# Decision Making in Fuzzy Environment Using Pythagorean Fuzzy Numbers

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Abstract
Pythagorean Fuzzy Sets are usually depicted by four parameters, membership, nonmembership, strength and direction. In this paper, we have used Minkowski's distance to rank Pythagorean Fuzzy Numbers and apply it in decision making problems. A numerical example is provided to illustrate the method.

**Keywords:** Decision Making, Distance Measures, Intuitionistic fuzzy Sets, Pythagorean Fuzzy Numbers, Pythagorean Fuzzy Sets.

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## 1. INTRODUCTION:

Decision making is a selection based on some criteria from two or more possible alternatives. Even though the society and economy has developed, decision making problems are becoming intricate. Hence it is hard for a verdict to come to a conclusion using crisp numbers. Due to this reason, Fuzzy set theory came into existence. To obtain better results, Intuitionistic fuzzy sets and Pythagorean Fuzzy Sets are used to solve decision making problems.

Zadeh[11] introduced Fuzzy Set theory in 1965. Fuzzy set theory was introduced to solve problems with uncertainty. Attanassov[1] introduced Intuitionistic Fuzzy Sets(IFS), characterized by a membership function and non membership function. Elements of Intutionisic Fuzzy sets are represented as an ordered pair $(\mu_I, v_I)$ where $\mu_I + v_I \leq 1$.

If the sum of the membership and nonmembership function is greater than one, then the condition for Intuitionistic Fuzzy Sets does not holds good. Hence, Yager[10] introduced Pythagorean Fuzzy sets(PFSs), where sum of the squares of the membership and nonmembership value does not exceed 1 (i.e.,) $0 \leq (\mu_P(x))^2 + (v_P(x))^2 \leq 1$.Pythagorean Fuzzy Sets are characterized by four parameters viz., membership degree, nonmembership degree, strength commitment about membership and direction commitment.

Yager and Abbasov[9,10] introduced Pythagorean Fuzzy weighted geometric average operator and defined the relationship between Pythagorean membership degrees and complex numbers. Zhang and Xu[13] defined order preference to compare Pythagorean Fuzzy Sets. Peng and Yang[5] ranked Pythagorean Fuzzy Numbers (PFNs) using a new method.

In Fuzzy environment, Distance measures and Similarity measures are used to solve Decision making problems. Thangaraj Beaula and Vijaya[9-13] defined new representations and ranking for different fuzzy numbers. Several distance measures are defined for fuzzy sets, IFSs and PFSs. Zhang and Xu[3] defined distance measures of Pythagorean Fuzzy Numbers. Li and Zeng[12] explored normalized Hamming distance and normalized Euclidean distance. Li Deqing, Had litery and Yindry[4] introduced Minkowski distance for PFN and PFS.

## 2.PRELIMINARIES

**Definition 2.1**[1]

Let X be the universe. An Intuitionistic Fuzzy Set (IFS) in X denoted by

$I = \{< x, \mu_I(x), v_I(x) >/x \in X\}$

where $\mu_I = X \to [0,1]$ denotes the degree of membership and $v_I = X \to [0,1]$ denotes the degree of nonmembership of $x \in X$, such that $0 \leq \mu_I(x) + v_I(x) \leq 1$.

The degree of indeterminacy for I is $\pi_I(x) = 1 - \mu_I(x) - v_I(x)$

If $\alpha =< \mu_\alpha, v_\alpha >$ is an Intuitionistic Fuzzy number(IFN), $\pi_\alpha = 1 - \mu_\alpha - v_\alpha$ is called the degree of indeterminacy.

**Definition 2.2**[8]

Let X be a nonempty set, and x , y , z $\in X$. A metric D on X is called the distance measure, if it satisfies the following three properties

$D(x, y) \geq 0$ $and D(x, y) = 0 \Rightarrow x = y$; (Non Negative property)

$D(x, y) = D(y, x)$;(Symmetric property)

$D(x, y) \leq D(x, z) + D(z, y)$ (Triangle Inequality)

**Definition2.3**[2]

Let $X = \{x_1, x_2, x_3, \dots\dots x_n\}$, the normalized Hamming distance between two IFS A and B is defined as follows:

$$D_H(A, B) = \frac{1}{n}\sum_{i=1}^{n}\frac{|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)|}{2}$$

The normalized Euclidean distance between two IFSs A and B is defined as follows:

$$D_E(A,B) = \left(\sum_{i=1}^{n} \frac{(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}{2n}\right)^{\frac{1}{2}}$$

**Definition 2.4**[7]

Let $X = \{x_1, x_2, x_3, \ldots\ldots x_n\}$. The normalized Hamming distance and normalized Euclidean distance between two IFSs A and B considering the degree of indeterminacy is defined respectively as

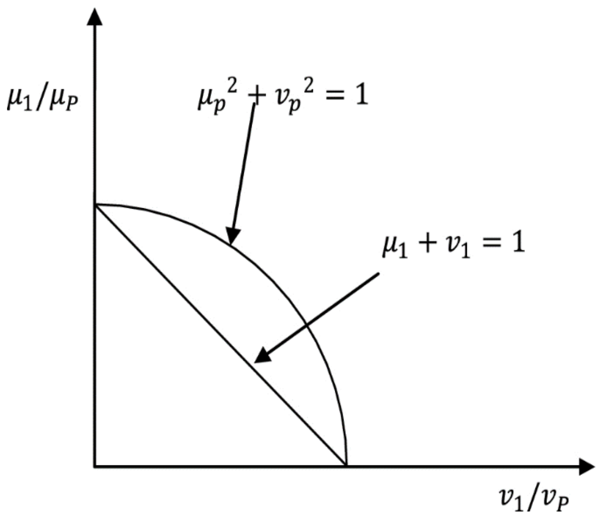
$$D_H(A,B) = \frac{1}{n}\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)| + |v(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|]$$

$$D_E(A,B) = \left(\frac{1}{2n}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]\right)^{1/2}$$

**Definition2.5**[10]

Let X be the universe of discourse. $P = \{\langle x, \mu_P(x), v_P(x)\rangle | x \in X\}$ represents a Pythagorean Fuzzy Set (PFS) in X, where $\mu_P : X \to [0,1]$ is the degree of membership and $v_P: X \to [0,1]$ is the degree of nonmembership of the element $x \in X$ to the set P, with the condition that $0 \leq (\mu_P(x)^2 + v_P(x)^2) \leq 1$. The degree of indeterminacy $\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 + (v_P(x))^2}$

Let $(\mu_P(x), v_P(x))$ be a Pythagorean Fuzzy Number(PFN) denoted by $p = (r_p, d_p)$, where $r_p$ is called the strength of p and $d_p$ is called the direction of the strength $r_p$.




**DIAGRAMMATIC REPRESENTATION OF IFS AND PFS**

The relationship between $p = (\mu_P, v_P)$ and $p = (r_p, d_p)$ is that

$\mu_p = r_p\cos(\theta_p)$, $v_p = r_p\sin(\theta_p)$,$d_p = 1 - \frac{2(\theta_p)}{\pi}$

The Pythagorean membership degrees are subclass of complex number called $\prod-\ i$ numbers. Hence they denote a PFN $p = (\mu_P, v_P)$as $p = r_p e^{-i\theta}$, where $\mu_p = r_p\cos(\theta)$and $v_p = r_p\sin(\theta)$.

Two vectors are used to view the membership degree $\mu_P$ and nonmembership degree $v_P$. The sum of the vectors $\mu_p$ and $v_p$ is the strength $r_P$. The axis of abscissas has the same direction as that of $\mu_p$ and the axis of ordinates has the direction of $v_p$.

**Definition 2.6**[10]

To compare two PFNs, for each PFN $p = (r_p, d_p)$, Value of the PFN is given by

$$V(p) = \frac{1}{2} + r_p\left(d_p - \frac{1}{2}\right) = \frac{1}{2} + r_p\left(\frac{1}{2} - \frac{2\theta_p}{\pi}\right)$$

Let $p_1 = \left(\mu_{p_1}, v_{p_1}\right)$and $p_2 = \left(\mu_{p_2}, v_{p_2}\right)$ be two PFNs, then

1) If $V(p_1) > V(p_2)$ , then $p_1 \succ p_2$;
2) If $V(p_1) = V(p_2)$ , then$p_1 \sim p_2$.

**Definition 2.7**

If $p = \left(\mu_p, v_p\right)$be a PFN, then score function of p is

$$s(p) = (\mu_P)^2 - (v_P)^2$$

Where $s(p) \in [-1,1]$. For any two PFNs $p_1, p_2$ if$s(p_1) < s(p_2)$, then$p_1 \prec p_2$. If $s(p_1) > s(p_2)$, then$p_1 \succ p_2$.If $s(p_1) = s(p_2)$,then $p_1 \sim p_2$.

**Definition 2.8**

If $p = \left(\mu_p, v_p\right)$ be a Pythagorean Fuzzy Number, then accuracy function of p is defined as follows:

$$a(p) = (\mu_p)^2 + (v_p)^2$$

For any two PFNs,$p_1, p_2$

1)If $s(p_1) > s(p_2)$, then $p_1 \succ p_2$.

2) If $s(p_1) = s(p_2)$, then $p_1 \sim p_2$

a)If $a(p_1) > a(p_2)$, then $p_1 \succ p_2$.

b)If $a(p_1) = a(p_2)$,then $p_1 \sim p_2$.

**Definition 2.9**

Let $p_1, p_2, p_3, \dots\dots, p_n$be collection of PFNs and if an importance weight $w_i$ such that $w_i \in [0,1]$(i=1,2….n) is associated for each $p_i = \left(\mu_{p_i}, v_{p_i}\right)$ such that $\sum_{i=1}^{n} w_i = 1$, then Pythagorean fuzzy weighted average is as follows:

$$C\left(p_1, p_{2,} \dots, p_n\right) = \left(\sum_{i=1}^{n} w_i \mu_{p_i}, \sum_{i=1}^{n} w_i v_{p_i}\right)$$

**Definition 2.10**

Let $p_1, p_2$ be two PFNs, the distance between $p_1$ and $p_2$ is defined as follows:

$$D(p_1, p_2) = \frac{1}{2}(|(\mu_{p_1})^2 - (\mu_{p_2})^2| + |(v_{p_1})^2 - (v_{p_2})^2| + |(\pi_{p_1})^2 - (\pi_{p_2})^2|)$$

**Definition 2.11**

Let $p_1$ and $p_2$ be two PFNs, the normalized Hamming distance $p_1$and $p_2$ is defined as follows

$$D_H(p_1, p_2) = \frac{1}{4}|\mu_{p_1} - \mu_{p_2}| + |v_{p_1} - v_{p_2}| + |r(p_1) - r(p_2)| + |d(p_1) - d(p_2)|$$

The normalized Euclidean distance $p_1$and $p_2$ is defined as follows

$$D_E(p_1, p_2) = [\frac{1}{4}((\mu_{p_1} - \mu_{p_2})^2 + (v_{p_1} - v_{p_2})^2 + \left(r(p_1) - r(p_2)\right)^2 + \left(d(p_1) - d(p_2)\right)^2)]^{\frac{1}{2}}$$

The normalized generalized distance between $p_1$and $p_2$ is defined as follows:

$$D_G(p_1, p_2) = [\frac{1}{4}(|\mu_{p_1} - \mu_{p_2}|^{\lambda} + |v_{p_1} - v_{p_2}|^{\lambda} + |r(p_1) - r(p_2)|^{\lambda} + |d(p_1) - d(p_2)|^{\lambda})]^{\frac{1}{\lambda}}$$

where$\lambda \geq 1$.

The Minkowski distance between $p_1$ and $p_2$ is

$$D_M(p_1, p_2) = [\frac{1}{4}(|\mu_{p_1} - \mu_{p_2}|^{q} + |v_{p_1} - v_{p_2}|^{q} + |r(p_1) - r(p_2)|^{q} + |d(p_1) - d(p_2)|^{q})]^{\frac{1}{q}}$$

Among them $\mu_{p_j}(x_i) = r_{p_j}(x_i)\cos\left(\theta_{p_j}\right)$ $and$ $v_{p_j}(x_i) = v_{p_j}(x_i)\sin\left(\theta_{p_j}\right)$

$$d_{p_j}(x_i) = 1 - \frac{2\theta_p}{\pi}, \theta_p \epsilon \left[0, \frac{\pi}{2}\right]$$

j = 1,2 , i = 1,2, ... n , and $q > 0$.

**Definition 2.12**

Let $p_1$ and $p_2$ be two PFSs $X = \{x_1, x_{2,\ldots\ldots} x_n\}$. Then the normalized Hamming distance $p_1$and $p_2$ is defined as follows

$$D_E(P_1, P_2) = \frac{1}{4n}\sum_{i=1}^{n}(\left|\mu_{p_1}(x_i) - \mu_{p_2}(x_i)\right| + \left|v_{p_1}(x_i) - v_{p_2}(x_i)\right| + \left|r_{p_1}(x_i) - r_{p_2}(x_i)\right| + \left|d_{p_1}(x_i) - d_{p_2}(x_i)\right|)$$

The normalized Euclidean distance between $P_1$ and $P_2$ is defined as

$$D_E(P_1, P_2) = [\frac{1}{4n}\sum_{i=1}^{n}((\mu_{p_1}(x_i) - \mu_{p_2}(x_i))^2 + (v_{p_1}(x_i) - v_{p_2}(x_i))^2 + \left(r_{p_1}(x_i) - r_{p_2}(x_i)\right)^2 + \left(d_{p_1}(x_i) - d_{p_2}(x_i)\right)^2)]^{\frac{1}{2}} \qquad (18)$$

The normalized generalized distance between $P_1$ and $P_2$ is defined as

$$D_G(P_1, P_2) = [\frac{1}{4n}\sum_{i=1}^{n}(|\mu_{p_1}(x_i) - \mu_{p_2}(x_i)|^{\lambda} + |v_{p_1}(x_i) - v_{p_2}(x_i)|^{\lambda} + \left|r_{p_1}(x_i) - r_{p_2}(x_i)\right|^{\lambda} + \left|d_{p_1}(x_i) - d_{p_2}(x_i)\right|^{\lambda})]^{\frac{1}{\lambda}} \qquad (19)$$

TheMinkowski distance between $P_1$ and $P_2$ is

$$D_M(P_1, P_2) = [\frac{1}{4n}\sum_{i=1}^{n}(|\mu_{p_1}(x_i) - \mu_{p_2}(x_i)|^{q} + |v_{p_1}(x_i) - v_{p_2}(x_i)|^{q} + \left|r_{p_1}(x_i) - r_{p_2}(x_i)\right|^{q} + \left|d_{p_1}(x_i) - d_{p_2}(x_i)\right|^{q}]^{\frac{1}{q}} \qquad (20)$$

Among them

$$\mu_{p_j}(x_i) = r_{p_j}(x_i)\cos\left(\theta_{p_j}\right) \, and \, v_{p_j}(x_i) = v_{p_j}(x_i)\sin\left(\theta_{p_j}\right)$$

$d_{p_j}(x_i) = 1 - \frac{2\theta_p}{\pi}$ , $\theta_p \epsilon \left[0, \frac{\pi}{2}\right]$, j = 1,2 , i = 1,2, ... n , and $q > 0$.

**3. ALGORITHM** For a PFN,

**Step1:** Calculate$(r_p, d_p)$, the corresponding strength and direction of PFNs.

**Step2:** Calculate the comparison value of $V(P_i)$,(i=1,2….m).

**Step3:** Using the value of PFN, $V(P_i)$ (i=1,2,….m), find the Pythagorean fuzzy positive ideal solution $A^+$ as in step 2.

**Step4:** Find the distance $D(A_i, A^+)$,(i=1,2…m)using Minkowski Distance

**Step5:** **T**he PFNs are ranked according to the priority that the number with least value will be ranked first.

## 4. NUMERICAL EXAMPLE

The purpose of this study is to assess the commercialization of new technological firms. A1, A2, A3, and A4 are four possible developing technological firms. Four primary criteria were chosen by the experts: technical advancement and financial conditions (C1), possible market risk (C2), industrialization infrastructure and human resources (C3), and employment and scientific development (C4). The criteria's weight vector is $w = (0.15, 0.25, 0.35, 0.25)^T$. Assume that the evaluation values of four companies in relation to each of the decision makers' criteria are expressed as Pythagorean Fuzzy numbers. Table I shows the Pythagorean fuzzy decision matrix.

**TableI.The Pythagorean Fuzzy Decision Matrix**

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | (0.9,0.3) | (0.7,0.6) | (0.5,0.8) | (0.6,0.3) |
| $A_2$ | (0.4,0.7) | (0.9,0.2) | (0.8,0.1) | (0.5,0.3) |
| $A_3$ | (0.8,0.4) | (0.7,0.5) | (0.6,0.2) | (0.7,0.4) |
| $A_4$ | (0.7,0.2) | (0.8,0.2) | (0.8,0.4) | (0.6,0.6) |

**Table II. Values of $V(P_i)$**

| | | | |
|---|---|---|---|
| **0.7801** | 0.5451 | 0.3638 | **0.6374** |
| 0.3669 | **0.8327** | **0.8393** | 0.5909 |
| 0.6832 | 0.5904 | 0.6867 | 0.6367 |
| 0.7350 | 0.7837 | 0.6832 | 0.5 |

Hence we select the Pythagorean positive ideal alternatives with the help of $V(P_i)$ values from table II.

The Pythagorean positive ideal solution $A^+$ as follows:

$A^+ = \{(0.9,0.3),(0.9,0.2),(0.8,0.1),(0.6,0.3)\}$

**Table III. The pair of ($r_p$,$d_p$) corresponding to the strength and direction of PFNs**

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | (0.9487,0.7952) | (0.9220,0.5489) | (0.9434,0.3556) | (0.6708,0.7048) |
| $A_2$ | (0.8062,0.3305) | (0.9220,0.8608) | (0.8062,0.9208) | (0.5831,0.6560) |
| $A_3$ | (0.8944,0.7048) | (0.8602,0.6051) | (0.6325,0.7952) | (0.8062,0.6695) |
| $A_4$ | (0.7280,0.8228) | (0.8246,0.8440) | (0.8944,0.9208) | (0.8485,0.5) |
| **$A^+$** | (0.9487,0.7952) | (0.9220,0.8608) | (0.8062,0.9208) | (0.6708,0.7048) |

**Table IV. Distance measure D($A_i$ ,$A^+$) and the ranking of alternatives**

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| D($A_i$ ,$A^+$) | 0.1632 | 0.1090 | 0.1314 | 0.1281 |
| Ranking | 4 | 1 | 3 | 2 |

Hence we obtained the least value will be ranked first. Then the ranking of alternative is $A_2>A_4>A_3>A_1$.

## 5.CONCLUSION:

Pythagorean Fuzzy Sets or Numbers have the advantage of being represented using parameters other than membership and nonmembership functions. We used a distance measure to rank the PFNs. The proposed strategy will make solving decision-making difficulties easier and more useful.

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