the modified QN method

Convergence analysis

Assumption: 1 [16]

i. The level set $\mathfrak{I} = \{x \mid Q(x) \leqslant Q(x_1)\}$ is bounded, namely, there exists a constant $B > 0$ such that $\| x \| \leqslant B$ for all $x \in \mathfrak{I}$ (43)
Denote $\bar{\mathfrak{I}}$ to be the closed convex hull

ii. In some neighborhood N of $\overline{\mathfrak{Z}}, Q$ is continuously differentiable, and its gradient is globally Lipschitz continuous, namely, there exists a constant $L > 0$ such that $\| g(x) - g(y) \| \leqslant L \| x - y \|$ for all $x, y \in N$. (44)
It is well known that the convex closure of a bounded set in $\mathbb{R}^n$ is still bounded.

As a result, when combined with Assumption i $\overline{\mathfrak{Z}}$ is a bounded convex subset in $\mathbb{R}^n$. As a result, Assumption ii holds for any function Q that meets Assumption i and has Lipschitz gradientg locally. Furthermore, we can see from (43) and (44) that there is a constant $\gamma > 0$ such that $\| g(x) \| \leqslant \gamma$ for all $x \in \mathfrak{J}$ (45)

Lemma 1.

Provided Assumptions i , ii, and the descent condition are accurate. Let α_k be found through the line search of strong Wolfe. Then there's

$|\theta_k| \leqslant \mathrm{L}\|s_k\|^2$ (46)

Wherever L is the same it is from Assumption ii.

Proof:

Because α_k derived from the S.W.C search eq. (19)

We know that $x_k \in \mathfrak{J} := \{x \mid Q(x) \leqslant Q(x_1)\}$ for $\forall\ k \geqslant 1$. (47)
From the other aspect, we know that there exists a mean value theorem. $\zeta_k \in [0,1]$ such that

$$Q_{k+1} - Q_k = g(x_k + \zeta_k(x_{k+1} - x_k))^{\mathrm{T}}(x_{k+1} - x_k) = g(x_k + \zeta_k s_k)^{\mathrm{T}} s_k. \quad (48)$$

From (47) we get :

$$x_k + \zeta_k s_k = x_k + \zeta_k(x_{k+1} - x_k) \quad (49)$$

As a result of the formulation of P_k

$$\theta = 2\lambda(Q_{k+1} - Q_{k+1}) - (g_k + g_{k+1})^T s$$
$$= 2\lambda g(x_k + \zeta_k s_k)^{\mathrm{T}} s_k - g_k^T s - g_{k+1}^T s$$

$$\|\theta\| \leq \|g_k - \lambda g(x_k + \zeta_k s_k)\|^T s_k - \|g_{k+1} - \lambda g(x_k + \zeta_k s_k)\|^T s_k$$

$$\leq [\|g_k - \lambda g(x_k + \zeta_k s_k)\| + \|g_{k+1} - \lambda g(x_k + \zeta_k s_k)\|]\|s_k\|$$

$$\leq [\|g_k - \lambda g(x_k + \zeta_k s_k)\| + \|g_{k+1} - \lambda g(x_k + \zeta_k s_k)\|]\|s_k\|$$

$$\leq [\|g_k - \lambda g(x_k + \zeta_k s_k)\| + \|g_{k+1} - \lambda g(x_k + \zeta_k s_k)\|]\|s_k\|$$

$$\leq [\lambda L\zeta_k\|s_k\| + \lambda L(1 - \zeta_k)\|s_k\|]\|s_k\|$$

$$\leq \lambda \mathrm{L}\|s_k\|^2$$

The initial inequality is derived of triangle inequality with Cauchy–Schwartz inequality, and the next inequality has derived of Assumption ii and(49). now we complete.

Corollary 1

Suppose that Assumption I and ii hold for $\hat{y}$ defined by

$$B_k s = \hat{y}$$

Where

$$\hat{y} = \lambda y_k^T + \frac{n\,\lambda\theta}{s^T y_k} y_k$$

$$\theta = 2\lambda(Q_{k+1} - Q_{k+1}) - (g_k + g_{k+1})^T s$$

We have

$\|\hat{y}\| \leq L\|s_k\|$ (50)

Proof:

Considering lemma 1 and assuptions I &ii hold and become $\lambda \in [0,1]$ we have

$\|\hat{y}_k\| = \|\lambda y_k^T + \frac{n\,\lambda\theta}{s^T y_k} y_k\|$ (51)

$\|y_k\| \leq L\|s_k\|$ (52)

$\leq \lambda\|y_k\| + \frac{n\,\lambda\|\theta\|\|y_k\|}{\|s^T y_k\|}$ (53)

By lemma $\|\theta\| \leq \lambda \mathrm{L}\|S_k\|^2$

$\leq \lambda L\|s_k\| + \frac{n\,\lambda\lambda \mathrm{L}\|s_k\|^2 L\|s_k\|}{\|s^T y_k\|}$ (54)

$\leq [\lambda L + \frac{n\,\lambda^2 \mathrm{L}^2\|s_k\|^2}{\|s^T y_k\|}]\|s_k\|$ (55)

$\leq N\|s_k\|$ (56)

Where $N = [\lambda L + \frac{n\,\lambda^2 \mathrm{L}^2\|s_k\|^2}{\|s^T y_k\|}]$

Theorem 1

If $s^T\hat{y}_k > 0 \quad \forall k$ then G is symmetric positive definite .

Proof

we have $\hat{y} = \lambda y_k^T + \frac{n\,\lambda\theta}{s^T y_k} y_k$

By multiply by s_k^T

$s_k^T y_k = \lambda s_k^T y_k + \frac{n\,\lambda\theta}{s^T y_k} s_k^T y_k$ (57)

$s_k^T y = \lambda s_k^T y_k + n\,\lambda\theta$ (58)

$\theta = 2\lambda(Q_{k+1} - Q_{k+1}) - (g_{k+1} + g_k)^T s$ (59)

$s_k^T y = \lambda s_k^T(g_{k+1} + g_k) + n\,\lambda 2\lambda(Q_{k+1} - Q_{k+1}) - (g_{k+1} + g_k)^T s$ (60)

$s_k^T y_k = 2n\lambda^2(Q_{k+1} - Q_{k+1}) + \lambda(1-n)s_k^T g_{k+1} - \lambda(1+n)s_k^T g_k^T$ (61)

By strong wolfe line search

$s_k^T y \le 2n\lambda^2\delta\alpha_k g_k^T d_k + \lambda(1-n)\alpha_k\sigma_2 g_k^T d_k \quad - \lambda(1+n)\alpha_k g_k^T d_k$ (62)

$s_k^T y \le [2n\lambda^2\delta + \lambda(1-n)\sigma_2 - \lambda(1+n)]\alpha_k g_k^T d_k$ (63)

Because $s_k^T g_k = \alpha_k g_k^T d_k < 0$

There exist constant $M < 0$

$M = 2n\lambda^2\delta + \lambda(1-n)\sigma_2 - \lambda(1+n) < 0$ (64)

$s_k^T y \le M\alpha_k g_k^T d_k \ge 0$ (65)

$s_k^T y \ge 0$ (66)

Theorem 2

Say the sufficiently Q (x) is smooth, and the $\|S_k\|$ is adequately tiny, then obtain

$s^T G_k s - s_k^T y_k = \frac{-1}{(n+1)\lambda^{n-1}} F^{n+1} + O\,\|s_k\|^{n+2}$ (67)

$s^T G_k s - s_k^T y_k = \frac{-1}{n\lambda^{n-1}} F^{n+1} + O\,\|s_k\|^{n+2}$ (68)

Proof

$$Q_{k+1} = Q_k + \frac{g_k^T s}{\lambda} + \frac{1}{2!}\frac{s^T G s}{\lambda^2} + \frac{1}{3!}\frac{s^T(Cs)s}{\lambda^3} + \frac{1}{4!}\frac{s^T(F(s)s)s}{\lambda^4} + \frac{1}{5!}\frac{s^T((U(s)s)s)s}{\lambda^5} +$$

For n=1

$s^T G_k s - s_k^T y_k = \frac{-1}{2} F^2 + O\,\|s_k\|^3$ (69)

$s^T G_k s - s_k^T y_k = -1\,F^2 + O\,\|s_k\|^3$

$F^2 = s^T G s$

n= 2

$s^T G_k s - s_k^T y_k = \frac{-1}{3\lambda}\, s^T(Cs)s + O\,\|s_k\|^4$ (70)

$$s^TG_k s - s^T_k y_k = \frac{-1}{2\lambda} s^T(Cs)s + O \|s_k\|^4$$

n=k

$$s^TG_k s - s^T_k y_k = \frac{-1}{(k+1)\lambda^{k-1}} V^{k+1} + O \|s_k\|^{k+2} \quad (71)$$

$$s^TG_k s - s^T_k y_k = \frac{-1}{k\lambda^{k-1}} V^{k+1} + O \|s_k\|^{k+2}$$

Now prove for n=k+1

$$s^TG_k s - s^T_k y_k = \frac{-1}{(k+2)\lambda^{k}} V^{k+2} + O \|s_k\|^{k+3} \quad (72)$$

$$s^TG_k s - s^T_k y_k = \frac{-1}{(k+1)\lambda^{k}} V^{k+2} + O \|s_k\|^{k+3}$$

5. Quality of global convergence

The quality global convergence of the QN technique with upgrades satisfying the modified QN equation is presented in this section. the BFGS matrix with line searches subject to the conditions of Wolfe (17) and (18). The result has global convergence on uniformly convex functions [19] . the BFGS technique be demonstrated The assumptions of next about the Q (x) and g (x) are required.

Assumption 2. [3]

(a)A double differentiable continuously $Q(x)$ has The objective function, and for a known pointx_0, the equal set $\Omega = \{x: Q(x) \le Q(x_0)\}$ is convex.
(b) Close by occur constants is positive m and M so

$$m \| z \|^2 \le z^TGz \le M \| z \|^2 \quad (73)$$

for all $z \in \Re^n$ and $x \in \Omega$.
(c) At be existent a constant $L > 0$ such that

$$\| g(x) - g(y) \| \le L \| x - y \| \; \forall x, y \in \Omega. \quad (74)$$

(d) $|s^T_{k-1}u| \ge \mu \| u \| \|s_{k-1}\|,\ \mu \in (0,1]$ (75)

If $u = s_{k-1}$ the form (iv) is happy with $\mu = 1$ and if $u = y_{k-1}$ it holds with $\mu = \sigma_1/\sigma_2$ so σ_1 and σ_2 positive coefficients so that $\sigma_1 \| v \|^2 \le v^TG(x)v \le \sigma_2 \| v \|^2$ clamps $\forall\, x$ close x^* & some vector v in $\Re^n$.

Now The present theorem is used to investigate the convergence of the BFGS is global for uniformly convex functions that obey conditions (17) and (18).

Theorem 3.

Assume for a given pointx_0, the function $Q(x)$ meets conditions (i)-(ii), and thatB_0 is symmetric positive definite. There are positive constants Å$_1$, Å$_2$, Å$_3$ and Å$_4$ if the sequence $\{x_k\}$

created by the BFGS technique with step length α_k satisfying requirements (1) and (2) is not terminated at some point x_k with $g_k = 0$, such that:

$$\frac{\|\hat{y}_k\|^2}{s_k^T \hat{y}_k} \leq Å_1$$
$$\frac{s_k^T B_k s_k}{s_k^T \hat{y}_k} \leq Å_2 \alpha_k$$
$$\frac{\|B_k s_k\|^2}{s_k^T B_k s_k} \geq Å_3 \frac{\alpha_k}{\cos^2 \theta_k}$$
$$\frac{|\hat{y}_k^T B_k s_k|}{s^T v_k} \leq Å_4 \frac{\alpha_k}{\cos \theta_k}$$

When all k are kept constant, the converges of sequence $\{x_k\}$ is unique minimizer $x*$ of f (x).

First : $\frac{\|\hat{y}_k\|^2}{s_k^T y_k} \leq Å_1$ (76)

Proof:

$\hat{y} = \lambda y_k^T + \frac{n\,\lambda\theta}{s^T y_k} y_k$ (77)

$\|\hat{y}\| = \lambda \|y_k\| + \frac{n\,\lambda\|\theta\|}{\| s^T y_k\|} \|y_k\|$ (78)

By assumes

$$\| s^T y_k\| \geq M\|s_k\|\|y_k\|$$
$$\|y_k\| \leq L\|s_k\|$$
$$\|\theta\| \leq N\|s_k\|^2$$

Then we have

$\|\hat{y}\| \leq \lambda L\|s_k\| + \frac{n\,\lambda N}{M}\|s_k\|$ (79)

$\|\hat{y}\| \leq [\lambda L + \frac{n\,\lambda N}{M}] \|s_k\|$ (80)

$\|\hat{y}\| \leq \partial\|s_k\|$ (81)

Where $\partial = [\lambda L + \frac{n\,\lambda N}{M}]$

By uniform convex

$s^T \hat{y} \geq \gamma\lambda\, s^T y_k \geq \gamma m\|s_k\|^2$ (82)

$\|\hat{y}\|^2 \leq \partial^2\|s_k\|^2$ (83)

$\|\hat{y}\|^2 \leq \frac{\partial^2}{\gamma m} s^T \hat{y}$ (84)

$\frac{\|\hat{y}\|^2}{s^T \hat{y}_k} \leq \frac{\partial^2}{\gamma m} \leq Å_1$ (85)

Second : $\frac{s_k^T B_k s_k}{s_k^T y_k} \leq Å_2 \alpha_k$ (86)

Proof :

From $cos\theta = \frac{\|s_k\| s_k^T B_k s_k}{\|B_k s_k\| \|s_k\|^2} = \frac{s_k^T B_k s_k}{\|B_k s_k\| \|s_k\|}$

We have

$s_k^T B_k s_k = \|B_k s_k\| \|s_k\| cos\theta$ .... a

$\|B_k s_k\| = \alpha_k \|g_k\|$ ....b

$- g_k{}^T s_k = \|g_k\| \|s_k\| cos\theta$.............................c

$\theta = 2\lambda(Q_{k+1} - Q_{k+1}) - (g_k + g_{k+1})^T s$ (87)

From wolf condition

$$-(Q_{k+1} - Q_{k+1}) \geq -\delta\alpha_k \, g_k{}^T d_k$$

$$\theta \geq -2\lambda - \delta\alpha_k \, g_k{}^T d_k - g_{k+1}{}^T s_k - g_k{}^T s_k$$

$\theta \geq 2\lambda\delta\alpha_k \, g_k{}^T d_k - \alpha_k \, g_{k+1}{}^T d_k - g_k{}^T s_k$ (88)

By wolf condition

$$\sigma_1 g_k^T d_k \leq g_{k+1}^T d_k \leq -\sigma_2 g_k^T d_k$$

$$\theta \geq 2\lambda\delta\alpha_k \, g_k{}^T d_k - \alpha_k \sigma_1 g_k^T d_k - g_k{}^T s_k$$

$$\theta \geq 2\lambda\delta \, g_k{}^T s_k - \sigma_1 \, g_k{}^T s_k - g_k{}^T s_k$$

$$\theta \geq [2\lambda\delta - \sigma_1 - 1] \, g_k{}^T s_k$$

$\theta \geq N \, g^T s_k$ (89)

Where $N = [2\lambda\delta - \sigma_1 - 1]$

$$s_k^T y = \lambda s_k^T y_k + \frac{n \, \lambda\theta}{s^T y_k} s_k^T y_k$$

$$s_k^T y = \lambda s_k^T y_k + n \, \lambda\theta$$

From $y_k^T s_k = g_{k+1}^T s_k - g_k^T s_k \geq -(1 - B) g_k^T s_k$

$$-(1 - B) g_k^T s_k \leq y_k^T s_k$$

$$s_k^T y = \lambda s_k^T y_k + n \, \lambda\theta$$

$$s_k^T y \geq -\lambda(1 - B) g s_k + n \, \lambda N \, g_k{}^T s_k$$

$$s_k^T y \geq -\lambda[1 - B - n \, N] \, g_k{}^T s_k$$

Now by a,b,c we obtain

$s_k^T B_k s_k = -\alpha_k \, g_k{}^T s_k$

Then

$$\frac{s_k^T B_k s_k}{s_k^T y_k} \leq \frac{-\alpha_k g_k^T s_k}{-\lambda[1-B-n\,N]\, g_k^T s_k} \quad (90)$$

$$\frac{s_k^T B_k s_k}{s_k^T y_k} \leq Å_2\, \alpha_k$$

Where $Å_2 = \frac{1}{\lambda[1-B-n\,N]}$

Third : $\frac{\|B_k s_k\|^2}{s_k^T B_k s_k} \geq Ă_3 \frac{\alpha_k}{\cos^2\theta_k}$

Proof

$$\|B_k s_k\| = \alpha_k \|g_k\| \quad (91)$$

From $\cos\theta = \frac{\|s_k\| s_k^T B_k s_k}{\|B_k s_k\| \|s_k\|^2} = \frac{s_k^T B_k s_k}{\|B_k s_k\| \|s_k\|}$

$$s_k^T B_k s_k = \|B_k s_k\| \|s_k\| \cos\theta \quad (92)$$

$$\frac{\|B_k s_k\|^2}{s_k^T B_k s_k} = \frac{\alpha^2 \|g_k\|^2}{\|B_k s_k\| \|s_k\| \cos\theta} = \frac{\alpha^2 \|g_k\|^2}{\alpha_k \|g_k\| \ \|s_k\| \cos\theta} \quad (93)$$

$$\frac{\|B_k s_k\|^2}{s_k^T B_k s_k} = \frac{\alpha_k \|g_k\|}{\|s_k\| \cos\theta}$$

From $c_1 \|g_k\| \cos\theta \leq \|s_k\| \leq c_2 \|g_k\| \ \cos\theta$

$$\|s_k\| \leq c_2 \|g_k\| \ \cos\theta \quad (94)$$

$$\frac{1}{\|s_k\|} \leq \frac{1}{c_2 \|g_k\| \ \cos\theta}$$

$$\frac{\|B_k s_k\|^2}{s_k^T B_k s_k} \geq \frac{\alpha_k \|g_k\|}{c_2 \|g_k\| \ \cos\theta \cos\theta} \quad (95)$$

$$\frac{\|B_k s_k\|^2}{s_k^T B_k s_k} \geq \frac{\alpha_k}{c_2 \cos^2\theta} \quad (96)$$

Forth : $\frac{|y_k^T B_k s_k|}{y_k^T s_k} \leq Å_4 \frac{\alpha_k}{\cos\theta_k}$

Proof

$$\frac{|y_k^T B_k s_k|}{y_k^T s_k} = \frac{\|y_k\| \|B_k s_k\|}{y_k^T s_k} \quad (97)$$

$$\|y_k\| \leq N \|s_k\| \quad (98)$$

$$\|B_k s_k\| = \alpha_k \|g_k\| \quad (99)$$

$$s_k^T y \geq -\lambda[1 - B - n\,N]\, g_k^T s_k \quad (100)$$

From

$$-g_k^T s_k = \|g_k\| \|s_k\| \cos\theta$$

$$s_k^T y \geq \lambda[1 - B - n\,N] \|g_k\| \|s_k\| \cos\theta \quad (101)$$

$$\frac{|y_k^T B_k s_k|}{y_k^T s_k} = \frac{\|y_k\| \|B_k s_k\|}{y_k^T s_k} \quad (102)$$

$$\frac{1}{\hat{y}_k^T s_k} \leq \frac{1}{\lambda[1-B-n\ N]\|g_k\|\|s_k\|cos\theta} \tag{103}$$

$$\frac{|\hat{y}_k^T B_k s_k|}{\hat{y}_k^T s_k} \leq \frac{\alpha_k N\|g_k\|\|s_k\|}{\lambda[1-B-n\ N]\|g_k\|\|s_k\|cos\theta} \tag{104}$$

$$\leq \frac{\alpha_k N}{\lambda[1-B-n\ N]cos\theta} \tag{105}$$

$$\frac{|\hat{y}_k^T B_k s_k|}{\hat{y}_k^T s_k} \leq \frac{\alpha_k N}{\lambda[1-B-n\ N]cos\theta} \tag{106}$$

$$\frac{|\hat{y}_k^T B_k s_k|}{\hat{y}_k^T s_k} \leq Å_4 \frac{\alpha_k}{cos\theta}$$

Where $Å_4 = \frac{N}{\lambda[1-B-n\,N]}$

6. Calculated results:

We present some numerical results from our suggest model. The first we see how well our modified secant equation (35,37) performs in the updated SCALCG method, we ran the code against the 8 techniques listed below.

(1) M1: the quadratic Algorithm when n=2 consistent to eq. (30)

(2) M2: the conic Algorithm when n=3 consistent to eq (31)

(3) M3: Consider Algorithm 1 when n=4 consistent to eq (32)

(4) M4: Consider Algorithm 2 when n=5 consistent to eq (32)

(5) M5: Consider Algorithm 3 when n=6 consistent to eq (32)

(6) M5: Consider Algorithm 4 when n=7 consistent to eq (32)

(7) M5: Consider Algorithm 5 when n=8 consistent to eq (32)

(8) M5: Consider Algorithm 6 when n=10 consistent to eq (32)

It subsection details some calculations from the PC computer's implementation of test cases from collection [2]. The codes are written in Fortran 77 in double-byte format, with BFGS included. A software that comprises the general formula of the conic model and is implemented in cases n=2 quatratic n=3 conic n=4,5,6,7,8,10 was developed.

We consider the conditions below the discontinuation criterion [5]

For unconstrained part

$\|x_k - x_{k-1}\| < \xi ,$ $\qquad \xi = 10^{-5}$

For constrained part

$r_k \ \sum_{i=1} e_i + r_k \ \sum_{i=m+1} c_i < \xi$ $\qquad \xi = 10^{-5}$

The two Tables shows the numerical computations of these algorithms proposed to check their performance and we have used the following well-known measures or tools used normally for this type of comparison of algorithms:

NOI : the total number of iterations

NOF : the total number of function evaluation

NOC : the total number of constrained

Table 1 : Comparisons of the quadratic algorithm with conic algorithm , new1 and nwe2 algorithm

NO	M1 N=2 QE NOF(NOI)NOC	M2 N=3 conic NOF(NOI)NOC	M3 N=4 NEW1 NOF(NOI)NOC	M4 N=5 NEW2 NOF(NOI)NOC
1	167(32)696	135(26)550	146(29)659	164(32)659
2	81(22)472	104(23)570	89(23)489	89(23)489
3	182(46)872	175(44)877	157(41)794	171(45)826
4	58(9)169	39(7)143	43(9)232	39(9)188
5	369(90)608	369(90)608	369(90)608	369(90)608
6	401(114)269	258(57)256	184(56)200	165(49)185
7	104(26)289	103(26)290	101(26)272	102(26)279
8	121(36)246	109(32)263	102(29)295	98(28)295
9	119(31)414	116(31)384	119(31)409	99(27)330
10	162(54)**55**	156(52)53	120(40)41	75(24)28
11	337(99)370	205(59)290	256(73)351	156(46)264

Table 2 Comparisons of the new3 algorithm with new4, new5 and nwe6 algorithm

NO	M5 N=6 New3 NOF(NOI)NOC	M6 N=7 NEW4 NOF(NOI)NOC	M7 N=8 NEW5 NOF(NOI)NOC	M8 N=10 NEW6 NOF(NOI)NOC
1	672(147)1690	326(72)901	357(77)920	810(240)1516
2	89(23)489	89(23)489	89(23)489	89(23)489
3	167(43)830	171(45)807	167(43)767	172(46)814
4	No.	57(15)133	84(18)272	No.
5	369(90)608	369(90)608	369(90)608	356(89)581
6	169(51)182	157(46)180	151(45)191	165(49)191

7	106(26)283	105(26)269	105(26)268	101(26)281
8	124(37)280	107(31)298	115(33)298	122(35)274
9	117(31)372	116(31)375	123(32)402	116(31)383
10	72(24)25	99(33)34	18608(6196)6217	115(37)41
11	147(42)320	175(46)275	322(94)374	198(55)311

The second we demonstrate the computational performance of our strategy on a set of test issues restricted optimization. we selected (11) large-scale restricted optimization problems Each problem must be tested using a general conic model and. To demonstrate the usefulness of the suggested approach, we employed the Dolan and more'[10] technique.

The following figures (1-3) is illustrate the results using the Dolan and more'. Displays the Dolan-More performance profile for these methods, which are susceptible to the frequency of suitable performance when compared to the basic methods.

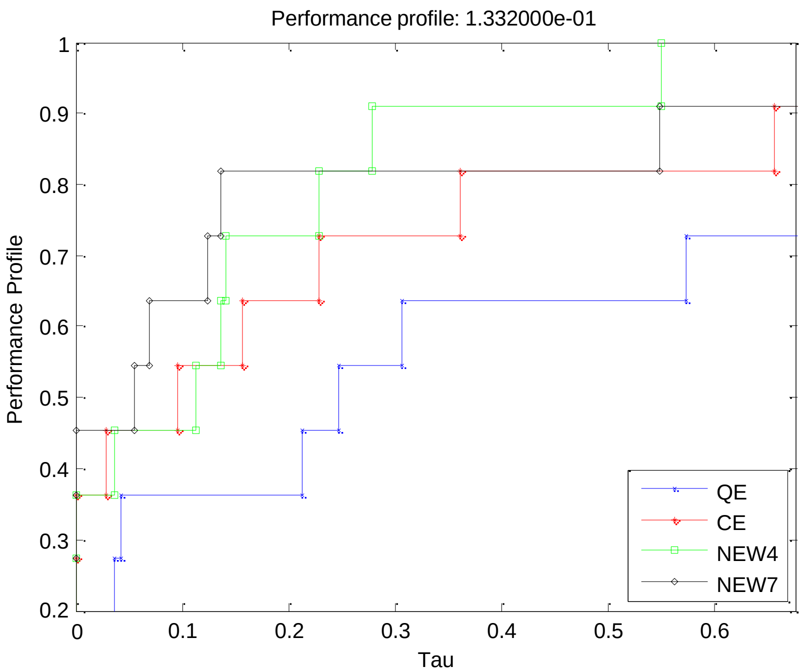

FIGURE 1 . data relating to how` well the aforementioned methods perform in terms of function evaluations

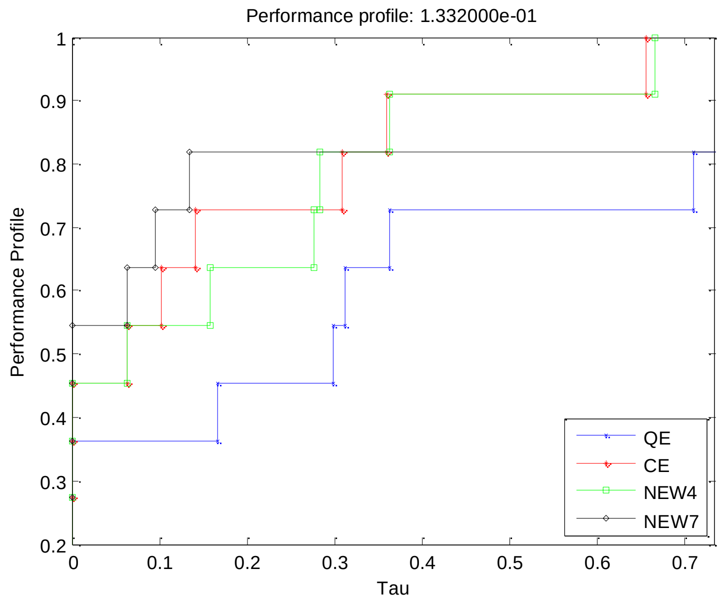

FIGURE 2 . data relating to how well the aforementioned methods perform in terms of iteration evaluations

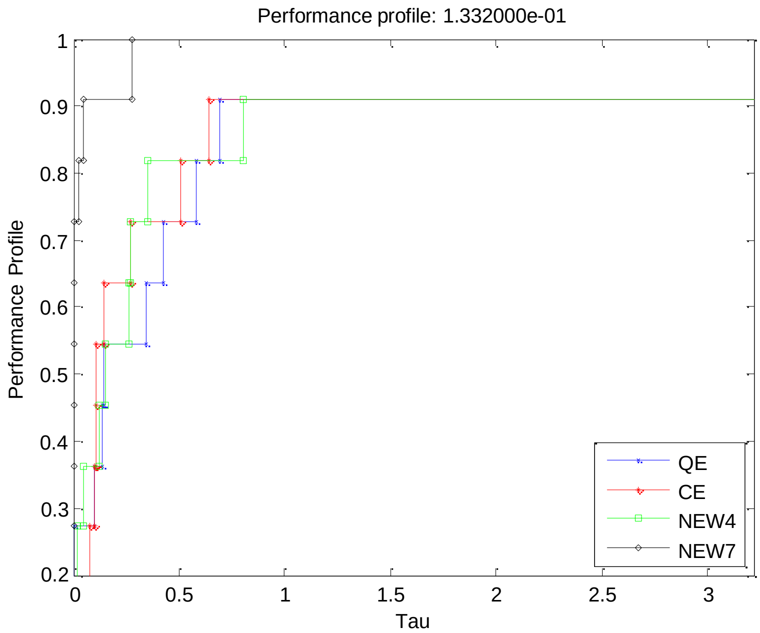

FIGURE 3 . data relating to how well the aforementioned methods perform in terms of gradients evaluations

By examining the Dolan-More performance profile, which is measured in CPU time, we may conclude from the three forms shown that the new method is particularly suitable for tackling problems with numerous dimensions

7. Conclusions

We present a new secant equation has been proposed by using the n-order Taylor expansion of the objective conic function. The suggest secant equation BFGS the Global convergence of it have be established. and the account of numerical results display the effectiveness of it

8. Recommendations

We recommend use the new extended area the application in metaheuristic algorithms.

Reference

1. Ahmed, Huda I., Rana Z. Al-Kawaz, and Abbas Y. Al-Bayati. "Spectral three-term constrained conjugate gradient algorithm for function minimizations." *Journal of Applied Mathematics* 2019 (2019).
2. Andrei, Neculai. "An unconstrained optimization test functions collection." *Adv. Model. Optim* 10.1 (2008): 147-161.
3. Biglari, Fahimeh, Malik Abu Hassan, and Wah June Leong. "New quasi-Newton methods via higher order tensor models." *Journal of computational and applied mathematics* 235.8(2011): 2412-2422.
4. Broyden, Charles George. "The convergence of a class of double-rank minimization algorithms 1. general considerations." IMA Journal of Applied Mathematics 6.1 (1970): 76-90.
5. Bunday, B. "Basic Optimization Methods, Edward Arnold." *Bedford square, London,UK* (1984).
6. D. SORENSEN, The Q-superlinear convergence of a collinear scaling algorithm for unconstrained optimization,SIAM J. Numer. Anal., 17 (1980), pp. 88-114.
7. Davidon, William C. "Conic approximations and collinear scalings for optimizers." *SIAM Journal on Numerical Analysis* 17.2 (1980): 268-281.
8. Dennis, John E., and Jorge J. Moré. "A characterization of superlinear convergence and its application to quasi-Newton methods." *Mathematics of computation* 28.126 (1974): 549- 560.
9. Dennis, John E., and Jorge J. Moré. "Quasi-Newton methods, motivation and theory." *SIAM review* 19.1 (1977): 46-89.
10. Dolan, Elizabeth D., and Jorge J. Moré. "Benchmarking optimization software with performance profiles." Mathematical programming 91.2 (2002): 201-213.
11. Fletcher, Roger. "A new approach to variable metric algorithms." *The computer journal* 13.3 (1970): 317-322.
12. Goldfarb, Donald. "A family of variable-metric methods derived by variational means." *Mathematics of computation* 24.109 (1970): 23-26.
13. Gourgeon, Hervé, and Jorge Nocedal. "A conic algorithm for optimization." *SIAM journal on scientific and statistical computing* 6.2 (1985): 253-267.
14. Griva, Igor. "Numerical experiments with an interior-exterior point method for nonlinear programming". Computational Optimization and Applications 29.2 (2004): 173-195.
15. K. A. ARIYAWANSA, Conic approximations and collinear scalings m algorithms for unconstrained minimization, Ph.D. thesis, University of Toronto, Canada, 1983.
16. L. GRANDINETTI, Some investigations m a new algorithm for nonlinear optimization based on conic models of the objective function, J. Optim. Theory Appl., 43 (1984), pp. 1-21.
17. Li, Guoyin, Chunming Tang, and Zengxin Wei. "New conjugacy condition and related new conjugate gradient methods for unconstrained optimization." *Journal of Computational and Applied Mathematics* 202.2 (2007): 523-539.
18. Liu, Dong C., and Jorge Nocedal. "Algorithms with conic termination for nonlinear optimization." *SIAM journal on scientific and statistical computing* 10.1 (1989): 1-17.

19. Powell, Michael JD. "Some global convergence properties of a variable metric algorithm for minimization without exact line searches. " *Nonlinear programming , SIAM-AMS proceedings*. Vol. 9. 1976.
20. Sumithra, M. (2022). Role of Embedded Systems in Industrial Section-By considering the Automotive Industry as an Example. *JOURNAL OF OPTOELECTRONICS LASER*, *41*(3).
21. Kumar, A., Bhatt, B. R., Anitha, P., Yadav, A. K., Devi, K. K., & Joshi, V. C. (2022, March). A new Diagnosis using a Parkinson's Disease XGBoost and CNN-based classification model Using ML Techniques. In *2022 International Conference on Advanced Computing Technologies and Applications (ICACTA)* (pp. 1-6). IEEE.
22. Preetha, M., Anil Kumar, N., Elavarasi, K., Vignesh, T., & Nagaraju, V. (2022). A Hybrid Clustering Approach Based Q-Leach in TDMA to Optimize QOS-Parameters. *Wireless Personal Communications*, *123*(2), 1169-1200.
23. Sumithra, M. (2022). Role of Embedded Systems in Industrial Section-By considering the Automotive Industry as an Example. *JOURNAL OF OPTOELECTRONICS LASER*, *41*(3).
24. Ashreetha, B., Devi, M. R., Kumar, U. P., Mani, M. K., Sahu, D. N., & Reddy, P. C. S. (2022). Soft optimization techniques for automatic liver cancer detection in abdominal liver images. *International journal of health sciences*, *6*.
25. Bosco Ekka, D. G., Prince Verma, D., & Harishchander Anandaram, D. (2022). A Review Of The Contribution Of Youth To Sustainable Development And The Consequences Of This Contribution. *Journal of Positive School Psychology*, 3564-3574.
26. Sharma, N., Yadava, A. K., Aarif, M., Anandaram, H., Alalmai, A., & Singh, C. (2022). Business Opportunities And Challenges For Women In The Travel And Tourism Industry During Pandemics Covid-19. *Journal of Positive School Psychology*, 897-903.
27. Yadava, A. K., Khan, I. A., Pandey, P., Aarif, M., Khanna, G., & Garg, S. (2022). Impact of marketing communication and information sharing on the productivity of India’s small and medium-sized businesses (SMEs). International Journal of Health Sciences, 6(S2), 12745–12755. https://doi.org/10.53730/ijhs.v6nS2.8352
28. Alalmai, A., Arun, A & ,.Aarif, M.تأثير إعلانات وسائل التواصل الاجتماعي على سلوك الشراء للمستهلك .
29. ALALMAI, A., ARUN, A., & AARIF, M. ROLE OF HAJJ AND UMRAH(PILGRIMAGE TOURISM) IN SAUDI ARABIAN ECONOMY.
30. Aarif, M. (2018). A STUDY ON THE ROLE OF HEALTHCARE INDUSTRY IN THE PROMOTING OF HEALTH TOURISM IN INDIA. *A CASE STUDY OF DELHI-NCR*.
31. Alalmai, A., & Fatma, D. G. A., Arun & Aarif, Mohd.(2022). Significance and Challenges of Online Education during and After Covid-19. Türk Fizyoterapi ve Rehabilitasyon Dergisi. *Turkish Journal of Physiotherapy and Rehabilitation*, *32*, 6509-6520.
32. MOURAD, H. M., KAUR, C., & AARIF, D. M. CHALLENGES FACED BY BIG DATAAND ITS ORIENTATION IN THE FIELD OF BUSINESS MARKETING.

33. Aarif, M., & Alalmai, A. (2019). Importance of Effective Business Communication for promoting and developing Hospitality Industry in Saudi Arabia. *A case study of Gizan (Jazan)*.
34. Alalmai, A. A., Arun, A., & Aarif, M. (2020). Implementing Possibilities and Perspectives of Flipped Learning in Hotel Management Institutions. *Test Engineering and Management*, *83*, 9421-9427.
35. Tripathi, M. A., Tripathi, R., Sharma, N., Singhal, S., Jindal, M., & Aarif, M. (2022). A brief study on entrepreneurship and its classification. *International Journal of Health Sciences*, *6*.
36. Arun, A. A., & Aarif, M. Student's Anticipation in Procuring Post Graduation Programme in Hotel Management through Distance Learning.
37. Rao, S. S., " Engineering optimization: theory and practice". (2019), John Wiley & Sons.
38. Shanno, David F. "Conditioning of quasi-Newton methods for function minimization." *Mathematics of computation* 24.111 (1970): 647-656.
39. Wolfe, Philip. "Convergence conditions for ascent methods. II: Some corrections." *SIAMreview* 13.2 (1971): 185-188.
40. Wolfe, Philip. "Convergence conditions for ascent methods." *SIAM review* 11.2 (1969):226-235.