On Soft Generalized ** Continuous Functions in Soft Topological Spaces

Ms.N.Gomathi[1] and Dr.T.Indira[2]

[1]Department of Mathematics,
Srimad Andavan Arts and Science College (Auto.),
No.7, Nelson Road, Tiruchurappalli, 620005,
Tamilnadu, India.
Email: gomathi@andavancollege.ac.in
[2]Department of Mathematics,
Seethalakshmi Ramaswami College (Auto.),
No.5, Sankaran Pillai Road, Tiruchurappalli, 620002,
Tamilnadu, India.
Email: drtindira.chandru@gmail.com

Article Info
Page Number: 912-931
Publication Issue:
Vol. 71 No. 4 (2022)

Article History
Article Received: 25 March 2022
Revised: 30 April 2022
Accepted: 15 June 2022
Publication: 19 August 2022

Abstract
The aim of this paper is to define and study the class of continuous function called soft generalized** continuous function (briefly Sg**-continuous function) and soft g**-irresolute function on soft topological spaces. We study the relationship between Soft g**-continuous function and soft g**-irresolute function with other existing soft continuous functions and soft irresolute functions. Also we studied some of their properties.
Keywords: Soft g** - continuous functions, Soft g** - irresolute functions, Soft g**- open map, Soft g**- closed map

I -Introduction

Molodtsov[11,12] introduced the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Soft set theory has a wider application and its progress is very rapid in different fields . Levine[9] introduced g-closed sets in general topology. Later Kharel.A, Ahmed.B[8] introduced the mappings of soft classes. Metin Akag, Alkan Ozkan [10] introduced soft b-open sets and soft b-continuous functions and Angelin Tidy.G, Francina Shalini[1] introduced soft sgb-continuous functions in soft topological spaces. In 2015, Ramadhan A.Mohammed, Tahir H.Ismail and A.A.Allam[15] introduced on soft $g\alpha$b-continuous functions in soft topological spaces and different kinds of soft continuous functions were studied and investigated by many authors[2,3,4,5,6,13,14,16,17]. Also in 2021 Sumer Al Ghour[17] introduced soft ω_p-open sets and soft ω_p-continuity in soft topological spaces . The aim of this paper is to introduce and study the concepts of soft generalized**-continuous functions and soft generalized**-irresolute functions. Also we discussed some properties of these functions

II- Preliminaries

In this section, we have presented basic definitions of soft set theory which may be found in earlier. Let X be an initial universe and E be the set of parameters, P(X) denote the power set of X and $(\mathrm{F_A})$ denotes a soft set in a soft topological space . Throughout this paper $(\mathrm{F_A}, \tilde{\tau})$ represents non-empty soft topological spaces on which no separation axioms are assumed.

Definition : 2.1 [18]

1. A soft set F_A on the universe U is defined by the set of ordered pairs, E be the set of parameters and $A \subseteq E$, then $F_A = \{(x, f_A(x)) : x \in E\}$ where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be aribitary. Some of them may be empty some may have non-empty intersection. Note that the set of all soft sets with the parameter set E over U will be denoted by $S(U)$.
2. Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A is called an empty soft set, denoted by $F_\emptyset$.
3. Let $F_A \in S(U)$. If $F_A(x) = U$ for all $x \in A$ then F_A is called a A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.
4. Let F_A , $F_B \in S(U)$. Then soft union $F_A \tilde{\cup} F_B$, Soft intersection$F_A \tilde{\cap} F_B$, and soft difference $F_A \backslash F_B$ of F_A and F_B are defined by respectively. $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, $f_{A\tilde{\backslash}B}(x) = f_A(x) \backslash f_B(x)$, and the soft complement $F^{\tilde{c}}{}_A$ of F_A is defined by $f^{\tilde{c}}{}_A(x) = f^c{}_A$ where $f^c{}_A(x)$ is complement of the set $f_A(x)$, that is $f^c{}_A(x) = U \backslash f_A(x)$ for all $x \in E$.
5. Let $F_A \in S(U)$. The relative complement of F_A is denoted by F'_A and is defined by $(F_A)' = (F'_A)$ where $F'_A : A \to P(U)$ is a mapping given by $F'_\propto = U \backslash F_\propto$ for all $\propto \in A$.

Definition : 2.2[1,2,4,5,7,10,14,15,17]

Let $(F_A, \tilde{\tau})$ and $(F_B, \tilde{\sigma})$ be two soft topological spaces. A function $f : (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ is said to be

1) Soft continuous, if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft closed (open) in $(F_A, \tilde{\tau})$.
2) Soft-Semi-continuous if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft-semi- closed (open) in $(F_A, \tilde{\tau})$.
3) Soft-Pre-continuous if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft-Pre- closed (open) in $(F_A, \tilde{\tau})$.
4) Soft-α-continuous if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft-α- closed (open) in $(F_A, \tilde{\tau})$.
5) Soft-β-continuous if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft-β- closed (open) in $(F_A, \tilde{\tau})$.
6) Soft Regular continuous if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft regular closed (open) in $(F_A, \tilde{\tau})$.

7) Soft generalized continuous (briefly soft g-continuous) if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft generalized closed (open) in $(F_A, \tilde{\tau})$.
8) Soft semi generalized continuous (briefly soft sg-continuous) if the inverse image of every soft closed (open) set in $F_B, \tilde{\sigma})$ is soft sg- closed (open) in $(F_A, \tilde{\tau})$.
9) Soft generalized continuous (briefly for g-continuous) if there inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft g- closed (open) in $(F_A, \tilde{\tau})$.
10) Soft generalized* continuous (briefly for g*-continuous) if there inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft g* closed (open) in $(F_A, \tilde{\tau})$.
11) Soft regular generalized continuous (briefly for rg-continuous) if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft rg- closed (open) in $(F_A, \tilde{\tau})$.
12) Soft regular α- generalized continuous (briefly for rαg-continuous) if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft rαg - closed (open) in $(F_A, \tilde{\tau})$.
13) Weakly Soft generalized continuous if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is weakly soft g-closed (open) in $(F_A, \tilde{\tau})$.
14) Soft generalized semi pre continuous (briefly soft gsp-continuous) if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft gsp-closed (open) in $(F_A, \tilde{\tau})$.
15) Soft generalized pre regular continuous (briefly soft gpr-continuous) if the inverse image of every soft closed (open) set in $(F_B, \tilde{\sigma})$ is soft gpr-closed (open) in $(F_A, \tilde{\tau})$.

III-Soft Generalized ** Continuous Functions

Definition 3.1

A Soft map $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ is called Soft g** -Continuous function, if the inverse image of every soft closed set in $(F_B, \tilde{\sigma})$ is soft g** closed (open) in $(F_A, \tilde{\tau})$.

Example: 3.2

Let $X = \{a, b, c\}$, $Y = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$, $A, B \subset E$ and

$A = B = \{e_1, e_2\} \subset E$

$F_A = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_5} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{A_6} = \{(e_2, \{c\})\}$

$F_{A_7} = F_A$

$F_{A_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_A, F_\varphi, F_{A_2}, F_{A_5}\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_A, F_{A_1}, F_{A_4}\}$

Soft g** - Closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g** - Open sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_5}, F_{A_6}, F_{A_2}\}$

$F_B = \{(e_1, \{a\}), (e_2, \{b, c\})\}$

$F_{B_1} = \{(e_1, \{a\})\}$

$F_{B_2} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{B_3} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{B_4} = F_B$

$F_{B_5} = \{(e_1, \{b\})$

$F_{B_6} = \{(e_2, \{c\})\}$

$F_{B_7} = \{(e_2, \{b, c\})\}$

$F_{B_8} = F_\varphi$

$\tilde{\sigma}$ (sos) $= \{F_\varphi, F_B, F_{B_7}, F_{B_2}\}$

$\tilde{\sigma}$ (scs) $= \{F_B, F_\varphi, F_{B_3}, F_{B_1}\}$

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be the identity mapping defined by

$f(a) = a, f(c) = c, f(b) = b$ and $f^{-1}(a) = a, f^{-1}(b) = b, f^{-1}(c) = c$

(i) Let $F_{B_3} = \{(e_1, \{a\}), (e_2, \{c\})\}$ is soft closed set in $(F_B, \tilde{\sigma})$. Then $f^{-1}(F_{B_5}) = \{(e_1, \{a\}), (e_2, \{d\})\} = F_{A_4}$ which is soft g** closed set in $(F_A, \tilde{\tau})$.

(ii) Let $F_{B_1} = \{(e_1, \{a\})\}$ is soft closed set in $(F_B, \tilde{\sigma})$. Then $f^{-1}(F_{B_2}) = \{(e_1, \{a\})\} = F_{A_1}$ which is soft g** closed set in $(F_A, \tilde{\tau})$.

THEOREM 3.3

Every Soft Continuous function is soft g** - Continuous function.

Proof:

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be a soft continuous function. Let F_B be a soft closed set over $(F_B, \tilde{\sigma})$. Since f is continuous, $f^{-1}(F_B)$ is soft closed set over $(F_A, \tilde{\tau})$. But we know that, Every soft closed set is soft g** - closed set. Therefore $f^{-1}(F_B)$ is soft g** - closed set over $(F_A, \tilde{\tau})$. Hence f is soft g** - Continuous function.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.4

Let $X = \{a, b, c\}$, $Y = \{a, b, c\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$, $A, B \subseteq E$ and $A = B = \{e_1, e_2\} \subseteq E$

$F_A = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_5} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{A_6} = \{(e_2, \{c\})\}$

$F_{A_7} = F_A$

$F_{A_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_A, F_\varphi, F_{A_2}, F_{A_5}\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_A, F_{A_1}, F_{A_4}\}$

Soft g** - closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}\}$

$F_B = \{(e_1, \{a, c\}), (e_2, \{b\})\}$

$F_{B_1} = \{(e_1, \{a\})\}$

$F_{B_2} = \{(e_1, \{c\})\}$

$F_{B_3} = \{(e_1, \{a, c\})\}$

$F_{B_4} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{B_5} = \{(e_1, \{c\}), (e_2, \{b\})\}$

$F_{B_6} = \{(e_2, \{b\})\}$

$F_{B_7} = F_B$

$F_{B_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_B, F_\varphi, F_{B_4}, F_6\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_B, F_{B_3}, F_{B_1}\}$

Let $f:(F_A,\tilde{\tau}) \to (F_B,\tilde{\sigma})$ be the mapping defined by

$f(a) = a, f(b) = c, f(c) = b$ and $f^{-1}(a) = a, f^{-1}(b) = c, f^{-1}(c) = b$

Here f is a soft g** - continuous function but not a soft continuous function.

Since $f^{-1}(F_{B_3}) = \{(e_1,\{a,b\})\} = F_{A_3}$ is soft g**-closed set over $(F_B,\tilde{\tau})$, and it is not soft closed over $(F_A,\tilde{\tau})$.

THEOREM:3.5

Every soft g**-continuous function is soft rg-continuous function.

Proof:

Let $f:(F_A,\tilde{\tau}) \to (F_B,\tilde{\sigma})$ be a soft g**- continuous function. Let F_B be a soft closed set over $(F_B,\tilde{\sigma})$. Since f is soft g**-continuous, $f^{-1}(F_{\mathrm{B}})$ is soft g** closed set over $(F_A,\tilde{\tau})$. But we know that, Every soft g**-closed set is soft rg - closed set. Therefore $f^{-1}(F_{\mathrm{B}})$ is soft rg - closed set over $(F_A,\tilde{\tau})$. Hence f is soft rg - continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example: 3.6

Let $X = \{a,b,c\}$, $Y = \{a,b,c\}$, $E = \{e_1,e_2,e_3\}$, $A,B \subset E$ and

$A = B = \{e_1,e_2\} \subset E$

$F_A = \{(e_1,\{a\}),(e_2,\{b,c\})\}$

$F_{A_1} = \{(e_1,\{a\})\}$

$F_{A_2} = \{(e_1,\{a\}),(e_2,\{b\})\}$

$F_{A_3} = \{(e_1,\{a\}),(e_2,\{c\})\}$

$F_{A_4} = F_A$

$F_{A_5} = \{(e_1,\{b\})$

$F_{A_6} = \{(e_2,\{c\})\}$

$F_{A_7} = \{(e_2,\{b,c\})\}$

$F_{A_8} = F_\varphi$

$\tilde{\sigma}$ (sos) $= \{F_\varphi,F_A,F_{A_7},F_{A_2}\}$

$\tilde{\sigma}$ (scs) $= \{F_A,F_\varphi,F_{A_3},F_{A_1}\}$

Soft g** - closed sets of $(F_A,\tilde{\tau}) = \{F_A,F_\varphi,F_{A_1},F_{A_3},F_{A_6}\}$

Soft rg - closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_5}, F_6, F_{A_7}\}$

$F_B = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$F_{B_1} = \{(e_1, \{a\})\}$

$F_{B_2} = \{(e_1, \{b\})\}$

$F_{B_3} = \{(e_1, \{a, b\})\}$

$F_{B_4} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{B_5} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{B_6} = \{(e_2, \{c\})\}$

$F_{B_7} = F_B$

$F_{B_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_B, F_\varphi, F_{B_1}, F_{B_4}\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_B, F_{B_5}, F_{B_2}\}$

Let $f: (F_A, \tilde{\tau}) \rightarrow (F_B, \tilde{\sigma})$ be the mapping defined by

$f(a) = B, f(b) = c, f(c) = a$ and $f^{-1}(a) = c, f^{-1}(b) = a, f^{-1}(c) = b$

Here f is soft rg-continuous but not a soft g** - Continuous function. Since $F_{B_5} = \{(e_1, \{b\}), (e_2, \{c\})\}$ is a closed set over $(F_B, \tilde{\sigma})$ and $f^{-1}(F_{B_5}) = \{(e_1, \{a\}), (e_2, \{b\})\}$

$= F_{A_2}$ is soft rg-closed set over $(F_A, \tilde{\tau})$ but it is not soft g** - closed set over $(F_A, \tilde{\tau})$.

THEOREM: 3.7

Every soft g** - Continuous function is weakly soft g – Continuous function.

Proof:

Let $f: (F_A, \tilde{\tau}) \rightarrow (F_B, \tilde{\sigma})$ be a soft g**- continuous function. Let F_B be a soft closed set over $(F_B, \tilde{\sigma})$. Since f is soft g**-Continuous, $f^{-1}(F_B)$ is soft g**-closed set over $(F_A, \tilde{\tau})$. We know that every soft g**-Closed set is weakly soft g-closed set. Therefore $f^{-1}(F_B)$ is weakly soft g-closed set over $(F_A, \tilde{\tau})$. Hence f is weakly soft g-continuous function.

The Converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.8

Let $X = \{a, b, c\}$, $Y = \{a, b, c\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$, $A, B \subseteq E$ and $A = B = \{e_1, e_2\} \subseteq E$

$F_A = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_5} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{A_6} = \{(e_2, \{c\})\}$

$F_{A_7} = F_A$

$F_{A_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_A, F_\varphi, F_{A_2}, F_{A_5}\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_A, F_{A_1}, F_{A_4}\}$

Soft g** - closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}\}$

Weakly soft g-closed sets of $(F_A, \tilde{\tau})$=$\{F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$

$F_B = \{(e_1, \{a, c\}), (e_2, \{b\})\}$

$F_{B_1} = \{(e_1, \{a\})\}$

$F_{B_2} = \{(e_1, \{c\})\}$

$F_{B_3} = \{(e_1, \{a, c\})\}$

$F_{B_4} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{B_5} = \{(e_1, \{c\}), (e_2, \{b\})\}$

$F_{B_6} = \{(e_2, \{b\})\}$

$F_{B_7} = F_B$

$F_{B_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_B, F_\varphi, F_{B_2}, F_5\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_B, F_{B_4}, F_6\}$

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be the identity mapping defined by

$f(a) = c, f(b) = a, f(c) = b$ and $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a$

Here f is weakly soft g-continuous but not a soft g** - Continuous function. Since $F_{B_6} = \{(e_2, \{b\})\}$ is a soft closed set over $(F_B, \tilde{\sigma})$, and $f^{-1}(F_{B_6}) = \{(e_2, \{c\})\} = F_{A_6}$ is weakly soft g-closed set over $(F_A, \tilde{\tau})$ but it is not a soft g** - closed set over $(F_A, \tilde{\tau})$.

THEOREM: 3.9

Every soft g**-continuous function is soft gpr-continuous function.

Proof:

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be a soft g**- continuous function. Let F_B be a soft closed set over $(F_B, \tilde{\sigma})$. Since f is soft g**-continuous, $f^{-1}(F_B)$ is soft g**-closed set over $(F_A, \tilde{\tau})$. We know that every soft g**-closed set is soft gpr-closed set. Therefore $f^{-1}(F_B)$ is soft gpr-closed set over $(F_A, \tilde{\tau})$. Hence f is soft gpr-continuous function.

The Converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.10

Let $X = Y = \{a, b, c\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$, $A, B \subseteq E$ and $\mathrm{A} = \mathrm{B} = \{\mathrm{e}_1, \mathrm{e}_2\} \subseteq \mathrm{E}$

$F_A = \{(e_1, \{a\}), (e_2, \{b, c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{A_3} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_4} = F_A$

$F_{A_5} = \{(e_1, \{b\})$

$F_{A_6} = \{(e_2, \{c\})\}$

$F_{A_7} = \{(e_2, \{b, c\})\}$

$F_{A_8} = F_\varphi$

$\tilde{\sigma}$ (sos) $= \{F_\varphi, F_A, F_{A_7}, F_{A_2}\}$

$\tilde{\sigma}$ (scs) $= \{F_A, F_\varphi, F_{A_3}, F_{A_1}\}$

Soft g** - closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_6}\}$

Soft gpr-Closed sets of $(F_A, \tilde{\tau})$ = $\{F_A, F_\varphi, F_{A_1}, F_{A_2}, F_A, F_{A_5}, F_{A_6}, F_{A_7}\}$

$F_B = \{(e_1, \{a, c\}), (e_2, \{b\})\}$

$F_{B_1} = \{(e_1, \{a\})\}$

ISSN: 2094-0343
2326-9865

$F_{B_2} = \{(e_1, \{c\})\}$

$F_{B_3} = \{(e_1, \{a, c\})\}$

$F_{B_4} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{B_5} = \{(e_1, \{c\}), (e_2, \{b\})\}$

$F_{B_6} = \{(e_2, \{b\})\}$

$F_{B_7} = F_B$

$F_{B_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_B, F_\varphi, F_{B_2}, F_5\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_B, F_{B_4}, F_6\}$

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be the identity mapping defined by

$f(a) = a, f(b) = b, f(c) = c$ and $f^{-1}(a) = a, f^{-1}(b) = b, f^{-1}(c) = c$

Here f is soft gpr-continuous function but not a soft g** - Continuous function. Since $F_{B_4} = \{(e_1, \{a\}), (e_2, \{b\})\}$ is a soft closed set over $(F_B, \tilde{\sigma})$ and $f^{-1}\left(F_{B_4}\right) = \{(e_1, \{a\}),$ $(e_2, \{b\})\} = F_{A_2}$ is soft gpr-closed set in $(F_A, \tilde{\tau})$ but it is not soft g** - closed set over $(F_A, \tilde{\tau})$.

THEOREM: 3.11

Every Soft g**- continuous function is soft αg-continuous function.

Proof:

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be a soft g**- continuous function. Let F_B be a soft closed set over $(F_B, \tilde{\sigma})$. Since f is soft g**-continuous, $f^{-1}(F_B)$ in soft g**-Closed set over $(F_A, \tilde{\tau})$. We know that every soft g**-closed set is soft αg-closed. Therefore $f^{-1}(F_B)$ is soft αg - closed set over $(F_A, \tilde{\tau})$. Hence f is soft αg -continuous function.

The Converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.12

Let us take example (3.8), Let $X = Y = \{a, b, c\}$, $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$, $\mathrm{A}, \mathrm{B} \subseteq \mathrm{E}$ and $\mathrm{A} = \mathrm{B} = \{\mathrm{e}_1, \mathrm{e}_2\} \subseteq \mathrm{E}$

$F_A = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$\tilde{\tau}$ (scs) $= \{F_A, F_\varphi, F_{A_4}, F_{A_1}\}$

$\tilde{\tau}$ (sos) $= \{F_\varphi, F_A, F_{A_2}, F_{A_5}\}$

Soft g** - closed sets are $= \{F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft αg - closed sets are = $\{F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_6}\}$

$F_B = \{(e_1, \{a, c\}), (e_2, \{b\})\}$

$\tilde{\sigma}$ (sos) = $\{F_B, F_\varphi, F_{B_5}, F_{B_2}\}$

$\tilde{\sigma}$ (scs) = $\{F_\varphi, F_B, F_4, F_{B_6}\}$

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be the mapping defined by

$f(a) = c, f(b) = a, f(c) = b$ and $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a$

Here f is soft αg-continuous but not a soft g** - continuous function. Since $F_{B_6} = \{(e_2, \{b\})\}$ is soft closed set over $(F_B, \tilde{\sigma})$ and $f^{-1}(F_{B_6}) = \{(e_2, \{c\})\} = F_{A_6}$ is αg - closed set over $(F_A, \tilde{\tau})$, but it is not a soft g** - closed set over $(F_A, \tilde{\tau})$.

THEOREM: 3.13

Every soft g**-Continuous function is soft gsp-continuous function.

Proof:

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be a soft g **- continuous function. Let F_B be a soft closed set over $(F_B, \tilde{\sigma})$. Since f is soft g**- continuous, $f^{-1}(F_B)$ in soft g**- closed set over $(F_A, \tilde{\tau})$. We know that, every soft g**- closed set is soft gsp-closed set. Therefore $f^{-1}(F_B)$ is soft gsp - closed set over $(F_A, \tilde{\tau})$. Hence f is soft gsp - continuous function.

The Converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.14

Let $X = Y = \{a, b, c\}$E = $\{e_1, e_2, e_3\}$, A, B ⊆ E and A = B = $\{e_1, e_2\}$ ⊆ E

$F_A = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_5} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{A_6} = \{(e_2, \{c\})\}$

$F_{A_7} = F_A$

$F_{A_8} = F_\varphi$

$\tilde{\tau}$ (sos) $= \{F_A, F_\varphi, F_{A_2}, F_{A_5}\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi, F_A, F_{A_1}, F_{A_4}\}$

Soft g** - closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft gsp-closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$

$F_B = \{(e_1, \{a\}), (e_2, \{b, c\})\}$

$F_{B_1} = \{(e_1, \{a\})\}$

$F_{B_2} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{B_3} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{B_4} = F_B$

$F_{B_5} = \{(e_1, \{b\})\}$

$F_{B_6} = \{(e_2, \{c\})\}$

$F_{B_7} = \{(e_2, \{b, c\})\}$

$F_{B_8} = F_\varphi$

$\tilde{\sigma}$ (sos) $= \{F_\varphi, F_B, F_{B_7}, F_{B_1}\}$

$\tilde{\sigma}$ (scs) $= \{F_B, F_\varphi, F_{B_2}, F_5\}$

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be the mapping defined by

$f(a) = c, f(b) = a, f(c) = b$ and $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a$

Here f is soft gsp - continuous but not a soft g** - Continuous function. Since $F_{B_5} = \{(e_1, \{b\})\}$ is a soft closed set over $(F_B, \tilde{\sigma})$ and $f^{-1}(F_{B_5}) = \{(e_2, \{b\})\} = F_{A_6}$ is s soft gsp-closed set over $(F_A, \tilde{\tau})$, but it is not a soft g** - closed set over $(F_A, \tilde{\tau})$.

THEOREM: 3.15

Every soft g**-continuous function is soft gp-continuous function.

Proof:

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be a soft g**- continuous function. Let F_B be a soft closed set over $(F_B, \tilde{\sigma})$. Since f is soft g**-continuous, $f^{-1}(F_B)$ in soft g**-closed set over $(F_A, \tilde{\tau})$. We know that every soft g**-closed set is soft gp-closed set. Therefore $f^{-1}(F_B)$ is soft gp -closed set over $(F_A, \tilde{\tau})$. Hence f is soft gp -continuous function.

The Converse of the above theorem need not be true as seen from the following example.

EXAMPLE: 3.16

Let us take example (3.14)

Let $X = Y = \{a, b, c\}$E = {e_1, e_2, e_3} , A, B ⊆ E and A = B = {e_1, e_2} ⊆ E

$F_A = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$\tilde{\tau}$ (sos) = $\{F_A, F_\varphi, F_{A_2}, F_{A_5}\}$

$\tilde{\tau}$ (scs) = $\{F_\varphi, F_A, F_{A_1}, F_{A_4}\}$

Soft g** - closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft gp-closed sets of $(F_A, \tilde{\tau}) = \{ F_A, F_\varphi, F_{A_2}, F_{A_3}, F_{A_5}\}$

$F_B = \{(e_1, \{a\}), (e_2, \{b, c\})\}$

$\tilde{\sigma}$ (sos) = $\{F_B, F_\varphi, F_{B_1}, F_{B_7}\}$

$\tilde{\sigma}$ (scs) = $\{F_\varphi, F_B, F_{B_2}, F_{B_5}\}$

Let $f: (F_A, \tilde{\tau}) \to (F_B, \tilde{\sigma})$ be the mapping defined by

$f(a) = c, f(b) = a, f(c) = b$ and

$f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a$

Here f is soft gp-continuous but not a soft g**- continuous function. Since $F_{B_2} = \{(e_1, \{a\}), (e_2, \{b\})\}$ and $f^{-1}(F_{B_2}) = \{(e_1, \{b\}), (e_2, \{c\})\} = F_{A_5}$ is soft gp-closed in $(F_A, \tilde{\tau})$ but it is not a soft g** - closed set over $(F_A, \tilde{\tau})$.

THEOREM: 3.17

Let $(F_A, \tilde{\tau})$ and $(G_A, \tilde{\sigma})$ be two soft topological spaces and $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ be a soft function, then the following conditions are equivalent

(i) f is soft g**-continuous
(ii) The inverse image of each soft open set over $(G_A, \tilde{\sigma})$ is soft g**-open set over $(F_A, \tilde{\tau})$.

Proof:

(i) ⇒ (ii)

Let $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ be soft g**-continuous function and let G_{A_i} be a soft open set over $(G_A, \tilde{\sigma})$. Then $G_A - G_{A_i}$ is soft closed set over $(G_A, \tilde{\sigma})$. Now by assumption, $f^{-1}(G_A - G_{A_i})$ is soft g**-closed set over $(F_A, \tilde{\tau})$. (ie) $f^{-1}(G_A) - f^{-1}(G_{A_i}) \Rightarrow F_A - f^{-1}(G_B)$, is soft g**-closed set over $(F_A, \tilde{\tau})$. Hence $f^{-1}(G_{A_i})$ is soft g**- open set over $(F_A, \tilde{\tau})$.

(ii) ⇒ (i)

Let G_{A_i} be any soft closed set over $(G_A, \tilde{\sigma})$. Then $G_A - G_{A_i}$is soft open set over $(G_A, \tilde{\sigma})$ and hence by hypothesis $f^{-1}(G_A - G_{A_i}) = f^{-1}(G_A) - f^{-1}(G_{A_i}) \Rightarrow F_A - f^{-1}(G_{A_i})$ is soft g**-open over $(F_A, \tilde{\tau})$. Therefore $f^{-1}(G_{A_i})$ is soft g**-closed over $(F_A, \tilde{\tau})$ and hence f is soft g**-continuous.

THEOREM: 3.18

Let $(F_A, \tilde{\tau})$ and $(G_A, \tilde{\sigma})$ be two soft topological spaces and $f: (F_A, \tilde{\tau}) \rightarrow (G_A, \tilde{\sigma})$ be a soft function. If f is soft g**- continuous then $f^{-1}(int(f(F_{A_i})) \tilde{\subseteq} Sg ** -int(F_{A_i})$ for every soft set F_{A_i} over $(F_A, \tilde{\tau})$.

Proof:

Let $f: (F_A, \tilde{\tau}) \rightarrow (G_A, \tilde{\sigma})$ be soft g**-continuous function and let F_{A_i} be a soft open set over $(F_A, \tilde{\tau})$. Then $int(f(F_{A_i}))$ is a soft open set over $(G_A, \tilde{\sigma})$. Now by assumption, $f^{-1}(int(f(F_{A_i}))$ is soft g**-open over $(F_A, \tilde{\tau})$. Since, $f^{-1}(int(f(F_{A_i})))\tilde{\subseteq} F_{A_i}$ and $Sg ** -int(F_{A_i})$ is the largest Sg**- open set contained in F_{A_i} . Therefore $f^{-1}(int(f(F_{A_i})) \tilde{\subseteq} Sg ** -int(F_{A_i})$.

THEOREM: 3.19

If a function $f: (F_A, \tilde{\tau}) \rightarrow (G_A, \tilde{\sigma})$ is soft g**- continuous, then $f\left(Sg ** -cl(F_{A_i})\right) \tilde{\subseteq} cl(f(F_{A_i}))$ for every soft set F_{A_i} of $(F_A, \tilde{\tau})$.

Proof:

Let $f: (F_A, \tilde{\tau}) \rightarrow (G_A, \tilde{\sigma})$ be a soft g**- continuous and let F_{A_i} be a soft set in $(F_A, \tilde{\tau})$. Then $cl(f(F_{A_i}))$ is soft closed set in $(G_A, \tilde{\sigma})$. Since f is soft g**-continuous, then $f^{-1}(cl(f(F_{A_i})))$ is soft g**- closed set in $(F_A, \tilde{\tau})$. Now by assumption $F_{A_i} \tilde{\subset} f^{-1}\left(cl(f(F_{A_i})\right)$ and $Sg ** - cl(F_{A_i})$ is the smallest soft g**- closed set containingF_{A_i}.

$\Rightarrow Sg ** -cl(F_{A_i}) \tilde{\subset} f^{-1}(cl(f(F_{A_i})))$. Therefore $f(Sg ** -cl(F_{A_i})) \tilde{\subset} (cl(f(F_{A_i})))$.

DEFINITION: 3.20

A mapping $f: (F_A, \tilde{\tau}) \rightarrow (G_A, \tilde{\sigma})$ is said to be soft g**-closed (briefly Sg**-closed) map if the image of every soft closed set in $(F_A, \tilde{\tau})$ is soft g**-closed set in $(G_A, \tilde{\sigma})$.

THEOREM: 3.21

Let $f: (F_A, \tilde{\tau}) \rightarrow (G_A, \tilde{\sigma})$ be a soft closed map and $g: (G_A, \tilde{\sigma}) \rightarrow (H_A, \tilde{\xi})$ be soft g**- closed map, then $g \circ f$ is soft g**- closed map.

Proof:

Let $f:(F_A,\tilde{\tau}) \to (G_A,\tilde{\sigma})$ be soft closed map and let F_{A_i} be a soft closed set in $(F_A,\tilde{\tau})$. Then $f(F_{A_i})$ is soft closed set in $(G_A,\tilde{\sigma})$. Since $g:(G_A,\tilde{\sigma}) \to (H_A,\tilde{\tilde{\xi}})$ is soft g**- closed map $g \circ f = g(f(F_{A_i}))$ is a soft g**- closed set in $(H_A,\tilde{\tilde{\xi}})$. Then $g \circ f$ is soft g**- closed map.

DEFINITION: 3.22

A mapping $f:(F_A,\tilde{\tau}) \to (G_A,\tilde{\sigma})$ is said to be soft g**-open (briefly Sg**-open) map if the image of every soft open set in $(F_A,\tilde{\tau})$ is soft g**-open set in $(G_A,\tilde{\sigma})$.

THEOREM: 3.23

Let $f:(F_A,\tilde{\tau}) \to (G_A,\tilde{\sigma})$ be a soft open map and $g:(G_A,\tilde{\sigma}) \to (H_A,\tilde{\tilde{\xi}})$ be soft g**- map, then $g \circ f$ is soft g**-open map.

Proof :

Let $f:(F_A,\tilde{\tau}) \to (G_A,\tilde{\sigma})$ be soft open map and let F_{A_i} be soft open set in $(F_A,\tilde{\tau})$. Then $f(F_B)$ is soft open set in $(G_A,\tilde{\sigma})$. Since $g:(G_A,\tilde{\sigma}) \to (H_A,\tilde{\tilde{\xi}})$ is soft g**- open map, $g \circ f = g(f(F_B))$. $g(f(F_{A_i}))$ is a soft g**- open set in $(H_A,\tilde{\tilde{\xi}})$. Then $g \circ f$ is soft g**- open map.

Remark: 3.24

The Composition of two soft g**-Continuous functions need not be soft g**-continuous function.

EXAMPLE: 3.25

Let $X = Y = \{a,b,c\}$, $\mathrm{E} = \{\mathrm{e}_1,\mathrm{e}_2,\mathrm{e}_3\}$, $A,B,C \subseteq E$

$F_A = \{(e_1,\{a\}),(e_2,\{b,c\})\}$

$\tilde{\tau}$ (sos) $= \{F_A,F_\varphi,F_{A_7},F_{A_2}\}$

$\tilde{\tau}$ (scs) $= \{F_\varphi,F_A,F_{A_3},F_{A_1}\}$

Soft g**- closed sets of $(F_A,\tilde{\tau})$ are = $\{F_A,F_\varphi,F_{A_1},F_{A_3},F_{A_6}\}$

$G_A = \{(e_1,\{a,b\}),(e_2,\{c\})\}$

$\tilde{\sigma}$ (sos) $= \{G_A,G_\varphi,G_{A_5},G_{A_2}\}$

$\tilde{\sigma}$ (scs) $= \{G_\varphi,G_A,G_{A_1},G_{A_4}\}$

$H_A = \{(e_1,\{a,c\}),(e_2,\{b\})\}$

$\tilde{\tilde{\xi}}$ (sos) $= \{H_A,H_\varphi,H_{A_4},H_{A_1}\}$

$\tilde{\tilde{\xi}}$ (scs) $= \{H_\varphi,H_A,H_{A_2},H_{A_5}\}$

Let $f:(F_A,\tilde{\tau}) \to (G_A,\tilde{\sigma})$ be the mapping defined by $f(a)=a, f(b)=c, f(c)=b$ and $f^{-1}(a)=a, f^{-1}(b)=c, f^{-1}(c)=b$

and $g:(G_A,\tilde{\sigma}) \to (H_A,\tilde{\xi})$ be the mapping defined by

$g(a)=c, g(b)=a, g(c)=b$ and $g^{-1}(a)=b, g^{-1}(b)=c, g^{-1}(c)=a$. Then f and g are soft g**-continuous function. Since $(g\circ f)^{-1}(H_{A_5}) = f^{-1}(g^{-1}(H_{A_5}))$, $=f^{-1}(\{(e_1,\{a\}),(e_2,\{c\})\} = f^{-1}(G_{A_4}) = F_{A_2}$ which is not soft g**-closed set in $(F_A,\tilde{\tau})$.

IV- SOFT g^{**}- IRRESOLUTE FUNCTION

DEFINITION: 4.1

A soft map $f:(F_A,\tilde{\tau}) \to (G_A,\tilde{\sigma})$ from a soft topological space $(F_A,\tilde{\tau})$ into a soft topological space $(G_A,\tilde{\sigma})$ is called soft g**- irresolute if the inverse image of every soft $g**$-closed set in $(G_A,\tilde{\sigma})$ is soft g^{**}- closed set in $(F_A,\tilde{\tau})$.

EXAMPLE: 4.2

Let $X=Y=\{a,b,c\}, E=\{e_1,e_2,e_3\}, A,B,C \subseteq E$

$F_A=\{(e_1,\{a,b\}),(e_2,\{c\})\}$

$F_{A_1}=\{(e_1,\{a\})\}$

$F_{A_2}=\{(e_1,\{b\})\}$

$F_{A_3}=\{(e_1,\{a,b\})\}$

$F_{A_4}=\{(e_1,\{a\}),(e_2,\{c\})\}$

$F_{A_5}=\{(e_1,\{b\}),(e_2,\{c\})\}$

$F_{A_6}=\{(e_2,\{c\})\}$

$F_{A_7}= F_A$

$F_{A_8}= F_\phi$

$\tilde{\tau}(sos)=\{F_A,F_\phi,F_{A_5},F_{A_2}\}$

$\tilde{\tau}(scs)=\{F_A,F_\phi,F_{A_1},F_{A_4}\}$

Soft g^{**}- closed sets of $(F_A,\tilde{\tau})$ are = $\{F_A,F_\phi,F_{A_1},F_{A_3},F_{A_4}\}$

$G_A=\{(e_1,\{a,c\}),(e_2,\{b\})\}$

$G_{A_1}=\{(e_1,\{a\})\}$

$G_{A_2}=\{(e_1,\{c\})\}$

$G_{A_3} = \{(e_1, \{a, c\})\}$

$G_{A_4} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$G_{A_5} = \{(e_1, \{c\}), (e_2, \{b\})\}$

$G_{A_6} = \{(e_2, \{b\})\}$

$G_{A_7} = G_A$

$G_{A_8} = G_\phi$

$\tilde{\sigma}(sos) = \{G_A, G_\phi, G_{A_4}, G_{A_1}\}$

$\tilde{\sigma}(scs) = \{G_A, G_\phi, G_{A_1}, G_{A_5}\}$

Soft g**- closed sets of $(G_A, \tilde{\sigma})$ are = $\{G_A, G_\phi, G_{A_2}, G_{A_3}, G_{A_5}\}$

Let $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ be the mapping defined by

$f(a) = c, f(b) = a, f(c) = b$ and $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a$

Let $G_{A_2} = \{(e_1, \{c\})\}$ be soft g**- closed set in $(G_A, \tilde{\sigma})$ and $f^{-1}(G_{A_2}) = \{(e_1, \{a\})\} = F_{A_1}$, which is also a soft g**- closed set in $(F_A, \tilde{\tau})$.

$Let\ G_{A_3} = \{(e_1, \{a, c\})\}$, be soft g**- closed set in $(G_A, \tilde{\sigma})$ and $f^{-1}(G_{A_3}) = \{(e_1, \{b, a\})\} = F_{A_3}$, which is also a soft g**- closed set in $(F_A, \tilde{\tau})$.

$Let\ G_{A_5} = \{(e_1, \{c\}), (e_2, \{b\})\}$, be soft g**- closed set in $(G_A, \tilde{\sigma})$ and $f^{-1}(G_{A_5}) = \{(e_1, \{a\}), (e_2, \{c\})\} = F_{A_4}$, which is also a soft g**- closed set in $(F_A, \tilde{\tau})$.

⇒f is a soft g**- irresolute function.

EXAMPLE: 4.3

If a soft map $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ is soft g**- irresolute function then it is soft g**-continuous function

Proof:

Let $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ is soft g**- irresolute function. Let G_{A_i} be a soft closed set in $(G_A, \tilde{\sigma})$, then G_{A_i} is soft g**- closed set in $(G_A, \tilde{\sigma})$. We know that, Every soft closed set is soft g**-closed set. Since f is soft g**- irresolute function $f^{-1}(G_{A_i})$ is a soft g**- closed set in $(F_A, \tilde{\tau})$. Hence f is soft g**- continuous mapping.

EXAMPLE: 4.4

Let $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$, $g: (G_A, \tilde{\sigma}) \to (H_A, \tilde{\xi})$ be two soft functions then

(i) $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is $Sg **$ - continuous, if f is $Sg **$- continuous and g is soft continuous.

(ii) $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is $Sg **$- irresolute, if f and g is $Sg **$- irresolute functions.

(iii) $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is $Sg **$- continuous, if f is $Sg **$- irresolute and g is $Sg **$- continuous.

Proof:

(i) Let H_{A_i} be soft closed set in $(H_A, \tilde{\xi})$. Since g is soft- continuous, We know that $g^{-1}(H_{A_i})$ is soft g**- closed set of $(G_A, \tilde{\sigma})$. Now $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ is $Sg **$- continuous and $g^{-1}(H_{A_i})$ is soft closed set in $(G_A, \tilde{\sigma})$. $\Rightarrow$ $f^{-1}\left(g^{-1}(H_{A_i})\right) = (g \circ f)^{-1}(H_{A_i})$ is $Sg **$ - closed in $(F_A, \tilde{\tau})$. Hence $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is $Sg **$- continuous.

(ii) Let $g: (G_A, \tilde{\sigma}) \to (H_A, \tilde{\xi})$ is $Sg **$ - irresolute and let H_{A_i} be $Sg **$ - closed set in $(H_A, \tilde{\xi})$. Since g is $Sg **$ - irresolute, We know that $g^{-1}(H_{A_i})$ is soft g**- closed set of $(G_A, \tilde{\sigma})$. Also f is $Sg **$- irresolute, so $f^{-1}\left(g^{-1}(H_{A_i})\right) = (g \circ f)^{-1}(H_{A_i})$ is $Sg **$- closed in $(F_A, \tilde{\tau})$. Hence $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is $Sg **$- irresolute.

(iii) Let $g: (G_A, \tilde{\sigma}) \to (H_A, \tilde{\xi})$ is $Sg **$- continuous and let H_{A_i} be soft closed set in $(H_A, \tilde{\xi})$. Since g is $Sg **$- continuous, $g^{-1}(H_{A_i})$ is $Sg **$ - closed set of $(G_A, \tilde{\sigma})$. Also f is $Sg **$- irresolute, so the inverse of every $Sg **$- closed set of $(G_A, \tilde{\sigma})$ is $Sg **$- closed set in $(F_A, \tilde{\tau})$. $\Rightarrow$ $f^{-1}\left(g^{-1}(H_{A_i})\right) = (g \circ f)^{-1}(H_{A_i})$ is $Sg **$ - closed in $(F_A, \tilde{\tau})$. Hence $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is soft g**- continuous.

THEOREM: 4.5

Let $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$, $g: (G_A, \tilde{\sigma}) \to (H_A, \tilde{\xi})$ be two functions such that $g \circ f : (F_A, \tilde{\tau}) \to (H_A, \tilde{\xi})$ is $Sg **$ - closed function.

(i) If f is soft continuous and surjective, then g is $Sg **$ - closed function.

(ii) If g is $Sg **$ - irresolute and injective, then f is $Sg **$ - closed function.

Proof:

(i) Let f and g be two function and let G_{A_i} be a soft closed set of $(G_A, \tilde{\sigma})$. Then $f^{-1}(G_{A_i})$ is Soft closed set in $(F_A, \tilde{\tau})$ as f is soft continuous. Since $g \circ f$ is $Sg **$ - closed function.$\Rightarrow$ $g \circ f\ \left(f^{-1}(G_{A_i})\right) = g\left(f\left(f^{-1}(G_{A_i})\right)\right)$.

$\Rightarrow g \circ f\ \left(f^{-1}(G_{A_i})\right) = g(G_{A_i})$ is soft g** - closed set in $(H_A, \tilde{\xi})$. Hence g is $Sg **$ - closed function.

(ii) Let H_{A_i} be $Sg**$ - closed set in $(H_A, \tilde{\xi})$. Then $(g \circ f)(H_{A_i})$ is $Sg**$ - closed set in $(H_A, \tilde{\xi})$.$\Rightarrow$ $g^{-1}(g \circ f)(H_{A_i}) = g^{-1}\left(g\left(f(H_{A_i})\right)\right)$. $\Rightarrow g^{-1}(g \circ f)(H_{A_i}) = f(H_{A_i})$ is $Sg**$ - closed set in $(G_A, \tilde{\sigma})$ (Since g is $Sg**$ - irresolute & injective). Hence f is $Sg**$ - Closed function.

THEOREM: 4.6

Let $(F_A, \tilde{\tau})$ and $(G_A, \tilde{\sigma})$ be two soft topological spaces and $f: (F_A, \tilde{\tau}) \to (G_A, \tilde{\sigma})$ be a soft function, then the following conditions are equivalent.

(i) f is soft $g**$ - irresolute
(ii) The inverse image of each soft $g**$ - open set over $(G_A, \tilde{\sigma})$ is soft $g**$ - Open over $(F_A, \tilde{\tau})$.

Proof:

(i) $\Rightarrow$ (ii)

Let f be soft $g**$ - irresolute and let G_{A_i} be a soft $g**$ - open set over $(G_A, \tilde{\sigma})$. Then $G_A - G_{A_i}$ is soft $g**$ - closed over $(G_A, \tilde{\sigma})$. By assumption, $f^{-1}(G_A - G_{A_i})$ is Soft $g**$ - closed set over $(F_A, \tilde{\tau})$. (ie) $f^{-1}(G_A) - f^{-1}(G_{A_i}) = F_A - f^{-1}(G_{A_i})$ is soft $g**$ - closed set over $(F_A, \tilde{\tau})$. Hence $f^{-1}(G_{A_i})$ is soft $g**$ - open set over $(F_A, \tilde{\tau})$.

(ii) $\Rightarrow$ (i)

Let G_B be any soft $g**$ - closed set over $(G_A, \tilde{\sigma})$. Then $G_A - G_{A_i}$ is soft $g**$ - open over $(G_A, \tilde{\sigma})$. By hypothesis, $f^{-1}(G_A - G_{A_i}) = f^{-1}(G_A) - f^{-1}(G_{A_i})$. $\Rightarrow F_A - f^{-1}(G_{A_i})$ is soft $g*$ $*$ - open over $(F_A, \tilde{\tau})$. Therefore $f^{-1}(G_{A_i})$ is soft $g**$ - closed over $(F_A, \tilde{\tau})$. $\Rightarrow$ f is soft $g**$ - irresolute.

Conclusion

In this paper we have introduced the notions of soft g**-continuous functions, soft g**-irresolute functions, soft g**-open map, soft g**-closed map and their basic properties are studied and investigated. In the extension of this work we will introduce soft connectedness, compactness, separation axioms etc. Also in future using this new type of soft set we would try to apply one of the real applications like data mining, image processing etc.

References

1) Angelin Tidy.G, Dr.A.Francina Shalini, On Soft sgb-continuous functions in soft topological spaces, IJESC, Volume : 7, Issue No : 4.
2) Christy.V, Mohan.K, On Soft πGP-continuous functions in soft topological spaces, IJESC, Volume : 6, Issue No : 8.

3) Gozde Yaylali, Bekir Tonay, Results on Soft continuous functions in soft topological spaces quipped with soft scott topology, Journal of non-linear sciences and applications, 10(2017), 1183-1194.
4) Indirani.K and Smitha.M.G, On Soft rgb-continuous functions in soft topological spaces, International Journal of Physics and Mathematical Sciences, 2014 Vol : 4(3) pp : 78-86.
5) Janaki.C, Jeyanthi.V, On Soft πgr-continuous functions in soft topological spaces, IJERT, Volume : 3, Issue No : 2(2014).
6) Kalaivani.N, Chandrasekaran.E, Fayaz Ur Rahman.K, On Soft $\alpha(\gamma,\beta)$-continuous functions in soft topological spaces, Fuzzy intelligent systems: Methodologies, Techniques and applications, August 2021.
7) Kannan.K, Rajalakshmi.D, Gopikrishnan.B, Soft s*g-continuous functions in soft topological spaces, International Journal of Pure and Applied Mathematics, Volume : 119, No: 6, 2018, 51-61.
8) Kharal.A, Ahmed.B, Mappings of Soft classes, New.Math.Nat.,Comput.,7(3), 2011, Vol: 37, No:1, 17-21.
9) Levine.N, On generalized closed sets in topology, Rend.Circ.Math.Palerno, 19(2) (1970) 89-96.
10) Metin Akdag, Alkan Ozkan, Soft b-open sets and Soft b-continuous functions, Math Sci(2014) 8: 124.
11) Molodtsov.D, Soft Set Theory-First results, Computers and Mathematics with applications, 37(1999), 19-31.
12) Molodtsov.D, Soft Set Theory-first results, Comput.Ath.Appl.Vol: 45(2003), pp : 555-562.
13) Muhammes Shabir, Munazza Naz, On Soft topological spaces, Computers and mathematics with applications, 61(2011), 1786-1799.
14) Pious Missier.S and Vijilius Helana Raj, Soft gs^ -continuous functions in soft topological spaces, Hindawi, Vol : 2013, Feb 2013.
15) Ramadhan A.Mohammed, Tahir H.Ismail and Allam.A.A, On Soft gαb-continuous functions in soft topological spaces, Assiut Univ.J of Mathematics and computer science, 44(2), pp: 13-29(2015).
16) Sabir Hussain, Bashir Ahmad, Some Properties of Soft topological spaces, Computers and mathematics with applications, 62(2011), 4058-4067.
17) Samer Al Ghour, Soft ω_p -open sets and soft ω_p -continuity in soft topological spaces, $\sum$Mathematics, 2021, 9, 2632.
18) Serder Enginoglu, Naim Cagman, Serkan Karatas, Tugce Aydm, On Soft topology , TUBIAD, Vol : 2, No: 3, 2015(23-38).