4-Square Sum E-Cordial Labeling for Some Graphs

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Abstract— Let G(V,E) be a simple graph and let $f: E(G) \to \{1,2,3,4\}$ be a mapping with the induced labeling $f^*: V(G) \to \{0,1\}$ defined by $f^*(u) = \sum\{(f(uv))^2/uv \in E(G)\}(mod\ 2)$ then f is called a 4-square sum E-cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ where $v_f(0)$ and $v_f(1)$ is the number of vertices labeled with 0 and labeled with 1; $e_f(i)$ and $e_f(j)$ is the number of edges labeled with i and labeled with j respectively. A graph which admits 4-square sum E-cordial labeling is called 4-square sum E-cordial graph.

Keywords: - Path graph, Bistar Graph, Square of Bistar Graph, Fan Graph, Semi Total Point Graph, E-cordial labeling, Square Sum E-cordial labeling.

1. Introduction

We begin with finite, connected and undirected graph without loops and multiple edges. Throughout this paper, $|V(G)|$ and $|E(G)|$ respectively denote the number of vertices and edges in G. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices or edges then the labeling is called a vertex labeling or an edge labeling. A mapping f is called binary vertex labeling of G and $f(u)$ is called the label of vertex of G under f. For an edge $e = uv,$ the induced edge labeling $f^*: E(G) \to \{0,1\}$ given by $f^*(e = vu) = |f(u) - f(v)|$ then $v_f(i)$ is the number of vertices of G having label 1 under f and $e_f(i)$ is the number of edges of G having label 1 under f and f^* for $i = 0,1$. A binary vertex labeling of a graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called cordial if it admits cordial labeling. Let G be a graph with vertex set V(G) and edge set $E(G)$ and $f: E(G) \to \{0,1\}$. Define f^* on V(G) by $f^*(v) = \sum\{f(uv)/uv \in E(G)\}$(mod2). The function f is called an E-cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called E-cordial if it admits E-cordial labeling. The concept of E-cordial labeling was introduced by Yilmaz and Cahit in 1997.

Let $G(V,E)$ be a simple graph and let $f: E(G) \to \{1,2,3,4\}$ be a mapping with the induced labeling $f^*: V(G) \to \{0,1\}$ defined by $f^*(u) = \sum\{(f(uv))^2/uv \in E(G)\}(mod\ 2)$ then f is called a 4-square sum E-cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ where $v_f(0)$ and $v_f(1)$ is the number of vertices labeled with 0 and labeled with 1; $e_f(i)$ and $e_f(j)$ is the number of edges labeled with i and labeled with j respectively.

Theorem 1.1. Path graph P_n is 4-square sum E-cordial graph.
Proof: Let $V(P_n) = \{u_1, u_2, \dots, u_n\}$ and

$$E(P_n) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_{n-1}u_n\}.$$

Define the labeling function $f: E(G) \to \{1,2,3,4\}$ as follows :

$$f(u_iu_{i+1}) = \begin{cases} 1 \;\; if\ i \equiv 1\ (mod\ 4) \\ 2 \;\; if\ i \equiv 3\ (mod\ 4) \\ 3 \;\; if\ i \equiv 2\ (mod\ 4) \\ 4 \;\; if\ i \equiv 0 (mod\ 4) \end{cases}$$

Define f^*on $V(G)$ by $f^*(u) = \sum\{(f(uv))^2\ /uv \in E(G)\}\ (mod\ 2)$.
The labeling pattern defined above covers all possible arrangement of edges.
The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.
Therefore path P_n admits 4-square sum E-cordial labeling.
Hence, path P_n is a 4-square sum E-cordial graph.

Example 1.2

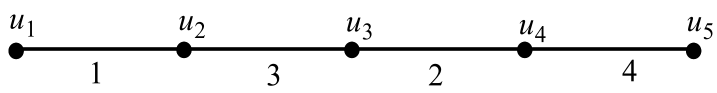

Figure 1: 4-square sum E-cordial labeling of P_5

In the above graph, $f^*(u_1) = 1, f^*(u_2) = 0, f^*(u_3) = 1, f^*(u_4) = 0, f^*(u_5) = 0$
Here, $v_f(0) = 3, v_f(1) = 2$ this implies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.
Hence, P_5 is a 4-square sum E-cordial graph.
Theorem 1.3. Bistar $B_{n,n}$ is 4-square sum E-cordial graph if n is odd
Proof: Let $V(B_{n,n}) = \{u, v, u_i, v_i, 1 \leq i \leq n\}$ and

$$E\left(B_{n,n}\right) = \{uv, uu_i, vv_i, 1 \leq i \leq n\}.$$

Define the labeling function $f: E(G) \to \{1,2,3,4\}$ as follows:

$$f(uv) = 3$$

$$f(uu_i) = \begin{cases} 1\ if\ i \equiv 1,3\ (mod\ 4) \\ \\ 3\ if\ i \equiv 2,0\ (mod\ 4) \end{cases}$$

$$f(vv_i) = \begin{cases} 2\ if\ i \equiv 1,3\ (mod\ 4) \\ \\ 4\ if\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \sum\{(f(uv))^2\ /uv \in E(G)\}(mod\ 2)$.
The labeling pattern defined above covers all possible arrangement of edges. The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.
Therefore, Bistar $B_{n,n}$ admits 4-square sum E-cordial labeling.
Hence, Bistar $B_{n,n}$ is a 4-square sum E-cordial graph.

Theorem 1.4. Square of Bistar $B_{n,n}^2$ is 4-square sum E-cordial graph if n is odd.

Proof: Consider Bistar $B_{n,n}^2$ with vertices $\{u, v, u_i, v_i, i = 1,2, \dots, n\}$ where u_i, v_i are pendent vertices, u and v are connected, u_i, v_i are adjacent to u and v respectively.

Then $|V(B_{n,n}^2)| = 2n + 2$ and $|E(B_{n,n}^2)| = 4n + 1$.

Define the labeling function $f: E(G) \to \{1,2,3,4\}$ as follows :

For $1 \leq i \leq n$,

$f(uv) = 3$

$$f(uu_i) = \begin{cases} 1 \ if \ i \equiv 1,3 \ (mod\ 4) \\ 2 \ if \ i \equiv 2,0 \ (mod\ 4) \end{cases}$$

$$f(vv_i) = \begin{cases} 3 \ if \ i \equiv 1,3 \ (mod\ 4) \\ 4 \ if \ i \equiv 0,2 \ (mod\ 4) \end{cases}$$

$$f(vu_i) = \begin{cases} 2 \ if \ i \equiv 1,3 \ (mod\ 4) \\ 1 \ if \ i \equiv 2,0 \ (mod\ 4) \end{cases}$$

$$f(uv_i) = \begin{cases} 3 \ if \ i \equiv 1,3 \ (mod\ 4) \\ 4 \ if \ i \equiv 0,2 \ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \sum\{(f(uv))^2 / uv \in E(G)\}(mod\ 2)$.

The labeling pattern defined above covers all possible arrangement of edges. The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.

Therefore, square of Bistar $B_{n,n}^2$ admits 4-square sum E-cordial labeling.

Hence, square of Bistar $B_{n,n}^2$ is a 4-square sum E-cordial graph.

Theorem 1.5. Fan graph $F_{1,n}$ is 4-square sum E-cordial graph if $n \equiv 0,2,3(mod\ 4)$

Proof: Let $V(F_{1,n}) = \{u, u_1, u_2, u_3, \dots, u_n\}$ and

$$E(F_{1,n}) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_{n-1}u_n, uu_1, uu_2, uu_3, \dots, uu_n\}.$$

Here u is the apex vertex and $u_1, u_2, u_3, \dots, u_n$ be the vertices of path.

Define the labeling function $f: E(G) \to \{1,2,3,4\}$ as follows:

$$f(uu_i) = \begin{cases} 1 \ if \ i \equiv 1,3 \ (mod\ 4) \\ 2 \ if \ i \equiv 0,2 \ (mod\ 4) \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 3 \ if \ i \equiv 1,3 \ (mod\ 4) \\ 4 \ if \ i \equiv 0,2 \ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \sum\{(f(uv))^2 / uv \in E(G)\}(mod\ 2)$

The labeling pattern defined above covers all possible arrangement of edges. The labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.

Therefore, Fan graph $F_{1,n}$ admits 4-square sum E-cordial labeling.

Hence, Fan graph $F_{1,n}$ is a 4-square sum E-cordial graph.

Theorem 1.6. Semi total point graph of path $T_2(P_n)$ is 4-square sum E-cordial graph when n is odd.

Proof : Let the path P_n has the vertices $v_1, v_2, v_3, \dots, v_n$ and the edges $e_1, e_2, e_3, \dots, e_{n-1}$.

Construct $T_2(P_n)$ from path P_n. Join v_i and v_{i+1} to a new vertex w_i by edges $e'_{2i-1} = v_i w_i$ and $e'_{2i} = v_{i+1} w_i$, for $i = 1,2,3,\ldots,n-1$.
Now $|V[T_2(P_n)]| = 2n - 1$ and $|E[T_2(P_n)]| = 3n - 3$
Define the labeling function $f: E(G) \to \{1,2,3,4\}$ as follows:
For $n \equiv 1,3\ (mod\ 4)$,

$$f(u_i u_{i+1}) = \begin{cases} 1\ if\ i \equiv 1 (mod\ 4) \\ 3\ if\ i \equiv 2\ (mod\ 4) \\ 2\ if\ i \equiv 3\ (mod\ 4) \\ 4\ if\ i \equiv 0\ (mod\ 4) \end{cases}$$

$$f(u_i v_i) = \begin{cases} 1\ if\ i \equiv 1,3 (mod\ 4) \\ 3\ if\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(u_{i+1} v_i) = \begin{cases} 2\ if\ i \equiv 1,3 (mod\ 4) \\ 4\ if\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \sum\left\{\left(f(uv)\right)^2 / uv \in E(G)\right\}(mod\ 2)$.
The labeling pattern defined above covers all possible arrangement of edges.
In each case the labeling defined as above satisfies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$.
Therefore, $T_2(P_n)$ admits 4-square sum E-cordial labeling.
Hence, $T_2(P_n)$ is a 4-square sum E-cordial graph if n is odd.

Example 1.7

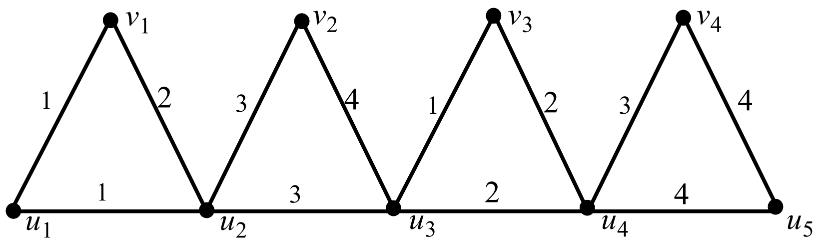

Figure 2: 4-square sum E-cordial labeling of $T_2(P_5)$

In the above graph, $f^*(u_1) = 0, f^*(u_2) = 1, f^*(u_3) = 0, f^*(u_4) = 1$, $f^*(u_5) = 0, f^*(v_1) = 1, f^*(v_2) = 1, f^*(v_3) = 1, f^*(v_4) = 1$.
Hence $v_f(0) = 5, v_f(1) = 4$ this implies $\left|v_f(0) - v_f(1)\right| \leq 1$and $|e_f(i) - e_f(j)| \leq 1$.
Hence $T_2(P_5)$ is a 4-square sum E-cordial graph.
Theorem 1.8. Ladder graph L_n is 4-square sum E-cordial graph when n is even.
Proof: Let $V(L_n) = \{u_1, u_2, u_3, \ldots, u_n, v_1, v_2, v_3, \ldots, v_n\}$ and

$$E(L_n) = \{u_1u_2, u_2u_3, u_3u_4, \ldots, u_{n-1}u_n, v_1v_2, v_2v_3, v_3v_4, \ldots, v_{n-1}v_n, u_1v_1, u_2v_2, u_3v_3, \ldots, u_nv_n\}$$

Define the labeling function $f: E(G) \to \{1,2,3,4\}$ by

$$f(u_i u_{i+1}) = \begin{cases} 3 \text{ if } i \equiv 1,3 (mod\ 4) \\ 4 \text{ if } i \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(v_i v_{i+1}) = \begin{cases} 1 \text{ if } i \equiv 1,3 (mod\ 4) \\ 2 \text{ if } i \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(u_i v_i) = \begin{cases} 1 \text{ if } i \equiv 1 (mod\ 4) \\ 2 \text{ if } i \equiv 2\ (mod\ 4) \\ 3 \text{ if } i \equiv 3\ (mod\ 4) \\ 4 \text{ if } i \equiv 0\ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \sum\left\{\left(f(uv)\right)^2 / \, uv \in E(G)\right\}(mod\ 2)$.

The labeling pattern defined above covers all possible arrangement of edges .

In each case the labeling defined as above satisfies $|v_f(0) - v_f(1)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$.

Therefore, L_n admits 4-square sum E-cordial labeling .

Hence, L_n is a 4-square sum E-cordial graph if n is even.

Theorem 1.9. Square of a path P_n^2 is a 4-square sum E-cordial graph.

Proof: Let $V(P_n^2) = \{u_1, u_2, u_3, \dots, u_n\}$ and

$$E(P_n^2) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_{n-1}u_n, u_1u_3, u_2u_4, u_3u_5, \dots, u_{n-2}u_n\}.$$

Define the labeling function $f: E(G) \to \{1,2,3,4\}$ as follows :

$$f(u_i u_{i+1}) = \begin{cases} 1 \text{ if } i \equiv 1\ (mod\ 4) \\ 2 \text{ if } i \equiv 2\ (mod\ 4) \\ 3 \text{ if } i \equiv 3\ (mod\ 4) \\ 4 \text{ if } i \equiv 0\ (mod\ 4) \end{cases}$$

$$f(u_i u_{i+2}) = \begin{cases} 1 \text{ if } i \equiv 1\ (mod\ 4) \\ 3 \text{ if } i \equiv 2\ (mod\ 4) \\ 2 \text{ if } i \equiv 3\ (mod\ 4) \\ 4 \text{ if } i \equiv 0\ (mod\ 4) \end{cases}$$

Define f^* on $V(G)$ by $f^*(u) = \sum\{(f(uv))^2 \, / uv \in E(G)\}(mod\ 2)$

The labeling pattern defined above covers all possible arrangement of edges. The labeling defined as above satisfies $|v_f(0) - v_f(1)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$.

Therefore, square of a path P_n^2 admits 4-square sum E-cordial labeling.

Hence, square of a path P_n^2 is a 4- square sum E-cordial graph.

Example 1.10.

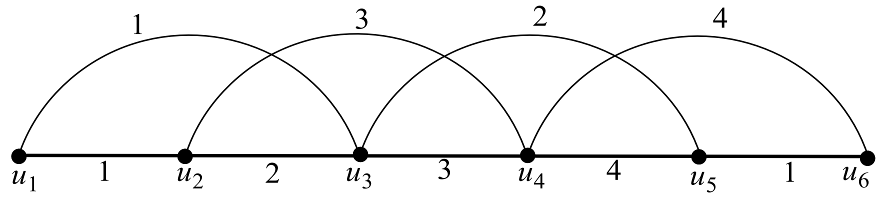

Figure 3: 4-square sum E - Cordial labeling of P_6^2

In the above graph, $f^*(u_1) = 0, f^*(u_2) = 1, f^*(u_3) = 0,$
$f^*(u_4) = 0, f^*(u_5) = 1, f^*(u_6) = 1.$
Hence $v_f(0) = 3, v_f(1) = 3$ this implies $|v_f(0) - v_f(1)| \leq 1$and $|e_f(i) - e_f(j)| \leq 1$.
Hence P_6^2 is a 4-square sum E-cordial graph.

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