Nano Regular B-Closed Sets and Nano Regular B-Open Sets

P. SRIVIDHYA[1], Dr. T. INDIRA[2],

[1]Research Scholar in Department of Mathematics,
Seethalakshmi Ramaswami College (Affiliated to Bharathidasan University),
Tiruchirappalli, TamilNadu, India.
psrividhya97@gmail.com
[2]Associate Professor in Department of Mathematics,
Seethalakshmi Ramaswami College (Affiliated to Bharathidasan University),
Tiruchirappalli, TamilNadu, India.
drtindira.chandru@gmail.com

Article Info
Page Number: 892-911
Publication Issue:
Vol. 71 No. 4 (2022)

Article History
Article Received: 25 March 2022
Revised: 30 April 2022
Accepted: 15 June 2022
Publication: 19 August 2022

Abstract
This research paper is to introduce a nano closed and nano open sets in nano topological spaces namely, Nano regular b-closed sets and Nano regular b-open sets and to compare them with some other Nano closed sets such that Nano regular closed, Nano generalized closed, etc. Then the Nano regular b-interior, Nano regular b-closure are introduced and discussed their properties.
Keywords: Nano regular b-open sets, Nano regular b-closed sets, Nano regular b-interior and Nano regular b-closure.

§1. INTRODUCTION

In topological space, Levine [10] introduced the concept of generalized closed sets in 1970. Later, in 1990, N. Palaniappan [14] investigated the concept of regular generalized closed sets in a topological space. Sharmistha Bhattacharya [19] introduced the concept of generalized regular closed sets in topological space in 2011. In 1996, Andrijevic [2] introduced b-open sets, a new class of generalized open sets. Later that year, A. Al-omari and M. S. M. Noorani [1] proposed the generalized closed set class. In 2012, A. Narmadha and N. Nagaveni [11] introduced regular b-cloesd sets. A. Narmadha, N. Nagaveni, and T. Noiri [12] invented regular b-open sets in 2013. Lellis Thivagar [9] introduced the concept of nano topology in 2013, as well as certain weak forms of nano open sets such as nano α-open sets, nano semi-open sets, and nano pre open sets. We introduced a new class of open and closed sets in nano topological spaces called Nrb-open sets and Nrb-closed sets in this paper.

§2. PRELIMINARIES

Definition: 2.1 [9]

Let U denote a non-empty finite set of objects known as the universe, and R denote an equivalence relation on U known as the indiscernibility relation. After that, U is divided into disjoint equivalence classes. Elements in the same equivalence class are said to be indiscernible from one another. The approximation space is defined as the pair (U, R). Let X ⊆ U. Then

- The lower approximation of X with respect to R is the set of all objects that can be defined as X with respect to R and is denoted by $L_R(X)$.

$L_R(X) = \bigcup_{x\in U}\{R(x): R(x) \subseteq X\}$

- The upper approximation of X with respect to R is the set of all objects that can be characterized as X with respect to R and is denoted by $U_R(X)$.

$$U_R(X) = \bigcup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$$

- The boundary region of X with respect to R is the set of all objects that can be labelled neither as X nor as not -X with respect to R and is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$.

Example: 2.2

Let U = {1,2,3,4}, X = {1,2}, U/R = {{1},{3}, {2,4}}.

$L_R(X)$ = {1}, $U_R(X)$ ={1}∪{2,4}={1,2,4},

$B_R(X) = U_R(X) - L_R(X)$ = {1,2,4} - {1}={2,4}.

Definition: 2.3 [9]

Let U represents the universe, R represents the equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where X⊆U. Then $\tau_R(X)$ obeys the below axioms:

- U and $\emptyset \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of any finite sub collection of $\tau_R(X)$ elements is in $\tau_R(X)$. Then $\tau_R(X)$ is called the nano topology on U with respect to X, $(U,\tau_R(X))$ is known as the nano topological space.

Elements of the nano topology are renowned as nano open sets. Nano closed sets are the

complement of elements from nano open sets.

Definition: 2.4 [9]

If $(U,\tau_R(X))$ is a nano topological space with respect to X where X ⊆ U and if A ⊆ U , then

- Nint(A) denotes the nano interior of a set A, which is defined as the union of all nano open sets contained in A.
- The intersection of all nano closed sets containing A is defined as the nano closure of a set A, and it is denoted by Ncl(A).

Example: 2.5

Let U = {1,2,3,4}, X = {1,2}, U/R = {{1},{3}, {2,4}},$\tau_R(X)$ = {U, ∅,{1}, {1,2,4}, {2,4}}.

Nano open sets={U,∅,{1},{1,2,4},{2,4}}, Nano closed sets={U, ∅,{2,3,4},{3},{1,3}}.

Let A={1,2}, Nint(A) ={1}∪ ∅ ={1}, Ncl(A)=U.

Definition: 2.6

Let A be a subset of a nano topological space (U,(τ_R(X)).

- If Ncl(Nint(A)) ⊆ A then A is Nano pre closed [9]
- If Nint(Ncl(A)) ⊆ A then A is Nano semi closed [9]
- If A= Ncl(Nint(A)) then Ais Nano regular closed [20]
- If Ncl(Nint(Ncl(A))) ⊆ A then A is Nano α (pre semi) closed [9]
- If Nint(Ncl(Nint(A))) ⊆ A then A is Nano β (semi pre) closed [18]
- If Nint(Ncl(A))∩Ncl(Nint(A)) ⊆ A then A is Nano b-closed [15]
- If A=Nclδ(A), where Nclδ(A) = {x∈U, Nint(Ncl(G))∩A=φ,G∈ τ_R(X) and x∈G} then

A is Nano δ-closed [16]

Example : 2.7

Let U = {1,2,3,4}, X = {1,2}, U/R = {{1},{3}, {2,4}},τ_R(X)= {U, ∅,{1}, {1,2,4}, {2,4}}.

Nano open sets = {U, ∅,{1}, {1,2,4}, {2,4}}, Nano closed sets = {U, ∅,{2,3,4},{3}, {1,3}}.

Nano pre closed sets = {U, ∅,{2},{3},{4}, {1,3}, {2,3}, {3,4}, {1,2,3}, {1,3,4}, {2,3,4}}.

Nano semi closed sets = {U, ∅,{1},{3}, {1,3}, {2,4}, {2,3,4}}.

Nano regular closed sets = {U, ∅,{1,3}, {2,3,4}.

Nano α -closed sets = {U, ∅,{3}, {1,3}, {2,3,4}}.

Nano β -closed sets = {U, ∅,{1},{2},{3},{4}, {1,2}, {1,3}, {1,4}, {2,3},{2,4},{3,4},{1,2,3}, {1,3,4},{2,3,4}}.

Nano b-closed sets= {U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3}, {1,3,4}, {2,3,4}}.

Nano δ -closed sets = {U, ∅,{1},{2,4},{1,2,3},{1,3,4}}.

Definition : 2.8

Let A be a subset of a nano topological space (U,τ_R(X)).

1) Nano generalized closed ⟹ Ncl(A) ⊆ G whenever A ⊆ G and G is nano open in U.[5]
2) Nano generalized semi closed ⟹ Nscl(A) ⊆ G whenever A ⊆ G and G is nano open in U.[4]
3) Nano semi generalized closed ⟹ Nscl(A) ⊆ G whenever A ⊆ G and G is nano semi open in U.[4]
4) Nano generalized pre closed ⟹ Npcl(A) ⊆ G whenever A ⊆ G and G is nano open in U.[6]
5) Nano pre generalized closed ⟹ Npcl(A) ⊆ G whenever A ⊆ G and G is nano pre open in U.[6]

6) Nano α-generalized closed $\Longrightarrow$ Nαcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.[21]
7) Nano generalized α-closed $\Longrightarrow$ Nαcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano α-open in U.[21]
8) Nano generalized β-closed $\Longrightarrow$ Nβcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.[17]
9) Nano β-generalized closed $\Longrightarrow$ Nβcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano β-open in U.[17]
10) Nano generalized regular closed $\Longrightarrow$ Nrcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U. [20]
11) Nano regular generalized closed $\Longrightarrow$ Nrcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano regular open in U.[20]
12) Nano generalized b-closed $\Longrightarrow$ Nbcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.[7]
13) Nano b-generalized closed $\Longrightarrow$ Nbcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano b-open in U.[7]
14) Nano δ-generalized closed $\Longrightarrow$ Nδcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.[9]

Example: 2.9

Let U={1,2,3,4}, X={1,2},U/R = {{1},{3}{2,4}}, τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nano open sets ={U, ∅,{1},{1,2,4},{2,4}}, Nano closed sets={U, ∅,{2,3,4},{3},{1,3}}.

Ng-closed sets ={U, ∅,{3},{1,3},{2,3},{3,4}{1,2,3},{1,3,4},{2,3,4}}.

Ngs- closed sets ={U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nsg- closed sets = {U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Ngp- closed sets = {U, ∅,{2},{3},{4},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Npg- closed sets = {U, ∅,{2},{3},{4},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nαg- closed sets = {U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Ngα - closed sets = {U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Ngr- closed sets = {U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrg- closed sets = {U, ∅,{3},{1,3},{1,4},{2,3},{3,4},{1,2,3},{1,2,4},{1,3,4},{2,3,4}}.

Ngβ - closed sets = {U, ∅,{1},{2},{3},{4},{1,2}{1,3},{1,4},{2,3},{2,4},{3,4},{1,2,3},

{1,3,4},{2,3,4}}.

Nβg- closed sets = {U, ∅,{1},{2},{3},{4},{1,2}{1,3},{1,4},{2,3},{2,4},{3,4},{1,2,3},

{1,3,4},{2,3,4}}.

Ngb- closed sets = {U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},

{2,3,4}}.

Nbg- closed sets = {U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},

{2,3,4}}.

Nano δg-closed sets = {U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}},{2,3,4}.

§3. Nano regular b-closed sets (Nrb-closed sets)

Definition: 3.1

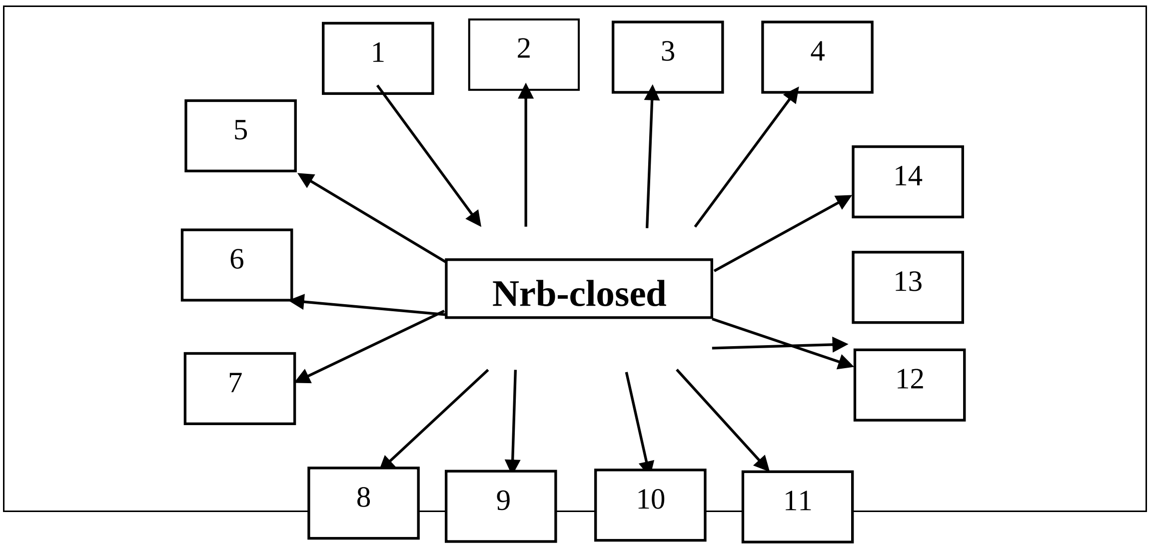
If Nrcl(A) ⊂ G whenever A ⊂ G and G is nano b-open in U then A subset A of a nano topological spaces (U,τ_R(X)) is said to be "Nano regular b-closed".

Example: 3.2

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Result: 3.3



1. Nano regular closed 2. Ng-closed 3. Ngs-closed 4. Nsg-closed

5. Ngp-closed 6. Npg-closed 7. Nαg-closed 8. Ngα -closed

9. Ngβ - closed 10. Nβg- closed 11. Ngr-closed 12. Nrg-closed

13. Nbg-closed 14. Ngb-closed.

Theorem: 3.4

Every nano regular closed set is Nrb-closed but not conversely.

Proof:

Let A be a nano regular closed set and G be a nano b-open set such that A⊂G. Then Nrcl(A)=A⊂G ⇒ Nrcl(A)⊂G whenever A⊂G and G is nano b-open.

Hence A is nano rb-closed.

Example 3.5:

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U,∅,{1},{1,2,4},{2,4}}.

Nano regular closed sets={ U, ∅,{1,3},{2,3,4}}, Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={3} is Nrb-closed set but not Nano regular closed.

Hence the converse part of the theorem 3.4 is proved.

Theorem: 3.6

Every Nrb-closed is Ng-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano open and let A⊂G. To prove that A is Ng-closed. But, Every nano open set is nano b-open.⇒ G is nano b-open. By our assumption A is Nrb-closed, we have Nrcl(A) ⊂ G. But Ncl(A) ⊂ Nrcl(A) ⊂ G.

⇒ Ncl(A) ⊂ G whenever A ⊂ G and G is nano open.

∴ A is Ng-closed.

Example: 3.7

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}}, τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}},

Ng closed sets={ U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Here A={{2,3},{3,4},{1,2,3},{1,3,4}} are Ng-closed sets but not Nrb-closed sets.

Hence the converse of the theorem 3.6 is proved.

Theorem: 3.8

Every Nrb-closed set is Ngs-closed but not conversely.

Proof:

Let A is Nrb-closed and G be nano open and let A ⊂ G.To prove that A is Ngs-closed. But, Every nano open set is nano b-open, we get G is nano b-open. By our assumption A is Nrb-closed, we have Nrcl(A) ⊂ G. We know that Nscl(A) ⊂ Nrcl(A).

$\Rightarrow$Nscl(A) $\subset$ Nrcl(A) $\subset$ G $\Rightarrow$ Nscl(A) $\subset$ G whenever A $\subset$ G and G is nano open.

$\therefore$ A is Ngs-closed.

Example: 3.9

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U,∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Ngs-closed sets={ U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Here A={{1},{2},{4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}} are Ngs-closed sets but not Nrb-closed sets. From this example the converse part of the theorem 3.8 is proved.

Theorem: 3.10

Every Nrb-closed set is Nsg-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano semi open and let A $\subset$ G. To prove that A is Nsg-closed. By the theorem, Every nano semi open set is nano b-open, we get G is nano b-open. Since A is Nrb-closed, we have Nrcl(A) $\subset$ G. We know that Nscl(A) $\subset$ Nrcl(A).

$\Rightarrow$ Nscl(A) $\subset$ Nrcl(A) $\subset$ G$\Rightarrow$Nscl(A) $\subset$ G whenever A $\subset$ G and G is nano semi open.

$\therefore$ A is Nsg-closed.

Example: 3.11

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}}, τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Nsg-closed sets={ U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4}{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Here A={{1},{2},{4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}} are Ngs-closed sets but not Nrb-closed sets. From this example the converse part of the theorem 3.10 is proved.

Theorem: 3.12

Every Nrb-closed set is Ngp-closed but not conversely.

Proof:

Let A is Nrb-closed and G be nano open and let A $\subset$ G.To prove that A is Ngp-closed. But, Every nano open set is nano b-open, we get G is nano b-open. By our assumption A is Nrb-closed, we have Nrcl(A) $\subset$ G. We know that Npcl(A) $\subset$ Nrcl(A).

$\Rightarrow$ Npcl(A) $\subset$ Nrcl(A) $\subset$ G $\Rightarrow$ Npcl(A) $\subset$ G whenever A $\subset$ G and G is nano open.

∴ A is Ngp-closed.

Example: 3.13

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}}, $\tau_R(X)$ = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Ngp closed sets={ U, ∅,{2},{3},{4},{1,3},{2,3},{2,4}{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Here A={{2},{4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}} are Ngp-closed sets but not Nrb-closed sets. Hence the converse part of the theorem 3.12 is proved.

Theorem: 3.14

Every Nrb-closed set is Npg-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano pre open and let A ⊂ G. To prove that A is Npg-closed. But, Every nano pre open set is nano b-open, we get G is nano b-open. Since A is Nrb-closed, we have Nrcl(A) ⊂ G. We know that Npcl(A) ⊂ Nrcl(A).

⇒ Npcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Npcl(A) ⊂ G whenever A ⊂ G and G is nano pre open.

∴ A is Npg-closed.

Example: 3.15

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}}, $\tau_R(X)$ = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Npg-closed sets={ U, ∅,{2},{3},{4},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Here A={{2},{4},{2,3},{3,4},{1,2,3},{1,3,4}} are Npg-closed sets but not Nrb-closed sets.

From the above example the converse part of the theorem 3.14 is proved.

Theorem: 3.16

Every Nrb-closed set is Nαg-closed but not conversely.

Proof:

Let A is Nrb-closed and G be nano open and let A ⊂ G.To prove that A is Ngp-closed. But, Every nano open set is nano b-open, we get G is nano b-open. By our assumption A is Nrb-closed, we have Nrcl(A) ⊂ G. We know that Nαcl(A) ⊂ Nrcl(A).

⇒ Nαcl(A) ⊂ Nrcl(A) ⊂ G⇒ Nαcl(A) ⊂ G whenever A ⊂ G and G is nano open.

∴ A is Nαg-closed.

Example: 3.17

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nαg-closed sets={ U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{2,3},{3,4},{1,2,3},{1,3,4}} are Nαg-closed sets but not Nrb-closed sets.

Hence the converse part of the theorem 3.16 is proved.

Theorem: 3.18

Every Nrb-closed set is Ngα-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano α -open and let A ⊂ G.To prove that A is Npg-closed. But, Every nano αopen set is nano b-open, we get G is nano b-open. Since A is Nrb-closed, we have Nrcl(A) ⊂ G. We know that Nαcl(A) ⊂ Nrcl(A).

⇒ Nαcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nαcl(A) ⊂ G whenever A ⊂ G and G is nano α-open.

∴ A is Ngα -closed.

Example: 3.19

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}}, τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Ngα -closed sets={ U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{2,3},{3,4},{1,2,3},{1,3,4}} are Ngα - closed sets but not Nrb-closed sets.

From the above example the converse part of the theorem 3.18 is proved.

Theorem: 3.20

Every Nrb-closed set is Ngβ-closed but not conversely.

Proof:

Let A is Nrb-closed and G be nano open and let A ⊂ G.To prove that A is Ngβ-closed. But, Every nano open set is nano b-open, we get G is nano b-open. By our assumption A is Nrb-closed, we have Nrcl(A) ⊂ G. We know that Nβcl(A) ⊂ Nrcl(A).

⇒ Nβcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nβcl(A) ⊂ G whenever A ⊂ G and G is nano open.

∴ A is Ngβ-closed.

Example: 3.21

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nano open sets={U, ∅,{1},{1,2,4},{2,4}}, Nano closed sets={U, ∅,{2,3,4},{3},{1,3}}.

Ngβ-closed sets ={U, ∅,{1},{2},{3},{4},{1,2},{1,3},{1,4},{2,3},{3,4},{1,2,3},{1,3,4},

{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{1},{2},{4},{1,2},{1,4},{2,3},{3,4},{1,2,3},{1,3,4}} are Ngβ-closed sets but not Nrb-closed sets. Hence the converse part of the theorem 3.20 is proved.

Theorem: 3.22

Every Nrb-closed set is Ngr-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano open and let A ⊂ G. To prove that A is Ngr-closed. But, Every nano open set is nano b-open, we get G is nano b-open. Since A is Nrb-closed. So we have , Nrcl(A) ⊂ G whenever A ⊂ G and G is nano open.

∴ A is Nαg-closed.

Example: 3.23

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Ngr-closed sets={ U, ∅,{3},{1,3},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{2,3},{3,4},{1,2,3},{1,3,4}} are Ngr-closed sets but not Nrb-closed sets. From the above example the converse part of the theorem 3.22 is proved.

Theorem: 3.24

Every Nrb-closed set is Nrg-closed but not conversely.

Proof:

Let A is Nrb-closed and G be nano open and let A ⊂ G.To prove that A is Nrg-closed. But, Every nano regular open set is nano b-open, So we get G is nano b-open. By our assumption A is Nrb-closed, and so we get Nrcl(A) ⊂ G whenever A ⊂ G and G is nano regular open. ∴ A is Nrg-closed.

Example: 3.25

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, φ,{1},{1,2,4},{2,4}}.

Nrg- closed sets={ U, ∅,{3},{1,3},{1,4},{2,3},{3,4},{1,2,3},{1,2,4},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{1,4},{2,3},{3,4},{1,2,3},{1,2,4},{1,3,4}} are Nrg-closed sets but not Nrb-closed sets. Hence the converse part of the theorem 3.24 is proved.

Theorem: 3.26

Every Nrb-closed set is Nbg-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano b-open and let A ⊂ G.To prove that A is Nbg-closed. Since A is Nrb-closed, So we get Nrcl(A) ⊂ G.

We know that Nbcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nbcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nbcl(A) ⊂ G whenever A ⊂ G and G is nano b-open.

∴ A is Nbg-closed.

Example: 3.27

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nbg closed sets={ U,∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{1},{2},{4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}} are Nbg-closed sets but not Nrg-closed sets. From the above example the converse part of the theorem 3.26 is proved.

Theorem: 3.28

Every Nrb-closed set is Ngb-closed but not conversely.

Proof:

Let A is Nrb-closed and G be nano open and let A ⊂ G.To prove that A is Ngb-closed. But, Every nano open set is nano b-open, we get G is nano b-open. By our assumption A is Nrb-closed, we have Nrcl(A) ⊂ G. We know that Nbcl(A) ⊂ Nrcl(A).

⇒ Nbcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nbcl(A) ⊂ G whenever A ⊂ G and G is nano open.

∴ A is Ngb-closed.

Example: 3.29

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Ngb closed sets={ U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{1},{2},{4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}} are Ngb-closed sets but not Nrb-closed sets. Hence the converse part of the theorem 3.28 is proved.

Theorem: 3.30

Every Nrb-closed set is Nδg-closed but not conversely.

Proof:

Assume that A is Nrb-closed and G be nano open and let A ⊂ G.To prove that A is Nδg-closed. Since A is Nrb-closed, So we get Nrcl(A) ⊂ G.

We know that Nδcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nδcl(A) ⊂ Nrcl(A) ⊂ G ⇒ Nδcl(A) ⊂ G whenever A ⊂ G and G is nano b-open.

∴ A is Nδg-closed.

Example: 3.31

Let U={1,2,3,4}, X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nδg closed sets={ U, ∅,{1},{2},{3},{4},{1,3},{2,3},{2,4},{3,4},{1,2,3},{1,3,4},{2,3,4}}.

Nrb-closed sets={ U, ∅,{3},{1,3},{2,3,4}}.

Here A={{1},{2},{4},{2,3},{2,4},{3,4},{1,2,3},{1,3,4}} are Nsg closed sets but not Nbg-closed sets. From the above example the converse part of the theorem 3.30 is proved.

Theorem: 3.32

If A and B are Nrb-closed set, then A∪B is Nrb-closed.

Proof:

Given that A and B are Nrb-closed. To Prove A∪B is nano rb-closed. Let A∪B ⊂ G and G is nano b-open set ⇒ A ⊂ G and B ⊂ G and G is nano b-open ⇒ Nrcl(A) ⊂ G and Nrcl(B) ⊂ G. (∵A and B are Nrb-closed). Then, we have Nrcl(A∪B) = Nrcl(A)∪Nrcl(B)⊂G. ∴ Nrcl(A∪B) ⊂ G. Hence A∪B is Nrb-closed.

Theorem: 3.33

If A and B are Nrb-closed, then A⋂B is Nrb-closed.

Proof:

Let A and B are any two Nrb-closed sets. Assume that A ⊆ V and B ⊆ V where V is a nano regular open. Then Nrcl(A)⊆V and Nrcl(B)⊆V ⇒ Nrcl(A∩B) = Nrcl(A)∩Nrcl(B) ⊆ V. (i.e) Nrcl(A∩B) ⊆ V, where V is nano regular open.

Hence A∩B is Nrb-closed.

Theorem: 3.34

If A is Nrb-closed set in $(U,\tau_R(X))$ then Nrcl(A)-A does not contain any non-empty Nano b-closed set.

Proof:

Assume that A is Nrb-closed set. To prove Nrcl(A)-A does not contain any non-empty nano b-closed set. Suppose F be any non-empty Nano b-closed set contained in Nrcl(A)-A.

(i.e) $F \subset \text{Nrcl}(A)\text{-}A = \text{Nrcl}(A) \cap A^C \Rightarrow F \subset \text{Nrcl}(A) \cap A^C$.

Then $F \subset \text{Nrcl}(A)$ and $F \subset A^C$ $\rightarrow$ (1)

Consider $F \subset A^C \Rightarrow F^C \subset A$. (i.e) $A \subset F^C$, where F^C is non-empty nano b-open. Since A is Nrb-closed, then $\text{Nrcl}(A) \subset F^C$, where F^C is non-empty nano b-open $\Rightarrow (\text{Nrcl}(A))^C \subset F$.

(i.e) $F \subset (\text{Nrcl}(A))^C$ $\rightarrow$ (2)

From (1) and (2), we get $F \subset \text{Nrcl}(A) \cap (\text{Nrcl}(A))^C = \emptyset$. Therefore F which implies $F = \emptyset$ which is a contradiction.

$\therefore$Nrcl(A) - A does not contain any non-empty nano b-closed set.

Theorem: 3.35

Let A be an Nrb-closed set in $(U,\tau_R(X))$. Then A is nano regular closed iff Nrcl(A) - A is nano b-closed.

Proof:

Necessary part:

Let A Nrb-closed in $(U,\tau_R(X))$. Suppose that A is nano regular closed. Then $A = \text{Nrcl}(A) \Rightarrow \text{Nrcl}(A) - A = \emptyset$. Since is nano b-closed, we get Nrcl(A)-A is nano b-closed.

Sufficient part:

Assume that Nrcl(A)-A is nano b-closed. To prove A is nano regular closed. By theorem 3.34 "If A is Nrb-closed in $(U,\tau_R(X))$ then Nrcl(A) - A does not contain any non-empty nano b-closed set" which implies Nrcl(A) - A = Nrcl(A) = A. $\therefore$ A is nano regular closed.

§4. Nano regular b-open sets (Nrb-open sets)

Definition: 4.1

If its complement is a nano regular b-closed, then a subset A of a nano topological space $(U,\tau_R(X))$ is said to be nano regular b-open (Nrb-open).

Example: 4.2

Let U={1,2,3,4}, X={1,2},U/R={{1},{3},{2,4}}, $\tau_R(X)$= {U, $\emptyset$,{1},{1,2,4},{2,4}}.

Nrb-closed sets= { U, ∅,{3},{1,3},{2,3,4}}, Nrb-open sets={U, ∅,{1},{2,4},{1,2,4}}.

Theorem: 4.3

If A and B are Nrb-open, then $A \cap B$ is Nrb-open.

Proof:

Given that A and B are Nrb-open. To prove $A \cap B$ is Nrb-open. Since A and B are Nrb-open sets, we have U-A and U-B are Nrb-closed. We know that the union of two Nrb-closed sets is also a Nrb-closed from the theorem 3.34. $\therefore$ $(U-A) \cup (U-B)$ is Nrb-closed $\Rightarrow$ $U - (A \cup B)$ is Nrb-closed. Hence $A \cap B$ is Nrb-open.

Theorem: 4.4

If A and B are Nrb-open, then $A \cup B$ is Nrb-open.

Proof:

Given that A and B are Nrb-open. To prove $A \cup B$ is Nrb-open. Since A and B are Nrb-open sets, we have U-A and U-B are Nrb-closed. We know that the intersection of two Nrb-closed sets is also a Nrb-closed from the theorem 3.35. $\therefore$ $(U-A) \cap (U-B)$ is Nrb-closed

$\Rightarrow$ $U - (A \cap B)$ is Nrb-closed. Hence $A \cup B$ is Nrb-open.

Theorem: 4.5

Let $A \subset U$ is Nrb-open if and only if $F \subset Nrint(A)$ whenever F is Nb-closed and $F \subset A$.

Proof:

Necessary part:

Let A be an Nrb-open set and suppose $F \subset A$, where F is Nb-closed. Then U-A is an Nrb-closed contained in the Nb-open set U - F. Hence $Nrcl(U-A) \subset U-F$.

$\Rightarrow U-Nrint(A) \subset U-F \Rightarrow F \subset Nrint(A)$.

Sufficient part:

To prove A is Nrb-open, it is enough to prove U-A is Nrb-closed. Let $U-A \subset G$ and G be Nb-open. Then $U-G \subset A$ and U-G is Nb-closed. By hypothesis, $U-G \subset Nrint(A)$ and hence $Nrcl(U-A) = U-Nrint(A) \subset G$. Hence U-A is an Nrb-closed set.

$\therefore$ A is an Nrb-open set.

Theorem 4.6:

A set A is Nrb-closed in U iff Nrcl(A)-A is Nrb-open in U.

Proof:

Neccesary Part:

Let A be an Nrb-closed. To prove Nrcl(A)-A is Nrb-open. Let F be a Nb-closed set such that F $\subset$ Nrcl(A)-A. $\therefore$ F=$\emptyset$. (By the theorem Nrcl(A)-A will not contain any non-empty b-closed set). By theorem 4.3, F $\subset$ Nrint(Nrcl(A)-A).Thus Nrcl(A)-A is Nrb-open.

Sufficient Part:

Let A$\subset$G and G be a Nb-open set $\Rightarrow$ $G^C \subset A^C$. Then Nrcl(A) $\cap$ $G^C \subset$ Nrcl(A) $\cap$ A^C = Nrcl(A) - A. $\therefore$ Nrcl(A) $\cap$ $G^C \subseteq$ Nrcl(A) $\cap$ A^C . Since Nrcl(A) $\cap$ G^C is Nb-closed and Nrcl(A) - A is Nrb-open. By theorem 4.3, Nrcl(A) $\cap$ $G^C \subset$ Nrint(Nrcl(A)-A) = $\emptyset$.

Hence Nrcl(A) $\subset$ G and so A is Nrb-closed in U.

§5. Nano regular b-interior (Nrb-interior) and Nano regular b-closure (Nrb-closure)

Definition: 5.1

The nano regular b-interior of A is the union of all nano regular b-open sets contained in A, which is denoted by Nrbint(A). In particular, Nrbint(A) is the biggest Nrb-open set in A.

Definition: 5.2

The nano regular b-closure of A is the intersection of all nano regular b-closed sets containing A, and it is denoted by Nrbcl (A). Also, the smallest Nrb-closed set containing A is Nrcl(A).

Example: 5.3

Let U={1,2,3,4},X={1,2}, U/R = {{1},{3}{2,4}},τ_R(X)= {U, $\emptyset$,{1},{1,2,4},{2,4}}.

Nrb-closed sets={ U, $\emptyset$,{3},{1,3},{2,3,4}}, Nrb-open sets={ U, $\emptyset$,{1},{2,4},{1,2,4}}.

Let A={1,2}, Nrbint(A)={1}, Nrbcl(A)=U.

Theorem: 5.4

With respect to X, (U,τ_R(X))) be the nano topological space, where X$\subseteq$U, let A$\subseteq$U, then

(i) U - Nrbint(A) = Nrbcl(U-A)
(ii) U - Nrbcl(A) = Nrbint(U-A)

Proof:

(i) Let x $\in$ U - Nrbint(A). (i.e) x $\notin$ Nrbint(A). Therefore G$\not\subseteq$A for every Nrb-open set G containing x.
(i.e) G $\cap$ (U-A) $\neq \varphi$ for every Nrb-open set containing x $\Rightarrow$ x $\in$ Nrbcl(U-A).

$\therefore$ U - Nrbint(A) $\subseteq$ Nrbcl(U-A) $\rightarrow$ (1)

Conversely if $x \in$ Nrbcl(U-A), then $G \cap (U\text{-}A) \neq \emptyset$ for every Nrb-open set G containing x $\Rightarrow$ $G \not\subseteq A$ for every Nrb-open set G containing x $\Rightarrow$ $x \notin$ Nrbint(A).

(i.e) $x \in$ U-Nrbint(A)

$\therefore$ Nrbcl(U-A) $\subseteq$ U-Nrbint(A) $\rightarrow$ (2)

From (1) and (2), we get U - Nrbint(A) = Nrbcl(U-A).

(ii) Let $x \in$ U-Nrbcl(A). (i.e) $x \notin$ Nrbcl(A), then $G \cap A = \emptyset$ for some Nrb-open set G containing x. (i.e) $G \cap (U\text{-}A) \neq \emptyset$ for some Nrb-open set containing x.
$\Rightarrow$ $x \in$ Nrbint(U-A).

$\therefore$ U - Nrbcl(A) $\subseteq$ Nrbint(U-A) $\rightarrow$ (3)

Conversely if $x \in$ Nrbint(U-A), then $G \subset U\text{-}A$ for some Nrb-open set G containing x. (i.e) G $\not\subseteq$ A for some Nrb-open set G containing x $\Rightarrow$ $G \cap A = \emptyset$ $\Rightarrow$ $x \notin$ Nrbcl(A).

(i.e) $x \in$ U-Nrbcl(A)

$\therefore$ Nrbint(U-A) $\subseteq$ U-Nrbcl(A) $\rightarrow$ (4)

From (1) and (2), we get U - Nrbint(A) = Nrbcl(U-A).

Result 5.5:

Taking complements of both side of (i) and (ii) in theorem 5.4, we get

(i) Nrbint(A) = U - Nrbcl(U-A)
(ii) Nrbcl(A) = U - Nrbint(U-A).

Theorem 5.6:

With respect to X, $(U,\tau_R(X))$ be the nano topological space, where $X \subseteq U$, let A, B $\subseteq$ U, then

(i) A $\subseteq$ Nrbcl(A).
(ii) A is Nrb-closed iff Nrbcl(A) = A.
(iii) Nrbcl($\emptyset$) = $\emptyset$ and Nrbcl(U) = U.
(iv) A $\subseteq$ B $\Rightarrow$ Nrbcl(A) $\subseteq$ Nrbcl(B).
(v) Nrbcl(A$\cup$B) = Nrbcl(A) $\cup$ Nrbcl(B).
(vi) Nrbcl(A$\cap$B) $\subseteq$ Nrbcl(A) $\cap$ Nrbcl(B).
(vii) Nrbcl(Nrbcl(A)) = Nrbcl(A).

Proof:

(i) From the definition of Nrb-closure, we get A $\subseteq$ Nrbcl(A).

(ii) If A is Nrb-closed, then A is the smallest Nrb-closed containing itself and hence

Nrbcl(A) =A. Conversely, if Nrbcl(A) = A, since Nrbcl(A) is Nrb-closed set and then A is Nrb-closed.

(iii) Since φ and U are Nrb-closed in (U,τ_R(X)) then Nrbcl(∅) = ∅ and Nrbcl(U) = U.

Let A ⊆ B. To prove Nrbcl(A) ⊆ Nrbcl(B). Then A ⊆ Nrbcl(B). Since B ⊆ Nrbcl(B).

But Nrbcl(A) is the smallest Nrb-closed containing A ⇒ A ⊆ Nrbcl(A).

∴ Nrbcl(A) ⊆ Nrbcl(B).

(iv) Let A ⊆ A∪B and B ⊆ A∪B.
Then Nrbcl(A) ⊆ Nrbcl(A∪B) and Nrbcl(B) ⊆ Nrbcl(A∪B).

∴ Nrbcl(A) ∪ Nrbcl(B) ⊆ Nrbcl(A∪B) → (1)

Suppose that A∪B ⊆ Nrbcl(A) ∪ Nrbcl(B). We know that Nrbcl(A∪B) is the smallest Nrb-closed set containing A∪B.

∴ Nrbcl(A∪B) ⊆ Nrbcl(A) ∪ Nrbcl(B) → (2)

From (1) and (2), we get Nrbcl(A∪B) = Nrbcl(A) ∪ Nrbcl(B).

(v) A∩B ⊆ A and A∩B ⊆ B then Nrbcl(A∩B) ⊆ Nrbcl(A) and Nrbcl(A∩B) ⊆ Nrbcl(B)
∴ Nrbcl(A∩B) ⊆ Nrbcl(A) ∩ Nrbcl(B).
(vi) Since Nrbcl(A) is Nrb-closed, we get by (ii) Nrbcl(A)=A.

∴ Nrbcl(Nrbcl(A)) = Nrbcl(A).

Example 5.7 :

Let U={1,2,3,4}, X={1,2},U/R={{1},{3},{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets= { U, ∅,{3},{1,3},{2,3,4}},

Nrb-open sets={U, ∅,{1},{2,4},{1,2,4}}.

Let A = {1}, B = {2,3}. Then A∩B = ∅ . Here Nrbcl(A∩B) = ∅ , Nrbcl(A) = {1,3},

Nrbcl(B) = {2,3,4} which implies Nrbcl(A) ∩ Nrbcl(B) = {3}.

∴ Nrbcl(A∩B) ≠ Nrbcl(A) ∩ Nrbcl(B).

From this example the equality does not holds for (vi).

Theorem 5.8:

Let (U,τ_R(X)) be a nano topological space with respect to X where X ⊆ U, let A, B ⊆ U, then

(i) Nrbint(A) ⊆ A.
(ii) A is Nrb-open iff Nrbint(A) = A.
(iii) Nrbint(∅) = ∅ and Nrbint(U) = U.
(iv) A ⊆ B ⇒ Nrbint(A) ⊆ Nrbint(B).
(v) Nrbint(A∪B) ⊆ Nrbint(A) ∪ Nrbint(B)
(vi) Nrbint(A∩B) = Nrbint(A) ∩ Nrbint(B).

(vii) Nrbint(Nrbint(A)) = Nrbint(A).

Proof

(i) From the definition of Nrb-interior , we get Nrbint(A) ⊆ A.
(ii) We know that A is Nrb-open iff U-A is Nrb-closed. Since U-A is Nrb-closed we have Nrbcl(U-A) = U-A which implies U-Nrbcl(U-A) = A ⇒ Nrbint(A) =A (By using (i) of result 5.5).
∴ A is Nrb-open iff Nrbint(A) =A.
(iii) Since and U are Nrb-open in (U,τ_R(X)), Nrbint(∅) = ∅ and Nrbint(U) = U.
(iv) Let A ⊆ B. To prove Nrbint(A) ⊆ Nrbint(B). From A ⊆ B we get U-B ⊆ U-A
⇒ Nrbcl(U-B) ⊆ Nrbcl(U-A) ⇒ U-Nrbcl(U-A) ⊆ U-Nrbcl(U-B)
∴ NrbintA ⊆ NrbintB (by using the result 5.5)
(v) Let A ⊆ A∪B and B ⊆ A∪B.
Then Nrbint(A) ⊆ Nrbint(A∪B) and Nrbint(B) ⊆ Nrbint(A∪B).
∴ Nrbint(A) ∪ Nrbint(B) ⊆ Nrbint(A∪B).
(vi) Let A∩B ⊆A and A∩B ⊆ B.
Then Nrbint(A∩B) ⊆ Nrbint(A) and Nrbint(A∩B) ⊆ Nrbint(B).

∴ Nrbint(A∩B) ⊆ Nrbint(A) ∩ Nrbint(B) → (1)

Suppose that Nrbint(A) ∩ Nrbint(B) ⊆ A∩B. We know that Nrbint(A∩B) is the largest Nrb-open set contained in A∩B.

∴ Nrbint(A) ∩ Nrbint(B) ⊆ Nrbint(A∩B) → (2)

From (1) and (2), we get Nrbint(A) ∩ Nrbint(B) = Nrbint(A∩B).

(vii) Since Nrbint(A) is Nrb-open, we get by (ii) Nrbint(A)=A.

∴ Nrbint(Nrbint(A)) = Nrbint(A).

Example 5.9 :

Let U={1,2,3,4}, X={1,2},U/R={{1},{3},{2,4}},τ_R(X) = {U, ∅,{1},{1,2,4},{2,4}}.

Nrb-closed sets= { U, ∅,{3},{1,3},{2,3,4}}, Nrb-open sets={U, ∅,{1},{2,4},{1,2,4}}.

Let A = {2}, B = {3,4}. Then A∪B ={2,3,4}. Here Nrbint(A∪B) = {2,4}, Nrbint(A) = ∅ , Nrbint(B) = ∅ which implies Nrbint(A)∪Nrbint(B) = ∅.

∴ Nrbint(A∪B) ≠Nrbint(A)∪Nrbint(B).

From this example the equality does not holds for (v).

§ 5. CONCLUSION

In this study, we describe the Nrb-open and Nrb-closed sets, a new class of nano open and nano closed sets in nano topological spaces. Some of their features are also investigated in terms of Nrb interior and Nrb closure. This work will be expanded in the future with some real life applications.

REFERENCES

[1] Ahmad Al. Omari and MD. Mohd. Salmi Noorani, On generalized b-closed sets, *Bulletin of the Malaysian Mathematical Sciences Society,* 32 (2009) 19-30.

[2] D. Andrijevic, On b-open sets, *Mat.Vesnik,* 48 (1996) 59-64.

[3] Bakeir, Nδg-closed sets in nano topology , *17th International conference on Differential geometry and Application, ICDGA* (2015).

[4] K. Bhuvaneshwari and A. Ezhilarasi , On Nsg and Ngs-closed sets in nano topological spaces, *International Journal of Mathematics and computer applications Research (IJMCAR),* 4 (2014) 117-124.

[5] K. Bhuvaneshwari and K. Mythili Gnanapriya , Ng-closed sets in nano topological spaces, *International journal scientific and research publications,* 4 (2014) 1-3.

[6] Bhuvaneshwari Kandaswamy, K. Mythili Gnanapriya , On Ngp and Npg- closed sets in nano topological spaces , *International Journal of Innovative Research in science,* Engineering and Technology, 3 (2014) 16829-16835.

[7] A. Dhanis Arul mary, I. Arockiarani , On Ngb-closed sets in nano topological spaces, *International journal of Mathematical Archieve,* 6 (2015) 2229-5046.

[8] Hosny. M, Nano $\hat{\beta}$ sets and Nano $\hat{\beta}$ continuity , *Journal of the Egyptian Mathematical society,* 28 (2020) 1-11. https://doi.org/10.1186/s42787-020-0070-5

[9] M. Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets , *International Journal of Mathematics and Statistics Invention,* 1 (2013) 31-37.

[10] N. Levine, Generalized closed sets in topology, *Rendiconti del circolo Matematico di Palermo,* 19 (1970) 89-96.

[11] A. Narmatha and N. Nagaveni On rb-closed sets in topological spaces, *Heber International Conference on Applications of Mathematics and Statistics*, (2012) 81-87.

[12] A. Narmatha, N. Nagaveni and T. Noiri, On rb-open sets, *International Journal of Mathematical Analysis*, 7 (2013) 937-948.

[13] A. A. Nasef, A. I. Aggour, S. M. Darwesh, On some classes of nearly open sets, *Journal of the Egyptian Mathematical Society,* 24, (2016) 585-589.

[14] N. Palaniappan and K. Chandrasekhara Rao, Regular generalized closed sets, *Kyungpook Mathematics Journal*, 33 (1993) 211-219.

[15] A. Parimala, G. Indirani and S. Jafari, On Nb-open sets, *Jordan journal of Mathematics and statistics (JJMS)*, 9 (2016) 173-184.

[16] S. Parimala and V. Chandrasekar, On Nδ-open sets and their notions, *Malaya journal of Matematik,* 1 (2019) 664-672.

[17] I. Rajasekaran, M. Menarin and O. Nethaji, On Ngβ-closed sets in nano topological spaces, *International journal of Mathematics and its applications,* 5 (2017) 377-382.

[18] A. Revathy and Gnanambal Ilango , On Nβ-open sets in nano topological spaces, *International Journal of engineering contemporary Mathematics and sciences,* 1 (2015) 2250-3099.

[19] Sharmistha Bhattacharya, On generalized regular closed sets, *International Journal of Mathematical sciences,* 6 (2011) 145-152.

[20] P. Sulochana Devi and K. Bhuvaneshwari, On Nrg and Ngr closed sets in nano topological spaces, *International journal of Engineering Trends and Technology (IJETT),* 13 (2014) 386-390.

[21] Thanga Nachiyar & K. Bhuvaneshwari, On Ngα and Nαg closed sets in nano topological spaces, *International Journal of Engineering Trends and Technology, (IJETT),* 13 (2014) 257-260.