

New Log Type Estimator in Simple Random Sampling

Kusum^{1,a}, Sangeeta Malik^{1,b}

*** Corresponding author :**

Dr.Sangeeta Malik, Associate Professor, Dept. of Mathematics

BMU, AsthalBohar, Rohtak (Haryana) Pin: 124021

Email : sangeetastat@gmail.com

Kusum, Research Scholar, Dept. of Mathematics

BMU, AsthalBohar, Rohtak (Haryana) Pin: 124021

Emails :kusumsansanwal19@gmail.com

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Abstract

The current study using a log class class estimates uses a helpful variant to estimate the number of people under a simple random sample. To find out the bias and square error of the proposed measurement scales take up to the first degree scale. Proposed categories of estimates are compared with the description of each unit with other emotional estimates and work better than comparative estimates.

INTRODUCTION

In a sample survey the purpose of the survey statistician is to estimates some functions of the population parameters, by choosing a sample and by observing the value of y only on the units selected in sample. To estimating the problem of finite population mean in a subsidiary variance has been discussed in the limited population. In the theory of sample survey it is antiquated event that the additional information is always used to upgrade the exactness of estimators. The use of additional information in sample sampling is widely practiced in the case of measuring human

parameters with known and unknown of auxiliary information. In this area the research work was initiated by Bhal and Tuteja (1991). If there is a very good correlation between the significant difference y and the auxiliary variation x the ratio is recommended but if there is a negative correlation between y and x then the product ratio is used effectively. Cochran (1940) discussed the measurement of the type of product, while Murthy (1967) suggested proposing the type of product. Many authors have used a set of specific human limitations to resolve certain values. The work of Kadilar and Cingi (2004), Onekya (2012), Chaun Singh (2014), Subramani and Ajith (2016), Madhulika et al. (2017) on the use of auxiliary variable. In order to improve the accuracy of population estimates of the dynamic definition of research by presenting a logarithmic estimate and a product type estimate we use additional information in the current work.

In the present work we consider a finite population $U = U_1, U_2, U_3, \dots, U_n$ of N units. Let \bar{y} and \bar{x} be the sample mean estimator of the population mean \bar{Y} and \bar{X} of the study variable y and auxiliary variable x . To obtain the bias and MSE. We define:

$$\bar{y} = \bar{Y}(1 + e_0) \text{ and } \bar{x} = \bar{X}(1 + e_1) \text{ such that } E(e_0) = E(e_1) = 0$$

$$E(e_0^2) = \frac{1-f}{n} c_y^2, \quad E(e_1^2) = \frac{1-f}{n} c_x^2, \quad E(e_0 e_1) = \frac{1-f}{n} c_{yx}$$

$$\text{Where } c_y^2 = \frac{s_y^2}{\bar{Y}^2}, \quad c_x^2 = \frac{s_x^2}{\bar{X}^2}, \quad \rho = \frac{s_{xy}}{s_x s_y}$$

Bhal and Tuteja (1991) present the exposure measure and the product type of the population \bar{Y} :

$$t_r = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)$$

$$t_p = \bar{y} \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right)$$

The square root error detected up to the first level of measurement of the product type scale and product type rating:

$$\text{MSE}(t_r) = \left(\frac{1-f}{n}\right) \left(S_y^2 + \frac{1}{4} R^2 S_x^2 - R S_{yx}\right)$$

$$\text{MSE}(t_p) = \left(\frac{1-f}{n}\right) \left(S_y^2 + \frac{1}{4} R^2 S_x^2 + R S_{yx}\right)$$

The current study introduces a logarithmic estimate and a product type estimate based on the natural logarithmic of the known population of the benefit variant. Searching for such an effective scale leads us to consider logarithmic type estimates.

The proposed logarithmic estimator:

$$\bar{y}_{klg} = \bar{y} + \log \left(\frac{\bar{X}}{\bar{x}} \right)^\beta \quad (2.1)$$

$$\bar{y} = \bar{Y} (1+e_0), \quad \bar{x} = \bar{X} (1+e_1)$$

$$= \bar{Y} (1+e_0) + \log \left(\frac{\bar{X}}{\bar{X} (1+e_1)} \right)^\beta$$

$$E(\bar{y}_{klg}) = \bar{Y} + \beta \frac{e_1^2}{2}$$

$$\text{Bias}(\bar{y}_{klg}) = E(\bar{y}_{klg}) - \bar{Y}$$

$$= \beta \frac{e_1^2}{2}$$

$$\text{Var.}(\bar{y}_{klg}) = E[\bar{y}_{klg} - E(\bar{y}_{klg})]^2$$

$$= E \left[\bar{y} + \log \left(\frac{\bar{X}}{\bar{x}} \right)^\beta - \bar{Y} - \beta \frac{e_1^2}{2} \right]^2$$

$$= E[\bar{Y}e_0 - \beta e_1]^2$$

$$= [\bar{Y}^2 e_0^2 + \beta^2 e_1^2 - 2\bar{Y}\beta e_0 e_1]$$

$$= \left(\frac{1-f}{n} \right) [s_y^2 + \beta^2 \frac{s_x^2}{\bar{X}^2} - 2\beta \frac{s_x s_y}{\bar{X}}] \quad (2.2)$$

Efficiency Comparison:

Comparison of \bar{y}_{klg} with mean per unit estimator:

We see that \bar{y}_{klg} is more efficient than whenever $\text{var.}(\bar{y}_{klg}) - \text{var.}(\bar{Y}) < 0$

i.e.

$$\left(\frac{1-f}{n}\right)[sy^2 + \beta^2 \frac{sx^2}{\bar{X}^2} + 2\beta \frac{\rho s_x s_y}{\bar{X}}] < \left(\frac{1-f}{n}\right)(sy^2)$$

$$\rho < \frac{1}{2} \beta \frac{s_x}{s_y}$$

Comparison of \bar{y}_{klg} with ratio estimator:

We see that $\text{var.}(\bar{y}_{klg}) < (\bar{y}_r)$

Or whenever

$$\rho < \frac{1}{2} (\beta - R) \frac{s_x}{s_y}$$

Optimum value of proposed estimator:

Optimum value of β at which MSE is minimum is given by:

$$\beta = -\rho \frac{s_x}{s_y} \bar{X}$$

The minimum MSE of the estimates \bar{y}_{klg} is given by:

$$\left(\frac{1-f}{n}\right)[s_y^2(1-\rho^2)]$$

Generalize form of proposed estimator:

$$\bar{y}_{klg2} = \bar{y} + \log\left(\frac{\bar{x}}{\bar{X}}\right)^\beta \quad (3.1)$$

$$\bar{y} = \bar{Y}(1+e_0), \quad \bar{x} = \bar{X}(1+e_1)$$

$$= \bar{Y}(1+e_0) + \log\left(\frac{\bar{X}(1+e_1)}{\bar{X}}\right)^\beta$$

$$E(\bar{y}_{klg2}) = \bar{Y} - \beta \frac{e_1^2}{2}$$

$$\text{Bias}(\bar{y}_{klg2}) = E(\bar{y}_{klg2}) - \bar{Y}$$

$$= -\beta \frac{e_1^2}{2}$$

$$\text{Var.}(\bar{y}_{klg2}) = E[\bar{y}_{klg2} - E(\bar{y}_{klg2})]^2$$

$$\begin{aligned}
&=E\left[\bar{y} + \log\left(\frac{\bar{X}}{\bar{x}}\right)^\beta - \bar{Y} + \beta \frac{e_1^2}{2}\right]^2 \\
&=E[\bar{Y}e_0 + \beta e_1]^2 \\
&=[\bar{Y}^2 e_0^2 + \beta^2 e_1^2 + 2\bar{Y}\beta e_0 e_1] \\
&=\left(\frac{1-f}{n}\right)[sy^2 + \beta^2 \frac{sx^2}{\bar{X}^2} + 2\beta \frac{s_x s_y}{\bar{X}}] \quad (3.2)
\end{aligned}$$

Efficiency comparison:

Comparison of \bar{y}_{klg2} with mean per unit estimator:

We see that \bar{y}_{klg2} is more efficient than whenever $\text{var.}(\bar{y}_{klg2}) - \text{var.}(\bar{Y}) < 0$

i.e.

$$\left(\frac{1-f}{n}\right)[sy^2 + \beta^2 \frac{sx^2}{\bar{X}^2} + 2\beta \frac{\rho s_x s_y}{\bar{X}}] < \left(\frac{1-f}{n}\right)(sy^2)$$

$$\rho < \frac{1}{2} \beta \frac{s_x}{s_y}$$

Comparison of \bar{y}_{klg2} with ratio estimator:

We see that $\text{var.}(\bar{y}_{klg2}) < (\bar{y}_r)$

Or whenever

$$\rho < \frac{1}{2} (\beta - R) \frac{s_x}{s_y}$$

Optimum value of generalize form of proposed estimator:

Optimum value of β at which MSE is minimum is given by:

$$\beta = -\rho \frac{s_x}{s_y} \bar{X}$$

The minimum MSE of the estimates \bar{y}_{klg2} is given by:

$$\left(\frac{1-f}{n}\right)[s_y^2(1 - \rho^2)]$$

Empirical study:

For empirical study we consider the following data:

Population1. [source: Murthy(1967,p.228)]

$N=106$, $\bar{Y}=15.37$, $\bar{X}=243.76$, $s_y^2=4127.626$, $s_x^2=242453$, $s_{xy}=25940.47$

Population2. $N=176$, $n=16$, $\bar{Y}=282.6136$, $\bar{X}=6.9943$,

$$s_y^2=24114.67, \quad s_x^2=8.76,$$

$$s_{xy}=400.3233015,$$

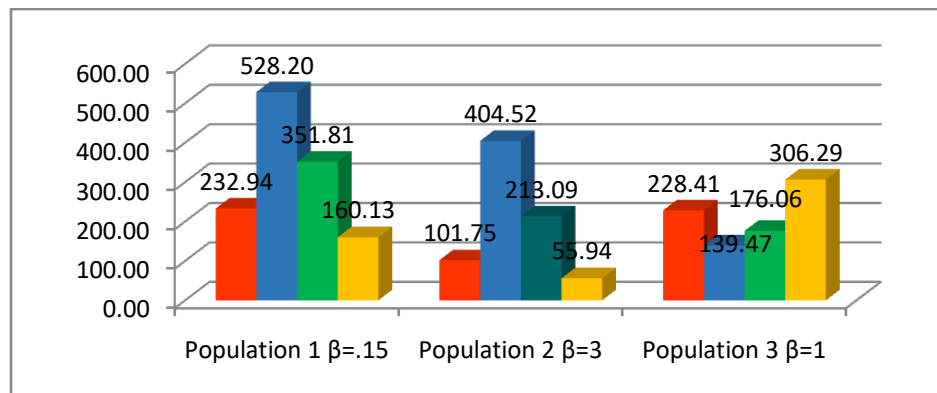
Population3. [source: Johnston(1982,p.171)]

$N=10$, $\bar{Y}=52$, $\bar{X}=200$, $s_y^2=65.97338$, $s_x^2=84.01556$, $s_{xy}=-69.98292$,

Table1. PRE of proposed estimator with usual and different estimators :

Estimators	Population 1	Population 2	Population 3
	$\beta=.15$	$\beta=3$	$\beta=1$
\bar{y}	232.9364	101.751	228.4084
\bar{y}_r	528.2028	404.5201	139.4699
\bar{y}_{tr}	351.8063	213.0933	176.0613
\bar{y}_{tp}	160.1321	55.93636	306.2924

Chart 1



Conclusion:

Finally, we conclude that our proposed measure is more efficient than the different population measurement I and population 3 but not with the average of 2 population. The result of the current operation is supported and displayed numerically .

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