Interval Valued $\kappa -$ Kernel Symmetric Fuzzy Matrices

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Abstract
We enhance the equal characterization of interval valued $\kappa -$ kernel symmetric fuzzy matrices in this study. Necessary and enough situations are decided for an interval valued matrix to be $\kappa -$ kernel symmetric. We deliver few end result of interval valued kernel symmetric matrices. This leads to an interval valued $\kappa -$ symmetric matrices implies interval valued $\kappa -$ kernel symmetric matrices, but the converse is not required. Fundamental properties of interval valued $\kappa -$ kernel symmetric fuzzy matrices are derived.

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I .INTRODUCTION

The approach of an interval valued fuzzy matrix is the one of the current subjects evolved for managing unreliability found in maximum of our actual lifestyles circumstance [5]. The premise of an IVFM as an abstract principle of fuzzy matrix became delivered and evolved through Shyamal and Pal [7], by enlarging the max. min operation on fuzzy algebra $\mathcal{F} = [0,1]$ $c + d = max\,\{c, d\}$ and $c.d = min\,\{c, d\}$ for every element $c, d \in \mathcal{F}$ .Let $\mathcal{F}_{mn}$ be the collection of all $m \times n$ fuzzy matrices with the support $[0 \quad 1]$over the fuzzy algebra. In a nutshell, $\mathcal{F}_{nn}$is indicate as $\mathcal{F}_n$ .For $A \in \mathcal{F}_n$, Let$A^T, A^+, R(A), C(A), N(A), \rho\,(A)$ indicate the transpose of matrix A ,generalization of inverse matrix A , row space of A , column space of A , null space of A and the rank of A respectively [1]. We have constitute IVFM $A = \left(a_{ij}\right) = \left(\left[a_{ijL}, a_{ijU}\right]\right)$.where in every $a_{ij}$is the subinterval of interval[0,1],as the interval matrix $A = [A_L, A_U]$ whose ij^{th}entry in the interval $\left[a_{ijL}, a_{ijU}\right]$ .where the lower limit $A_L = a_{ijL}$ and the upper limit $A_U = a_{ijU}$ are fuzzy matrices such that $A_L \leq A_U$ [4].For a fuzzy matrix $A \in \mathcal{F}_n$, If A is range symmetric [9] especially $R(A) = R(A^T)$ implies $N(A) = N(A^T)$. The contrary on the other hand does not have to be true. [2].Letting K be the related permutation matrix [8] and $\kappa -$ be a fixed product of disjoint transpositions in $S_n = 1,2,3, \dots . n$.[6].By the use of this illustration we have provided the method to study the interval valued $\kappa -$kernel symmetric matrices and characterization of interval valued $\kappa -$ kernel symmetric matrices acquired which consist of the end result discovered in [3].

II. PRELIMINARIES

A few theoretical foundations and attributes pertaining to the development of essential outcomes are provided in this study. The collection of all interval valued fuzzy matrices whose entries are the subinterval of the interval $[0,1]$ is referred to as IVFM.

Definition 2.1

For a couple of fuzzy matrices $G = (g_{ij})$and $H = (h_{ij})$in $\mathcal{F}_{mn} \ni G \leq H$, Now let us describe the interval matrix indicate as $[G, H]$, where ij^{th}entry is the interval with lower (bottom) limit g_{ij} and upper (higher) limit h_{ij}, it can be represent that$[g_{ij}, h_{ij}]$.

In specific, For $\mathrm{G} = \mathrm{H}$, IVFM $[\mathrm{G}, \mathrm{G}]$ simplifies to the fuzzy matrices $\mathrm{G} \in \mathcal{F}_{\mathrm{mn}}$ .For $\mathrm{A} = (\mathrm{a_{ij}}) = ([\mathrm{a_{ijL}}, \mathrm{a_{ijU}}]) \in (\mathrm{IVFM})_{\mathrm{mn}}$.Assume that $\mathrm{A_L} = \mathrm{a_{ijL}}$ and $\mathrm{A_U} = \mathrm{a_{ijU}}$ are defined. Clearly $\mathrm{A_L}$ and $\mathrm{A_U} \in \mathcal{F}_{\mathrm{mn}}$ so that$\mathrm{A_L} \leq \mathrm{A_U}$. As a result, A can be stated as $\mathrm{A} = [\mathrm{A_L}, \mathrm{A_U}]$ where $\mathrm{A_L}$and $\mathrm{A_U}$ are the lower and upper limits respectively.

The primary processing on IVFM as described in [7] has been performed here.

For$X = (x_{ij}) = ([x_{ijL}, x_{ijU}])$ and $Y = (y_{ij}) = ([y_{ijL}, y_{ijU}])$ of order $m \times n$

Here

$$X + Y = (x_{ij} + y_{ij}) = ([x_{ijL} + y_{ijL}, x_{ijU} + y_{ijU}]) \dots\dots\dots (2.1.1)$$

For $X = (x_{ij})_{m\times n}$ and $Y = (y_{ij})_{n\times p}$ their product denoted as XY defined as,

$$XY = (z_{ij})_{m\times p} = \left[\sum_{k=1}^{n} a_{ik}\ b_{kj}\right] \qquad i = 1,2,\dots..m \ and \ j = 1,2,\dots\dots p$$

$$= \left[\sum_{k=1}^{n}(a_{ik\,L} \cdot b_{kjL}), \sum_{k=1}^{n}(a_{ik\,U} \cdot b_{kjU}),\right] \dots\dots\dots\dots\dots\dots (2.1.2)$$

$X \leq Y$ if and only if $x_{ijL} \leq y_{ijL}$ and $x_{ijU} \leq y_{ijU}$

In specific, if $x_{ijL} = x_{ijU}$ and $y_{ijL} = y_{ijU}$ then (2.1.2) simplifies to the standard max. min composition of fuzzy matrix [1, 2]

Definition 2.2

For $x = x_1,, \dots\dots . x_{\mathrm{n}} \in \mathcal{F}_{1\times\mathrm{n}}$ Let us described the function

$\mathrm{K}(x) = x_{\mathrm{k(1)}}, x_{\mathrm{k(2)}}, \dots\dots . x_{\mathrm{k(n)}} \in \mathcal{F}_{\mathrm{n}\times 1}$.

Given that K is involuntary, the related permutation matrix may be shown to satisfy the conditions listed in [2]. K is already a permutation matrix, $\mathrm{KK^T} = \mathrm{K^TK} = \mathrm{I}_{\mathrm{n}\times\mathrm{n}}$ and K is an involution, especially $\mathrm{K}^2 = \mathrm{I}$ therefore $\mathrm{K^T} = \mathrm{K}$

$K^T = \mathrm{K}$ and $K(x) = \mathrm{K}\,x$ for $\mathrm{A} \in \mathcal{F}_{\mathrm{n}}$ ……….. (C.2.1)

$N(A) = \mathrm{N(AK)}$ ……..….. (C.2.2)

If A^{+}exist then $(KA^{+}) = A^{+}\mathrm{K}$ and$(AK)^{+} = KA^{+}$ ……….. (C.2.3)

If A^{+} exist ⟺ A^T is a generalize inverse of A ……….. (C.2.4)

Definition 2.3

For$A \in \mathcal{F}_n$ is a kernel symmetric if $N(A) = N(A^T)$ where $N(A) = \{x\,/xA = 0 \ and \ x \in \mathcal{F}_{1\times n}\}$

Definition 2.4

For$A \in \mathcal{F}_n$ is a range symmetric if $R(A) = R(A^T)$. where $R(A) = \{(x, 0\,)\ x \in \mathcal{F}\ \}$

Lemma: 2.5

For$A\,, B \in \mathcal{F}_n$ and P being a permutation matrix $N(A) = N(B) \iff N(PAP^T) = N(PBP^T)$

Theorem: 2.6

For$A \in \mathcal{F}_n$ the subsequent statements are identical

1) $\mathrm{A} \in \mathcal{F}_{\mathrm{n}}$ is kernel symmetric fuzzy matrix
2) For a few permutation matrix P, PAP^T is kernel symmetric
3) There exist a permutation matrix P with $\det D > 0$ so that $PAP^T = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}$

III. INTERVAL VALUED $\kappa -$ KERNEL SYMMETRIC FUZZY MATRICES

The concepts of interval valued $\kappa -$ symmetric matrices and interval valued $\kappa -$kernel symmetric matrices are introduced in this section. Some results on interval valued $\kappa -$kernel symmetric matrices are also discussed.

Definition 3.1

For a matrix $A = [A_L, A_U] \in IVFM_{nn}$is stated to be interval valued kernel symmetric if $N(A_L) = N({A_L}^T)$ where $N(A_L) = \{x\,/xA_L = 0 \ and \ x \in \mathcal{F}_{1\times n}\}$.

$N(A_U) = N({A_U}^T)$,where $N(A_U) = \{x\,/xA_U = 0 \ and \ x \in \mathcal{F}_{1\times n}\}$

The principles that follow will also be used wisely.

Remark 3.2

For a matrix$A = [A_L, A_U] \in IVFM_{nn}$with $\det A > 0$.By the definition, $A = [A_L, A_U]$ has no $'\,0\,'$ row and no $'\,0\,'$ columns.

Hence $N(A_L) = \{0\} = N({A_L}^T)$,$N(A_U) = \{0\} = N({A_U}^T)$

Further for a symmetric matrix $A = A^T$, hence $N(A_L) = N({A_L}^T)$, $N(A_U) = N({A_U}^T)$

Thus, a kernel symmetric matrix is a generalization of interval valued symmetric fuzzy matrix with positive determinant.

Remark 3.3

In general, there is no relation between regular, kernel symmetric and range symmetric interval value fuzzy matrices

Example 3.4

Let$A = [A_L, A_U] = \begin{bmatrix} [1,1] & [0,0] \\ [1,1] & [0,0] \end{bmatrix}$, ${A_L}^2 = A_L$, ${A_U}^2 = A_U$, A is the idempotent. Hence A is regular.

$$Here\ [A_L] = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, [A_U] = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, [{A_L}^T] = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\ and\ [{A_U}^T] = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$R(A_L) = R(A_U) = \{(x, 0) \ / \ x \in \mathcal{F}\}$$

$R(A_L) = R(A_U) = R({A^T}_U) = \{(x, x) \ / \ x \in \mathcal{F}\}$

$R(A_L) \neq R(A_L)$

Hence $A = [A_L, A_U]$ is not interval valued range symmetric matrix.

$$N(A_L) = N(A_U) = \{0\}$$

$N({A^T}_L) = N\ ({A^T}_U) = \{(0, x) \ / \ x \in \mathcal{F}\}$

$N(A_L) \neq N(A_L)$

Hence $A = [A_L, A_U]$ is not interval valued kernel symmetric matrix.

Example 3.5

Let $A = [A_L, A_U] = \begin{bmatrix} [1,1] & [1,1] \\ [1,1] & [1,1] \end{bmatrix}$, A is symmetric idempotent .Hence A is regular .Also interval valued range symmetric and interval valued kernel symmetric matrix.

$$Here\ [A_L] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, [A_U] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, [{A_L}^T] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\ and\ [{A_U}^T] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$R(A_L) = R({A_L}^T) = \{0\} \Rightarrow R(A_U) = R({A_U}^T) = \{0\}$

$$N(A_L) = N({A_L}^T) = \{0\} \Rightarrow N(A_U) = N({A_U}^T) = \{0\}$$

Hence $A = [A_L, A_U]$ is an interval valued range symmetric and interval valued kernel symmetric matrix.

Lemma 3.6

For $A, B \in IVFM_{n\times n}$ and P being a Permutation matrix

$N(A_L) = N(B_L) \iff N(PA_LP^T) = N(PB_LP^T)$

$N(A_U) = N(B_U) \iff N(PA_UP^T) = N(PB_UP^T)$

Proof

For$A = [A_L, A_U]$, $B = [B_L, B_U] \in IVFM_{n\times n}$, P being a Permutation matrix, $if\ N(A) = N(B)$

Let $x \in N(PA_LP^T)$

$\Rightarrow x(PA_LP^T) = 0$

$\Rightarrow yP^T = 0$ where $y = xPA_L$

$\Rightarrow y \in N(P^T)$

Since, $\det \mathrm{P} = \det P^T > 0, N(P^T) = \{0\}$ .Hence $y = 0$

$\Rightarrow yPA_L = 0$

$\Rightarrow xP \in N(A_L) = N(B_L)$

$\Rightarrow x(PB_LP^T) = 0$

$\Rightarrow x \in N(PB_LP^T)$

$\Rightarrow (PA_LP^T) \subseteq N(PB_LP^T)$

Similarly it can be prove that $N(PA_LP^T) \subseteq N(PB_LP^T)$

Thus $N(PA_LP^T) = N(PB_LP^T)$

Conversely if $N(PA_LP^T) = N(PB_LP^T)$ then by the above proof $N(A_L) = N(B_L)$.

Similarly, we can prove that $N(A_U) = N(B_U) \iff N(PA_UP^T) = N(PB_UP^T)$

Hence, in general for, $A = [A_L, A_U]$, $B = [B_L, B_U] \in IVFM_{n\times n}$

$$N(A) = N(B) \iff N(PAP^T) = N(PBP^T)$$

Theorem 3.7

The subsequent statements are equivalent for $A \in IVFM_{n\times n}$

1) $[A_L, A_U] = A$ is an interval valued kernel symmetric

2) For few permutation matrix P, PA_LP^T, PA_UP^T is an interval valued kernel symmetric

3) There exist a permutation matrix P,with det $D > 0$ so that $PA_LP^T = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$, $PA_UP^T = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$

Proof

(1) ⇔(2), this equivalence follow from by the definition 3.1

$N(A_L) = N(A_L{}^T), N(A_U) = N(A_U{}^T)$and

Lemma 3.6,

$$N(A_L) = N(B_L) \iff N(PA_LP^T) = N(PB_LP^T)$$

$$N(A_U) = N(B_U) \iff N(PA_UP^T) = N(PB_UP^T)$$

(1) ⇒ (3)

Let $A = [A_L, A_U]$ be an interval valued kernel symmetric. If $\det A > 0$ then A have no $'\,0\,'$ row and no $'\,0\,'$ columns.

Hence (3) routinely holds by taking $\mathrm{P = I}$ and $\mathrm{D = A}$ itself.If $\det \mathrm{A} = 0$ then

$$\mathrm{N(A_L) = N(A_L{}^T) \neq \{0\}\,, N(A_U) = N(A_U{}^T) \neq \{0\}}$$

For $x \neq 0, x \in N(A_L)$ corresponding to each non zero coefficient$x_i\ of\ x$, the fuzzy sum $\sum x_i\, a_{ik} = and\ \sum x_i\, a_{k\,i} = 0$ for all k

Hence the i^{th} column of A_L and i^{th} row of A_L are full of zeros. Now by permutating the rows and columns suitably, we can move all the zero rows to the bottom and all zero column to the left. Thus $A = [A_L, A_U]$ is of the form $PA_LP^T = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$,$PA_UP^T = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$ where D is a square matrix is. D has no $'\,0\,'$ row and no $'\,0\,'$ columns.

$$\therefore \det D > 0$$

Hence (3)holds.

(3) ⇒ (2)

Since $\det D > 0$, by Remark (3.2) D is kernel symmetric,$\begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix} = PA_LP^T = PA_UP^T$ is also an interval valued kernel symmetric.

Hence (2) holds.

Example 3.8

Let $A = [A_L, A_U] = \begin{bmatrix} [0.2,0.5] & [0,0] & [0.3,0.4] \\ [0,0] & [0,0] & [0,0] \\ [0.2,0.6] & [0,0] & [0.1,0.4] \end{bmatrix}$

Where $A_L = \begin{bmatrix} 0.2 & 0 & 0.3 \\ 0 & 0 & 0 \\ 0.2 & 0 & 0.1 \end{bmatrix}$ and $A_U = \begin{bmatrix} 0.5 & 0 & 0.4 \\ 0 & 0 & 0 \\ 0.6 & 0 & 0.4 \end{bmatrix}$

$i)\ N(A_L) = N({A_L}^T)$ Where $N(A) = \{(0 \quad x \quad 0) \;/\; x \in \mathcal{F}\}$ is an interval valued kernel symmetric

$ii)\ N(A_U) = N({A_U}^T)$ Where $N(A) = \{(0 \quad x \quad 0) \;/\; x \in \mathcal{F}\}$ is an interval valued kernel symmetric

For a Permutation matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Consider $PA_LP^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0 & 0.3 \\ 0 & 0 & 0 \\ 0.2 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

. $$= \begin{bmatrix} [0 \quad 0 \quad 0.2] & [0 \quad 0 \quad 0.1] & [0 \quad 0 \quad 0] \\ [0.2 \quad 0 \quad 0] & [0.3 \quad 0 \quad 0] & [0 \quad 0 \quad 0] \\ [0 \quad 0 \quad 0] & [0 \quad 0 \quad 0] & [0 \quad 0 \quad 0] \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$=. \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.2 & 0.3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} [0 \quad 0 \quad 0] & [0.2 \quad 0 \quad 0] & [0 \quad 0.1 \quad 0] \\ [0 \quad 0 \quad 0] & [0.2 \quad 0 \quad 0] & [0 \quad 0.3 \quad 0] \\ [0 \quad 0 \quad 0] & [0 \quad 0 \quad 0] & [0 \quad 0 \quad 0] \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.2 & 0.1 \\ 0 & 0.2 & 0.3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$$

$PA_LP^T = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$,$\det D > 0$Where $D = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}$

$$N(PA_LP^T) = \{(0 \quad x) \;/\; x \in \mathcal{F}\}$$

PA_LP^T is an interval valued kernel symmetric for a few permutation matrix P

Similarly, for few permutation matrix, $PA_UP^T = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & D \\ 0 & 0 \end{bmatrix}$, PA_UP^T is an interval valued kernel symmetric matrix.

Definition 3.9

If $\mathrm{N(A_L)} = \mathrm{N}\left(\mathrm{KA_L}^{\mathrm{T}}\mathrm{K}\right)$, $\mathrm{N(A_U)} = \mathrm{N}\left(\mathrm{KA_U}^{\mathrm{T}}\mathrm{K}\right)$ a matrix $A = [A_L, A_U] \in IVFM_{nn}$ is defined as an interval valued κ −kernel symmetric matrix.

Remark 3.10

In specific, while $k(j) = j$ for every $j = 1\ to\ n$, the related permutation matrix K becomes the unit matrix, and definition 3.1 becomes to $N(A_L) = N\left({A_L}^T\right)$, $N(A_U) = N\left({A_U}^T\right)$ this is $A = [A_L, A_U]$ is interval valued kernel symmetric. If A is symmetric, then $[A_L, A_U]$ is interval valued κ −kernel symmetric for all transposition of $\kappa \in S_n$

Likewise, an interval valued matrix A is an interval valued κ −symmetric implies it is an interval valued $\kappa -$ kernel Symmetric, for $A_L = K{A_L}^TK$, $A_U = K{A_U}^T\mathrm{K}$ routinely implies $N(A_L) = N\left(K{A_L}^TK\right), N(A_U) = N\left(K{A_U}^TK\right)$. The inverse, on the other hand, does not have to be true. The following examples demonstrate this.

Example 3.11

Let $A = [A_L, A_U] = \begin{bmatrix} [0,0] & [0,0] & [0.2,0.6] \\ [0.3,0.5] & [0.4,0.5] & [0,0] \\ [0.2,0.6] & [0.1,0.3] & [0,0] \end{bmatrix}$

Where $A_L = \begin{bmatrix} 0 & 0 & 0.2 \\ 0.3 & 0.4 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}$ and$A_U = \begin{bmatrix} 0 & 0 & 0.6 \\ 0.5 & 0.4 & 0 \\ 0.6 & 0.3 & 0 \end{bmatrix}$, $K = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$K{A_L}^TK = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0.2 \\ 0.3 & 0.4 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.3 & 0.2 & 0 \\ 0.4 & 0.1 & 0 \end{bmatrix}$$

$$A_L \neq K{A_L}^TK$$

$\therefore A_L$ is not an interval valued κ −symmetric matrix.

For this $N(A_L) = \{0\}$. here A_L has no $'\,0\,'$ row and no $'\,0\,'$ columns.

$$N\left(K{A_L}^TK\right) = \{0\}$$

$$\Rightarrow \ N(A_L) = N\left(K{A_L}^TK\right)$$

$\therefore A_L$ is an interval valued κ −kernel symmetric, but $\therefore A_L$ is not an interval valued $\kappa -$ symmetric

Similarly $\therefore A_U$ is also an interval valued k -kernel symmetric, but $\therefore A_U$ is not $\kappa -$ symmetric

Hence, the interval valued matrix $A = [A_L, A_U]$ is an interval valued κ −kernel symmetric, but not an interval valued $\kappa -$ symmetric.

Lemma 3.12

For $A = [A_L, A_U] \in IVFM_{nn}$, ${A_L}^+$, ${A_U}^+$ exists if and only if $(KA_L)^+, (KA_U)^+$ exits

Proof

For $A = [A_L, A_U] \in IVFM_{nn}$ if ${A_L}^+$ exists then ${A_L}^+ = {A_L}^T$ which ${A_L}^T$ is a generalize inverse of A_L.

Contrarily, while ${A_L}^T$ is a generalize inverse of $A_L$then

$$A_L{A_L}^T A_L = A_L \Longrightarrow {A_L}^T A_L A^T = {A_L}^T.$$

Hence ${A_L}^T$ is a generalize inverse of A_L .

Both $A_L A^T$ *and* ${A_L}^T A_L$ are symmetric.

Hence ${A_L}^T = {A_L}^+$.

$$\begin{aligned} {A_L}^+ \text{ exists } &\Leftrightarrow A_L{A_L}^T A_L = A_L \\ &\Leftrightarrow KA_L{A_L}^T A_L = KA_L \\ &\Leftrightarrow (KA_L)(KA_L)^T(KA_L) = KA_L \\ &\Leftrightarrow (KA_L)^T \in KA_L \\ &\Leftrightarrow (KA_L)^+ exist \qquad \text{(By C.2.4)} \end{aligned}$$

Similarly we can prove that,

$${A_U}^+ \text{ exists } \Leftrightarrow (KA_U)^+ exist$$

Theorem 3.13

The subsequent sentences are equal for $A = [A_L, A_U] \in IVFM_{nn}$

$i)$ $\mathrm{A} = [\mathrm{A_L}, \mathrm{A_U}]$ is an interval valued κ −kernel symmetric

$ii)$ KA_L , KA_U is an interval valued kernel symmetric

$iii)$ $A_L K$, $A_U K$ is an interval valued kernel symmetric

$iv)$ $N(A_L) = N(KA_L), N(A_U) = N(KA_U)$

$$v)\ N(A_L) = N((A_L K)^T)\ , N(A_U) = N((A_U K)^T)$$

Lemma 3.14

Any two of the subsequent conditions imply each other in the case of $A = [A_L, A_U] \in IVFM_{nn}$

(1) $A = [A_L, A_U]$ is kernel symmetric with interval value.

(2) $A = [A_L, A_U]$ is an interval valued κ −kernel symmetric

$$(3)\ N\left(A_L{}^T\right) = N((A_L K)^T)\ , N\left(A_U{}^T\right) = N((A_U K)^T)$$

Proof

Although (1) and (2) $\Longrightarrow$ (3)

$A = [A_L, A_U]$ is an interval valued $\kappa -$ kernel symmetric

$$\Longrightarrow N(A_L) = N\left(KA_L{}^T K\right), N(A_U) = N\left(KA_U{}^T K\right),$$

$$\Longrightarrow N(A_L) = N\left(KA_L{}^T\right), N(A_U) = N\left(KA_U{}^T\right) \qquad \text{(By C.2.2)}$$

Hence (1) and (2) $\Longrightarrow N\left(A_L{}^T\right) = N(A_L) = N((A_L K)^T)$,

$$N\left(A_U{}^T\right) = N(A_U) = N((A_U K)^T)$$

As follows (3) holds.

In addition (1)and (3) $\Longrightarrow$ (2)

$A = [A_L, A_U]$ is an interval valued kernel Symmetric

$$\Longrightarrow N(A_L) = N\left(A_L{}^T\right), N(A_U) = N\left(A_U{}^T\right)$$

Hence (1) and (3) $\Longrightarrow N(A_L) = N((A_L K)^T), N(A_U) = N((A_U K)^T)$

$$\Longrightarrow N(A_L K) = N((A_L K)^T), N(A_U K) = N((A_U K)^T)$$

(By C.2.2)

$\Longrightarrow AK = [A_L K\ , A_U K]$ is an interval valued kernel Symmetric

$\Longrightarrow A = [A_L, A_U]$ is an interval valued $\kappa -$ kernel Symmetric

(By theorem 3.13)

As a result (2) holds.

On other hands (2) and (3) $\Longrightarrow$ (1):

$A = [A_L, A_U]$ is an interval valued $\kappa -$ kernel Symmetric

$$\Longrightarrow\ N(A_L) = N\left(KA_L{}^T K\right), N(A_U) = N\left(KA_U{}^T K\right)$$

$$\Longrightarrow N(A_L) = N((A_L K)^T), N(A_L) = N((A_L K)^T) \qquad \text{(By C. 2.2)}$$

Hence (1) and (3) $\Longrightarrow\ N(A_L) = N\left(A_L{}^T\right), N(A_U) = N\left(A_U{}^T\right)$

As a result (1) holds.

Hence the proof

To characterize a matrix as interval valued $\kappa -$kernel symmetric, we initially show the subsequent lemma.

Lemma 3.15

Letting $B = [B_L, B_U] = \begin{bmatrix} 0 & [D_L, D_U] \\ 0 & 0 \end{bmatrix}$ where $D = [D_L, D_U]$ is $p \times p$ interval valued fuzzy matrix with no $'\ 0\ '$ row and no $'\ 0\ '$ columns, then the subsequent equivalent condition holds:

(1) $B = [B_L, B_U]$ is an interval valued $\kappa -$ kernel symmetric

(2) $N\left(B_L{}^T\right) = N((B_L K)^T), N\left(B_U{}^T\right) = N((B_U K)^T)$

(3) $K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$ where K_1 *and* K_2 are permutation matrixes of order p *and* $n - p$ respectively.

(4) $k = k_1 k_2$ Where k_1 is the combination of disjointed inversion matrices on $S_n = 1,2,3, \dots .n$ leaving $p + 1, p + 2, \dots .., n$ invariant and k_2 is the combination of disjointed inversion leaving $1,2, \dots ..p$ invariant.

Proof.

Let $D = [D_L, D_U] = \begin{bmatrix} 0 & [D_L, D_U] \\ 0 & 0 \end{bmatrix}$ have no $'\ 0\ '$ row and no $'\ 0\ '$ columns.

$N(D_L) = N(D_L)^T = \{0\}\ \ N(D_U) = N(D_U)^T = \{0\}$

$\therefore\ N(B_L) = N(B_L)^T \neq \{0\}\ \ N(B_U) = N(B_U)^T \neq \{0\}$, and

$B = [B_L, B_U]$ is an interval valued kernel symmetric

We will now illustrate the equivalence of (1), (2) and (3)

Here $B = [B_L, B_U]$ is an interval valued $\kappa -$ kernel symmetric

$$\Leftrightarrow N\left(B_L{}^T\right) = N((B_L K)^T), N\left(B_U{}^T\right) = ((B_U K)^T) \qquad \text{(By lemma 3.14)}$$

Select $z = [0 \quad x]$ with each component of $x \neq 0$ and partitioned in conformity with that of

$B = [B_L, B_U] = \begin{bmatrix} 0 & [D_L, D_U] \\ 0 & 0 \end{bmatrix}$.

Clearly, $z \in N(B_L) = N\left({B_L}^T\right) = N((B_L K)^T)$

$z \in N(B_U) = N\left({B_U}^T\right) = N((B_U K)^T)$

Let $K = \begin{bmatrix} K_1 & K_3 \\ {K_3}^T & K_2 \end{bmatrix}$ then

$$K{B_L}^T = \begin{bmatrix} K_1 & K_3 \\ {K_3}^T & K_2 \end{bmatrix}\begin{bmatrix} 0 & {D_L}^T \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & K_1{D_L}^T \\ 0 & {K_3}^T{D_L}^T \end{bmatrix}$$

Now

$$z = [0 \quad x] \in N(B_L) = N((B_L K)^T)$$

$$\Rightarrow [0 \quad x]\begin{bmatrix} 0 & K_1{D_L}^T \\ 0 & {K_3}^T{D_L}^T \end{bmatrix} = 0$$

$$\Rightarrow [0 \quad x{K_3}^T{D_L}^T]$$

$$\Rightarrow x\,{K_3}^T{D_L}^T = 0$$

Since$N(D_L)^T = 0$, in such a way $\quad x\,{K_3}^T = 0$

Consequently every component of $\ x \neq 0$ under max- min composition

$$x\,{K_3}^T = 0$$

$$\Rightarrow {K_3}^T = 0$$

$$\Rightarrow K_3 = 0$$

$$\therefore K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$$

As a result, (3) is established.

Versus, if (3) obtains, on the other hand,

$$K{B_L}^T = \begin{bmatrix} 0 & K_1{D_L}^T \\ 0 & 0 \end{bmatrix}$$

$$N\left(K{B_L}^T\right) = N\left({B_L}^T\right)$$

In the same way, we can prove that, $N\left(K{B_U}^T\right) = N\left({B_U}^T\right)$

As a result $(1) \Leftrightarrow (2) \Leftrightarrow (3)$ holds.

Moreover $(3) \Leftrightarrow (4)$ the definition of κ shows the equivalence of(3) and (4).

IV. CONCLUSION

We defined the interval valued kernel symmetric and interval valued $\kappa -$ kernel symmetric fuzzy matrices in this work. In addition we have investigated into some Proposition of interval valued $\kappa -$ kernel symmetric fuzzy matrices with examples.

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