M/M/2 Heterogeneous Server Queue with Variant Breakdown and with Discouraged Arrivals

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Abstract
The content of this article is a two server Markovian queue with different service rates, two different breakdowns and discouraged arrivals. We compute the system size probabilities in steady state with stability condition using Matrix Analytic Technique. Also we compute some performance measures. Finally, we present some numerical illustrations.
Keywords: - Working vacation; Breakdown; Matrix Analytic method.

Introduction

In real life, it can be seen that all arriving customers need not join the queue for service, if the service is not immediate, or the server is not available for service or etc., such a queueing situation is called queue with discouraged arrivals. This behaviour of the customer, in general, is called impatient customer. Wang et al have given an extensive review an queue with impatient customers [28]. Kapodistria has analyzed a Markovian queue with impatient customers [10]. Ammar et al analyzed a finite capacity. single server queue with discouraged arrivals and reneging using matrix method [2].

Queues with more than one server constitute an important class of queueing system and have wide practical applications. For example, in bank, in telephone exchange, in calling centre and in similar situations we can see such systems. A large number of researchers have contributed to the system with more than one server but with identical service rates. Another class of system, which was considered by a huge number of researchers is queues with unreliable servers due to the reason that this system can be encountered in the field of computer operating systems, communication networks and manufacturing systems.

The first work on queue with breakdown was by White and Christie [30]. Most of the works on the queueing system with breakdown deals with the single-server models [4, 5, 6, 13, 23, 25, 26, 32, 34]. The works on unreliable multiserver systems is not sufficient. Mitrany and Avi-Itzhak analyzed the model with N servers and the same amount of repairmen, they obtained the steady-state mean queueing length of customers [15]. Vinod studied the same model using the matrix geometric solution method [24]. Neuts and Lucantoni [16], and Wartenhorst [29] extended the models of Mitrany and Avi-Itzhak, and Vinod by restricting the repair capacity.

Alam and Mani studied recursive solution technique in a multi-server bi-level queueing system with server breakdowns [3]. Wang and Chang considered finite waiting line M/M/R model with balking, reneging and breakdown [27]. They obtained the steady state probabilities in matrix form. A multi-server queueing system with identical unreliable servers with phase type distributed service times is considered by Yang and Alfa [33]. In this paper, the authors assumed that the servers are subject to random breakdowns and repair is carried out when a repair person is available. They considered both the cases of infinite and finite buffers and solved the models. Wu et al studied an infinite capacity multi server queueing system with unreliable servers [31]. They introduced a controllable repair policy and solved the model using matrix analytic method. Klimenok investigated an infinite buffer model with, two unreliable heterogeneous servers, they fails alternatively [11].

The models discussed above all assumed that the servers are homogeneous, this may be valid only when the service system is mechanically or electronically operated. But, if the service system has human servers, the service rate may not be identical. It can be seen in checkout counters of departmental stores, in banks, in hospitals, etc. Researchers in the field of Engineering and Technology, Statistics and Mathematics work in the field of queues with servers and with different service rates, called queue with heterogeneous servers.

In 1981, Neuts and Takahashi have pointed out that analytical results are intractable for the queueing system with two heterogeneous servers [17]. Even though, some researchers focused their studies on queue with two heterogeneous servers. Krishnamoorthy consider a Poisson queue with two heterogeneous servers and with violation of the first in first out principle [12]. A Markovian queueing system with balking and two heterogeneous servers has been considered in Singh [21]. The author determines the capacity of the slower server and obtains the optimal service rate. Singh discussed a Markovian queue with the number of servers depending upon the queue length [22]. Lin and Kumar has analyzed the optimal control of a queueing system with two heterogeneous servers [14]. Rubinovitch studied the problem of a heterogeneous two channels queueing systems [19, 20]. In his first paper he discussed three simple models and gave the condition when to discard the slower server depending on the expected number of customers in the system. In the second paper he studied a queueing model with a stalling concept. In 1999, Abou-Elu-Ata and Shawky introduced a simpler approach to find the condition when to discard the slower server in a heterogeneous two channel queue [1].

Kalyanaraman and Senthilkumar analyzed a two heterogeneous server Markovian queue with switching of service mode [7]. The same authors discussed heterogeneous server Markovian queue with restricted admissibility and with reneging [8]. Kalyanaraman and Senthilkumar analyzed a two heterogeneous server queue with restricted admissibility [9]. Matrix-geometric method approach is a useful tool for solving various queueing problems in different frameworks. Neuts explained various matrix geometric solutions of stochastic models [18]. Matrix geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic.

In the present work, we consider an M/M/2 heterogeneous servers queue with two different types of breakdowns and with discouraged arrivals. Section 2 discusses the model and the main results. Section 3 discusses some numerical results and in Section 4 a conclusion has been given.

The Model

A two server queueing system with the following assumptions has been considered:

- The arrival follows Poisson process with parameter λ.
- The two servers are heterogeneous servers.
- An arriving customer joins the first server, if there is no customers in the system. Otherwise, the customer waits in the queue for a free server.
- There is a waiting line of infinite capacity. First come First served rule is used for service.
- During service, Server1 may breakdown, the breakdown occurs in two modes, called partial and total mode.
- The number of total breakdowns (partial breakdowns) follows Poisson process with rate $\alpha_1(\alpha_2)$. The server is immediately sent for repair, the time period follows exponential distribution with parameter $\beta_1(\beta_2)$.
- During normal period, the service times follows negative exponential distribution with rates μ_1(Server 1), μ_2(Server 2). Also $\mu = \mu_1 + \mu_2 \ \ and \ \ \mu_1 > \mu_2$.
- During partial breakdown period the servers serve the customers with service times follows exponential distribution with rates μ_3(server 1), μ_2(server 2). Also $\mu' = \mu_2 + \mu_3$.
- At the time of total breakdown, if the service of the customer in the service station is not completed, the customer enter in to the head of the queue and the service of this customer starts afresh.
- At the time of partial breakdown, if the customer in service is not completed, the customer's service rate reduces to μ_3 and the service starts afresh.
- During breakdown (partial breakdown), the arriving customer become discouraged and fail to join the queue with probability $1-\theta_1(1-\theta_2)$.

The quasi-birth-and-death (QBD) Process

The notations used to define the model are:

Let X(t) be the number of customers present in the system, Y(t) be the server state 1 and Z(t) be the server state 2 at time t.

$$Y(t) = \begin{cases} 0 & \text{server1 is in idle state} \\ 1 & \text{server1 is in busy state} \\ 2 & \text{server1 is in breakdown state} \\ 3 & \text{server1 is in partial breakdown state} \end{cases}$$

$$Z(t) = \begin{cases} 0 & \text{server2 is in idle state} \\ 1 & \text{server2 is in busy state} \end{cases}$$

The process $\{(X(t), Y(t), Z(t)): t \geq 0\}$ is a quasi-birth-death process with states. $S = S_1 U S_2 U S_3$, Where

$S_1 = \{(0,0,0), (0,2,0), (0,3,0)\}$

$S_2 = \{(1,1,0), (1,0,1), (1,3,0), (1,2,1), (1,3,1)\}$ and

$S_3 = \{(x, y, z): x \geq 2, y = 1,2,3, z = 1\}$.

The above set of states are written in lexicographical order to form the rate matrix Q. We defined the following auxillary matrices for the formation of the rate matrix of the corresponding Markov process.

The Markov process $\{(X, Y, Z)\}$ has the following generator matrix:

$$Q = \begin{bmatrix} B_0 & C_0 & \dots & & & & \\ B_1 & C_1 & D_0 & \dots & & & \\ . & B_2 & A_1 & A_0 & \dots & & \\ . & . & A_2 & A_1 & A_0 & \dots & \\ . & . & . & A_2 & A_1 & A_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Where,

$$B_0 = \begin{bmatrix} -\lambda & 0 & 0 \\ \beta_1 & -(\lambda+\beta_1) & 0 \\ \beta_2 & 0 & -(\lambda+\beta_2) \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & \lambda\theta_1 & 0 & 0 \\ 0 & 0 & 0 & \lambda\theta_2 & 0 \\ & & & & \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \mu_2 & 0 & 0 \\ \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \\ 0 & 0 & \mu_2 \\ & & \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -\mu_2 & 0 & 0 & 0 & 0 \\ 0 & -(\mu_1+\alpha_1+\alpha_2+\lambda) & \alpha_1 & \alpha_2 & 0 \\ \beta_1 & 0 & -(\mu_2+\beta_1+\lambda\theta_1) & 0 & 0 \\ 0 & \beta_2 & 0 & -(\mu_3+\lambda\theta_2+\beta_2) & 0 \\ \beta_2 & 0 & 0 & 0 & -(\mu_2+\beta_2+\lambda\theta_2) \\ & & & & \end{bmatrix}$$

$$D_0 = \begin{bmatrix} 0 & 0 & 0 \\ \lambda & 0 & 0 \\ 0 & \lambda\theta_1 & 0 \\ 0 & 0 & \lambda\theta_2 \\ 0 & 0 & \lambda\theta_2 \\ & & \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \mu_1 & \mu_2 & 0 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 & 0 \\ 0 & 0 & 0 & \mu_2 & \mu_3 \\ & & & & \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu' \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda+\mu+\alpha_1+\alpha_2) & \alpha_1 & \alpha_2 \\ \beta_1 & -(\lambda\theta_1+\mu_2+\beta_1) & 0 \\ \beta_2 & 0 & -(\lambda\theta_2+\beta_2+\mu_2+\mu_3) \end{bmatrix}$$

$$A_0 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda\theta_1 & 0 \\ 0 & 0 & \lambda\theta_2 \end{bmatrix}$$

The Steady-state Solution

In this subsection, the stability condition for the existance of steady state solution has been established first, then the steady state probabilities are obtained using matrix analytic solution method. Finally some performance measures are derived.

Stability Condition

Define $A = A_0 + A_1 + A_2$, it is obtained as $\begin{bmatrix} -(\alpha_1+\alpha_2) & \alpha_1 & \alpha_2 \\ \beta_1 & -\beta_1 & 0 \\ \beta_2 & 0 & -\beta_2 \end{bmatrix}$. It is easily seen that A is an irreducible generator of a Markov chain and let $\pi = (\pi_0, \pi_1, \pi_2)$ be the stationary probability vector satisfying the condition

$$\pi A = 0, \pi e = 1 \tag{1}$$

Solving equation(1), we have

$$\pi_0 = \frac{\beta_1\beta_2}{\alpha_1\beta_2 + \alpha_2\beta_1 + \beta_1\beta_2}$$

$$\pi_1 = \frac{\alpha_1\beta_2}{\alpha_1\beta_2 + \alpha_2\beta_1 + \beta_1\beta_2}$$

$$\pi_2 = \frac{\alpha_2\beta_1}{\alpha_1\beta_2 + \alpha_2\beta_1 + \beta_1\beta_2}$$

By theorem 3.1.1 in Neuts (1981) [18], the condition $\pi A_0 e < \pi A_2 e$ is the equilibrium condition for the QBD process, where e is the column vector of one's of appropriate dimension. After simplification, the condition has been obtained as

$$\rho = \frac{\lambda(\alpha_1\beta_2 + \theta_1\alpha_2\beta_1 + \theta_2\beta_1\beta_2)}{\mu\beta_1\beta_2 + \mu_2\alpha_1\beta_2 + \mu'\alpha_2\beta_1} < 1$$

The Matrix Geometric Solution

Let $P = (p_0, p_1, p_2, \ldots)$ be the probability vector in steady state, associated with Q, with $PQ = 0$ and $Pe = 1$. Let $p_0 = (p_{000}, p_{020}, p_{030})$, $p_1 = (p_{110}, p_{101}, p_{130}, p_{121}, p_{131})$ and $p_n = (p_{n11}, p_{n21}, p_{n31}), n \geq 2$.

In steady state, the probabilities p_i are obtained from

$$p_0B_0 + p_1B_1 = 0 \tag{2}$$

$$p_0C_0 + p_1C_1 + p_2B_2 = 0 \tag{3}$$

$$p_1D_0 + p_2A_1 + p_3A_2 = 0 \tag{4}$$

$$p_iA_0 + p_{i+1}A_1 + p_{i+2}A_2 = 0, \quad i \geq 2 \tag{5}$$

$$p_i = p_2R^{i-2}, \quad i = 3,4,5,\ldots \tag{6}$$

where R is the rate matrix, is the minimal non-negative solution of the matrix quadratic equation (see Neuts(1981)).

$$R^2A_2 + RA_1 + A_0 = 0 \tag{7}$$

Substituting the equation (6) in (4), we have

$$p_1D_0 + p_2(A_1 + RA_2) = 0 \tag{8}$$

and the total probability condition is

$$p_0e + p_1e + p_2(I-R)^{-1}e = 1 \tag{9}$$

Theorem 2.1

If $\rho < 1$, the stationary probability vectors $p_0 = (p_{000}, p_{020}, p_{030})$, $p_1 = (p_{110}, p_{101}, p_{130}, p_{121}, p_{131})$ and $p_n = (p_{n11}, p_{n21}, p_{n31}), n \geq 2$ are

$p_{000} = S_{13} p_{030}$

$p_{020} = S_{11} p_{030}$

$p_{030} = \frac{1}{S_{15}}$

$p_{110} = S_{12} p_{030}$

$p_{101} = (S_0 - S_1 S_6 S_{11} + S_1 S_7) p_{030}$

$p_{130} = \frac{S_{11}(\lambda+\beta_1)}{\mu_2} p_{030}$

$p_{121} = S_{14} p_{030}$

$p_{131} = \frac{\mu_3(S_6 S_{11} - S_7)}{\lambda\theta_2 + \mu_2 + \beta_2} p_{030}$

$p_{211} = (S_4 + S_5 S_7 - S_8 S_{11}) p_{030}$

$p_{221} = (S_9 S_{11} - S_{10}) p_{030}$

$p_{231} = (S_6 S_{11} - S_7) p_{030}$

where

$$S_0 = \frac{(\lambda\theta_2 + \beta_2 + \mu_3)(\lambda + \beta_2) - \lambda\theta_2\mu_3}{\mu_3\alpha_3}$$

$$S_1 = \frac{\mu_2(\mu_2 + \mu_3 + 2\beta_2 + 2\lambda\theta_2)}{\alpha_2(\lambda\theta_2 + \mu_2 + \beta_2)}$$

$$S_2 = \frac{(\lambda\theta_1 + \mu_2 + \beta_1)(\lambda + \beta_1) - \lambda\theta_1\mu_2}{\mu_2^2}$$

$$S_3 = \frac{\mu_3\beta_2 - \mu_1 S_1(\lambda + \mu_2 + \beta_2)}{\lambda(\lambda + \mu_2 + \beta_2)}$$

$$S_4 = \frac{\mu_3 S_0(\lambda + \alpha + \mu_1) - \mu_3(\beta_2 + \mu_1 S_0) - \beta_2(\lambda + \beta_2)}{\mu\mu_3}$$

$$S_5 = \frac{S_1(\lambda + \mu_2 + \beta_2)(\lambda + \alpha + \mu_1) + \lambda S_3(\lambda + \mu_2 + \beta_2) - \mu_2\beta_2}{\mu(\lambda + \mu_2 + \beta_2)}$$

$$S_6 = \frac{\beta_1(\lambda + \mu_2 + \beta_1)[\mu r_{11} - (\lambda + \mu + \alpha)] - S_2\mu\mu_2(\beta_1 + \mu r_{21})}{\mu[\mu_2(\beta_2 + \mu r_{31}) - \alpha_1 S_1(\beta_1 + \mu r_{21}) - S_5\mu_2[\mu r_{11} - (\lambda + \mu + \alpha)] - \lambda\mu_2 S_1]}$$

$$S_7 = \frac{\lambda\mu_2 S_0 + \mu_2 S_4[\mu r_{11} - (\lambda + \mu + \alpha)] - \alpha_1 S_0(\beta_1 + \mu r_{21})}{\mu_2(\beta_2 + \mu r_{31}) - \alpha_1 S_1(\beta_1 + \mu r_{21}) - S_5\mu_2[\mu r_{11} - (\lambda + \mu + \alpha)] - \lambda\mu_2 S_1}$$

$$S_8 = \frac{\mu\mu_2 S_5 S_6 + \beta_1(\lambda + \mu_2 + \beta_1)}{\mu\mu_2}$$

$$S_9 = \frac{\mu_2 S_2 + \alpha_1 S_1 S_6}{\mu_2}$$

$$S_{10} = \frac{\alpha_1(S_0 + S_1 S_7)}{\mu_2}$$

$$S_{11} = \frac{\mu_2[\mu_2 r_{32} S_7 + S_{10}[\mu_2 r_{22} - (\lambda\theta_1 + \mu_2 + \beta_1)] - (\alpha_1 + \mu_2 r_{12})(S_4 + S_5 S_7)]}{\lambda\theta_1(\lambda + \beta_1) - \mu_2 S_8(\alpha_1 + \mu_2 r_{12}) + \mu_2 S_9[\mu_2 r_{22} - (\lambda\theta_1 + \mu_2 + \beta_1)] + \mu_2^2 r_{32} S_6}$$

$$S_{12} = \frac{\beta_1 S_{11}(\lambda+\beta_1)(\lambda\theta_2+\mu_2+\beta_2)+\mu_2\mu_3\beta_2(S_6S_{11}-S_7)+\mu_1\mu_2(\lambda\theta_2+\mu_2+\beta_2)(S_4+S_5S_7-S_8S_{11})}{\mu_2^2(\lambda\theta_2+\mu_2+\beta_2)}$$

$$S_{13} = \frac{\beta_1 S_{11}(\lambda+\mu_2+\beta_1)+\mu_2(\beta_2+\mu_1S_0)+\lambda\mu_2S_3(S_6S_{11}-S_7)+\mu_1\mu_2(S_4+S_5S_7-S_8S_{11})}{\lambda\mu_2}$$

$$S_{14} = \frac{(\lambda+\beta_2)(\lambda\theta_2+\mu_2+\beta_2)-\mu_2\mu_3(S_6S_{11}-S_7)}{\mu_3(\lambda\theta_2+\mu_2+\beta_2)}$$

$$S_{15} = 1+S_{11}+S_{12}+S_{13}+S_0-S_1S_6S_{11}+S_1S_7+\frac{S_{11}(\lambda+\beta_1)}{\mu_2}+\frac{\mu_3(S_6S_{11}-S_7)}{\lambda\theta_2+\mu_2+\beta_2} + S_{14}Z_1(S_4+S_5S_7-S_8S_{11})+Z_2(S_9S_{11}-S_{10})+Z_3(S_6S_{11}-S_7)$$

$$Z_1 = \frac{1-r_{22}-r_{23}r_{32}-r_{33}+r_{22}r_{33}+r_{13}+r_{13}r_{32}-r_{12}r_{33}+r_{12}-r_{13}r_{22}+r_{12}r_{23}}{1-r_{11}-r_{12}r_{21}-r_{22}+r_{11}r_{22}-r_{13}r_{31}+r_{13}r_{22}r_{31}-r_{12}r_{23}r_{31}-r_{13}r_{21}r_{32}-r_{23}r_{32}+r_{11}r_{23}r_{32}-r_{33}+r_{11}r_{33}+r_{12}r_{21}r_{33}+r_{22}r_{33}-r_{11}r_{22}r_{33}}$$

$$Z_2 = \frac{r_{21}+r_{23}r_{31}-r_{21}r_{33}+1-r_{11}-r_{13}r_{31}-r_{33}+r_{11}r_{33}+r_{13}r_{21}+r_{23}-r_{11}r_{23}}{1-r_{11}-r_{12}r_{21}-r_{22}+r_{11}r_{22}-r_{13}r_{31}+r_{13}r_{22}r_{31}-r_{12}r_{23}r_{31}-r_{13}r_{21}r_{32}-r_{23}r_{32}+r_{11}r_{23}r_{32}-r_{33}+r_{11}r_{33}+r_{12}r_{21}r_{33}+r_{22}r_{33}-r_{11}r_{22}r_{33}}$$

$$Z_3 = \frac{r_{31}-r_{22}r_{31}+r_{21}r_{22}+r_{12}r_{31}+r_{32}-r_{11}r_{32}+1-r_{11}-r_{12}r_{21}-r_{22}+r_{11}r_{22}}{1-r_{11}-r_{12}r_{21}-r_{22}+r_{11}r_{22}-r_{13}r_{31}+r_{13}r_{22}r_{31}-r_{12}r_{23}r_{31}-r_{13}r_{21}r_{32}-r_{23}r_{32}+r_{11}r_{23}r_{32}-r_{33}+r_{11}r_{33}+r_{12}r_{21}r_{33}+r_{22}r_{33}-r_{11}r_{22}r_{33}}$$

Proof:

$p_{000}, p_{020}, p_{030}, p_{110}, p_{101}, \mathrm{p}_{130}, p_{121}, p_{131}, p_{211}, p_{221}$ and p_{231} follows from the equations (2), (3), (8) and (9).

Remark.

The matrix R, is obtained using the computational procedure

$R(0) = 0$ (10)

$R(n+1) = -A_0A_1^{-1} - [R(n)]^2A_2A_1^{-1} \quad for \quad n \geq 0$ (11)

Some Performance Measures

The following performance measures are calculated using R:

1. Expected number of customers in the system $L_1 = \sum_{n=0}^{n} np_n = p_1 + 2p_2e + p_2R(I-R)^{-2}e$
2. Second moment of system length $L_2 = \sum_{n=0}^{n} n^2p_n$
3. Variance of system length $L_3 = L_2 - L_1^2$

Numerical Results

In this section, some examples are given to show the effect of the parameters $\lambda, \theta_1, \theta_2, \mu_1, \mu_2, \mu_3, \beta_1, \beta_2, \alpha_1, \alpha_2$ on the performance measures expected number of customers in the system, $E(L^2)$ and variance of L for the model analyzed in this paper. The corresponding results are presented as case(1), case(2) and case(3).

Case 1:

For $\lambda = 0.6$, $\theta_1 = 0.5$, $\theta_2 = 0.7$, $\mu_1 = 1.7$, $\mu_2 = 0.8$, $\mu_3 = 0.9$, $\beta_1 = 3.5$, $\beta_2 = 2.5$, $\alpha_1 = 2.5$, $\alpha_2 = 1.5$ the R matrix is (using equations (10) and (11))

$$R = \begin{bmatrix} 0.167282 & 0.095918 & 0.061801 \\ 0.068506 & 0.105725 & 0.025974 \\ 0.071828 & 0.041892 & 0.121715 \end{bmatrix}$$

and the invariant probability vectors are $P = (p_0, p_1, p_2, p_3, \ldots\ldots\ldots)$ where

p_0 = (0.574285567, 0.008415922, 0.009013617)

p_1 = (0.217060313, 0.075073436, 0.043131597, 0.039150875, 0.000328742)

p_2 = (0.012915956, 0.010246227, 0.001358801)

p_3 =(0.002960129, 0.002379081, 0.001229737)

p_4 =(0.000746486, 0.000586975, 0.000394410)

p_5 =(0.000193414, 0.000150182, 0.000109385)

p_6 =(0.000050500, 0.000039012, 0.000029168)

p_7 =(0.000013215, 0.000010190, 0.000007684)

p_8 =(0.000003461, 0.000002667, 0.000002017)

p_9 =(0.000000906, 0.000000698, 0.000000529)

p_{10} =(0.000000237, 0.000000183, 0.000000139)

p_{11} =(0.000000062, 0.000000048, 0.000000036)

p_{12} =(0.000000016, 0.000000013, 0.000000010)

p_{13} =(0.000000004, 0.000000003, 0.000000002)

p_{14} =(0.000000001, 0.000000001, 0.000000001)

p_{15} =(0.000000000, 0.000000000, 0.000000000)

p_{16} =(0.000000000, 0.000000000, 0.000000000)

The sum of the probabilities is 0.999891639.

The performance measures are

(i) Expected number of customers in the system $L_1 = 0.453689811$

(ii) Second moment of system length $L_2 = 0.577465677$

(iii) Variance of system length $L_3 = 0.371631232$

Case 2:

If $\lambda = 0.7$, $\theta_1 = 0.5$, $\theta_2 = 0.7$, $\mu_1 = 1.7$, $\mu_2 = 0.8$, $\mu_3 = 0.9$, $\beta_1 = 3.5$, $\beta_2 = 2.5$, $\alpha_1 = 2.5$, $\alpha_2 = 1.5$ the matrix R is obtained using the equations(10) and (11) as

$$R = \begin{bmatrix} 0.195033 & 0.111566 & 0.072450 \\ 0.079950 & 0.122633 & 0.030598 \\ 0.084147 & 0.049085 & 0.141391 \end{bmatrix}$$

and the invariant probability vector are $P = (p_0, p_1, p_2, p_3, \ldots\ldots\ldots)$ where

p_0 = (0.531730831, 0.008782281, 0.009255922)

p_1 = (0.237057537, 0.080460444, 0.046106972, 0.041709468, 0.000409222)

p_2 = (0.016095437, 0.012715642, 0.001723281)

p_3 =(0.004300775, 0.003439647, 0.001798843)

p_4 =(0.001265163, 0.000989931, 0.000671177)
p_5 =(0.000382372, 0.000295492, 0.000216849)
p_6 =(0.000116447, 0.000089541, 0.000067405)
p_7 =(0.000035542, 0.000027281, 0.000020707)
p_8 =(0.000010855, 0.000008327, 0.000006337)
p_9 =(0.000003316, 0.000002543, 0.000001937)
p_{10} =(0.000001013, 0.000000777, 0.000000592)
p_{11} =(0.000000310, 0.000000237, 0.000000181)
p_{12} =(0.000000095, 0.000000073, 0.000000055)
p_{13} =(0.000000029, 0.000000022, 0.000000017)
p_{14} =(0.000000009, 0.000000007, 0.000000005)
p_{15} =(0.000000003, 0.000000002, 0.000000002)
p_{16} =(0.000000001, 0.000000001, 0.000000000)
p_{17} =(0.000000000, 0.000000000, 0.000000000)

The sum of the steady state probability is found to be 0.999800980.

The performance measures are

(i) Expected number of customers in the system $L_1 = 0.514132011$

(ii) Second moment of system length $L_2 = 0.699360639$

(iii) Variance of system length $L_3 = 0.435028914$

Case 3:

If $\lambda = 0.6$, $\theta_1 = 0.5$, $\theta_2 = 0.7$, $\mu_1 = 1.7$, $\mu_2 = 0.8$, $\mu_3 = 0.9$, $\beta_1 = 3.5$, $\beta_2 = 2.5$, $\alpha_1 = 2.5$, $\alpha_2 = 1.5$ the matrix R is obtained using the equations(10) and (11) as

$$R = \begin{bmatrix} 0.222749 & 0.127116 & 0.083196 \\ 0.091402 & 0.139346 & 0.035305 \\ 0.096561 & 0.056330 & 0.160902 \end{bmatrix}$$

and the invariant probability vector are $P = (p_0, p_1, p_2, p_3, \ldots\ldots)$ where

p_0 = (0.491077751, 0.008941408, 0.009329740)
p_1 = (0.253775328, 0.084621854, 0.048355598, 0.043564871, 0.000476239)
p_2 = (0.019245517, 0.015147624, 0.002042538)
p_3 =(0.005868681, 0.004672226, 0.002464587)
p_4 =(0.001972280, 0.001535889, 0.001049761)
p_5 =(0.000681074, 0.000523861, 0.000387219)
p_6 =(0.000236981, 0.000181385, 0.000137462)
p_7 =(0.000082640, 0.000063143, 0.000048238)
p_8 =(0.000028837, 0.000022021, 0.000016866)
p_9 =(0.000010065, 0.000007684, 0.000005890)
p_{10} =(0.000003513, 0.000002682, 0.000002056)
p_{11} =(0.000001226, 0.000000936, 0.000000718)

p_{12} =(0.000000428, 0.000000327, 0.000000251)
p_{13} =(0.000000149, 0.000000114, 0.000000087)
p_{14} =(0.000000052, 0.000000040, 0.000000031)
p_{15} =(0.000000018, 0.000000014, 0.000000011)
p_{16} =(0.000000006, 0.000000005, 0.000000004)
p_{17} =(0.000000002, 0.000000002, 0.000000001)
p_{18} =(0.000000001, 0.000000001, 0.000000000)
p_{19} =(0.000000000, 0.000000000, 0.000000000)

The sum of the steady state probability is found to be 0.996587932.

The performance measures are

(i) Expected number of customers in the system $L_1 = 0.574404388$

(ii) Second moment of system length $L_2 = 0.842907814$

(iii) Variance of system length $L_3 = 0.512967413$

Conclusion

In this paper, we have successfully designed a Markovian queue with two hetrogeneous servers. In this model the server undergoes two different breakdowns and the arriving customer has impatient behaviour. A study state analysis has been carried out for this model and we derive some performance measures. Numerical results have been given to demonstrate the numerical tractability of the model.

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