Modeling of Communication Network with Queuing Theory under Fuzzy Environment

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Abstract
In the present study, quantitative model and methods that analyze dynamic system of communication flow have been developed in the domain of queuing theory under uncertain environment. The study approach supports model-based queue design as opposed to creative engineering. Some critical aspects and results of queue model and fuzzy set theory has been reviewed in brief and the application of bulk queue model to communication network under imprecise data has been discussed. We have assumed that the arrival pattern, service pattern as well covariance between incoming and transmitted packets all are fuzzy in nature. The system characteristics with defuzzification process have been explored on the basis of lower and upper bound at possibility level alpha and signed distance method. The validity of the results has been analyzed through numerical illustration and graphical study.

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Introduction

Bulk Queue Models are one of major application area for telecommunication system, computer networking, radio/satellite communication, manufacturing concern, inventory control, functioning of tele-traffic engineering of an automatic call distributor (operational management of airlines, ticket reservation, booking and confirmation, in and out time of flights, billing management and various other activities) (Sharma et al. 2010, 2011; Mittal & Sharma 2020) studied optimization of queuing models of service station, toll plaza, traffic and hospital scenario. If the operation in a manufacturing does not begin unless certain raw materials are accumulated during idle interval, this type of study is made through bulk input queue model which is a relevant tool for analysis of performance measure. In the present scenario the discovery of satellite and internet has turned the era as the era of, “Information and Communication Technology”. Hence, there is an increasing need to address and explain some fundamental tele-traffic issues in today’s and forthcoming communication system where the current trend is a change- over towards heterogeneous network types and Quality of Service (QoS) Provision is not solved up to the desired level. The network traffic observed in modern communication system, office automation has drastically influenced and is on changing continuously, due to the newly coming inventions like fax followed by internet, social networking through WhatsApp, facebook etc. and other features. The basic idea in office automation is to move information or ideas to people and not vice versa. This means that the secretaries, executives or assistants do not physically move, drive or fly but communicate their informations to each other with a variety of telecommunication system which involve voice facsimile, graphics, video conferences etc. Machines in an extended data processor environment communicate with one another over conventional shared telephone service. A digital signal processor (DSP), which is a communication device, is responsible for translating the electrical signal wave to correct analogue signal at one end of the connection and back to electronic circuits towards the other end of the connection. As a result, more efficient congestion management techniques are needed to solve the congestion crisis. The performance analysis of any system is done because of two reasons:

(1) One is to improve productivity i.e. more capacity of work is done in the same amount of time and cost.
(2) Or second adding functionality i.e. to introduce some new functions in place of old one that can offer the potential for new gains or new revenue.

Traffic delays in a telecommunications system can be described as a condition of reduced network efficiency. If a network sends signals at a faster rate than the incoming signals receiving capacity, it will form a long queue. A dynamic system would be required to handle the queue while maintain broadcast control protocols in determining either the device output is adequate or not. The mechanism begins dropping packets earlier and notify stages of congestion which is based on implicit feedback signal to reduce the congestion window.

Mathematical analysis of modern communication system and voice calls can be better explained on the basis of queueing theory as we find close similarity in the functioning of both queue networks and communication system. The messages arrive for execution to a transmitter which can be considered as customers or jobs. The transmitter buffer (the region which temporally hold data) has the infinite capacity which can be supposed as capacity of queue network to form long queues and all the respective activities in spreading of the message can be supposed as services. Each transmission consist of several parcel that deliver information. A network established using computers can accepts the information parcels from multiple lines and resend the signals using the same communication lines. Receiving packets are of two kinds such as data type and control type. To allow the proper reception of data packets, control packets are required when setting up a connection between the sender and the recipient pair. Data packets are comprised of segments of a message stream sent in between receiver and transmitter. The queue length numbers obey the cumulative Poisson theorem, and the module category average response cycles form a series of statistically independent increasing probability distributions. The sender performs packages in the sequence of appearance, which is known as queue discipline.

The buffer was considered as a storage devices component used to store data periodically when it is transferred from source to target. Buffer is often used where the receipt of the connection speeds and the transaction processing speed vary even if both factors are different (e.g. online video streaming). Also Buffer changes the duration by using a queue equation in storage to at once type and send information at a different speed in queue.

Due to unpredictable nature in demand of transmission lines conjunction generally occurs in communication networking. The analysis of modern communication system through queue models can be better explained. These models define a significant character in the simulation of phone calls. The essence of traffic has changed drastically since the invention of fax machines and the internet. As a result, the parcel of transferred networks have increased in popularity over circuit swapping networks. The transmission links are exchanged on an as-needed basis in packet switching. Each message is handled by the transmitter, propagated to the recipient through the network, handled by the processor and returned. A recognition promotes across to the transmitter which takes little time to evaluate. Although the recipient can reserve only maximum packet size, the transmission is closed down before space allows in the recipient if its recipient is complete i.e. each packet is sent as soon as the required connection becomes available, but a source holds no transmission facilities when it has nothing to send. As a result, connection utilisation is increased at the cost of model storage and control complexity. Using device control switches, packet switching separates data massages into tiny packets of information and transmits them via communication networks to their indented destination. The message average response times are an arbitrary probability distribution with an independent and identically distributed acceptable speed. Flow management is used to ensure that texts are not missed because the recipient does not have a memory or buffer. This can happen for a number of reasons, whether for the speed of two entities, or for receiver investments other important work relative to packet communication.

Network resources are managed by mathematical multiplexing or hierarchical processor speed in all packet node communication networks, which effectively splits a communications network into a random number of virtual bit rates or data streams. The use of statistical multiplexing in a communication system will minimise packet switching delay. Researchers have been researching communication networks and data traffic processes in the sense of queuing research in recent years. The majority of researchers in this area looked at the communication network as a series of interconnected queues, assuming that arrival and service patterns are independent. The credit goes to Jenq (1984), Hosida (1993), Sharma et al. (2012), Srinivasa Rao et al. (2000) who worked in this direction.

Few attempts have been made to build and analyse interdependent communication networks, which are most commonly used in packet radio systems, data or voice transmission, teleprocessing, tele-traffic engineering, airlines, hotel and motel management etc. In the seminal paper by Leland et al. (1993) showed that network traffic in many cases has properties characterized by long range dependency and variability at a wide range of time scale. These dependencies can have a significant impact on system efficiency and must be included in any practical review.

Srinivasa Rao et al. (2003, 2006) created a network of interdependent communication nodes based on the assumption that the arrival and transmission processes at each node are associated and follow a bi-variate Poisson method. Packet arrivals are believed to be single, meaning that each packet arrives on its own. Messages are separated into small packets of varying lengths in packet switching. As a result, message packets arrive in bulk at the buffer. Singh (2011), calculated the various queue characteristics in buffer using Chapman Kolmogrov equations. Singh et al. (2012, 2018) explored an interdependence fuzzy queue model of performance measures in communication system with voice packetize statistical multiplexing. This paper is extended and wider study of earlier authors.

Congestion arises in electronic data networking due to the erratic and volatile existence of demand at transmission lines. As a result, the texts, as well as the activities involved in the transmission of massages, such as service patterns, have been believed to be ambiguous. The correlation between the composite arrival and transmission completion have also been believed to be fuzzy. The main objective in this paper is to give a mathematical treatment of related topic and present some general results that are important for performance measure characterizing rate processes. The usual way of characterizing dependencies in stochastic processes is to introduce the second order statistics i.e. co-variance or autocorrelation function and then to fuzzify the model through alpha cut approach or other defuzzified formulae. Our model is more practical than previous authors work. We have made a focus on queue models to describe steady state behavior of system and end to end delay in packet networks which provide insight into some aspects of networking, as congestion, information loss and delay variation for a packet flow. We arrived at our conclusions based on a number of applied mathematical fields such as probability theory, queue modeling, differential difference equations and few results on fuzzy set when the environment is totally uncertain.

Model Discussion With Assumptions And Notations

The jobs of typical arrival were considered in bulk of packets or clumps, as there is either nothing or large number of jobs waiting in a queue for their execution. The every job is being performed in a sequence of arrival i.e. no priority is considered. There is correlation between tasks i.e. arrival of packets and transmission are correlated. Though each task on arrival can generate 'interrupt' of the processor which we have not count in our study. The measure of correlation is called crossvariance while covariance is special type of crossvariance, one that is crossed with itself. It is generated in the model by using a flow control mechanism that drops bits, inducing a dependency between messages arriving and service transmission completion. Following symbols and notations are used :

λ_x= Mean arrival rate of messages containing x packets or mean external rate of port x packets

$\lambda = \sum_x \lambda_x$Composite mean arrival rate of packets of size x or aggregate total network throughput rate

μ = Mean transmission rate

δ_x = Covariance between arriving and transmitted packets

$\delta = \sum_x \delta_x$ Covariance between composite arrival and transmitted packets of size x .

C_x = Probability of a batch of size x packets arriving in buffer = $\frac{\lambda_x}{\lambda}$

L_s = Mean queue length of system (waiting and execution)

Var = Variance (measure of fluctuation) in system

The arriving source of packages is assumed to be a poission operation. Now the whole machine mechanism comes from the overlap of poission processes, with rates $\{\lambda_x$, x = 1,2,3 ----------$\}$ is a multiple of complex poisson process. The queue network diagram can be depicted in figure 1.

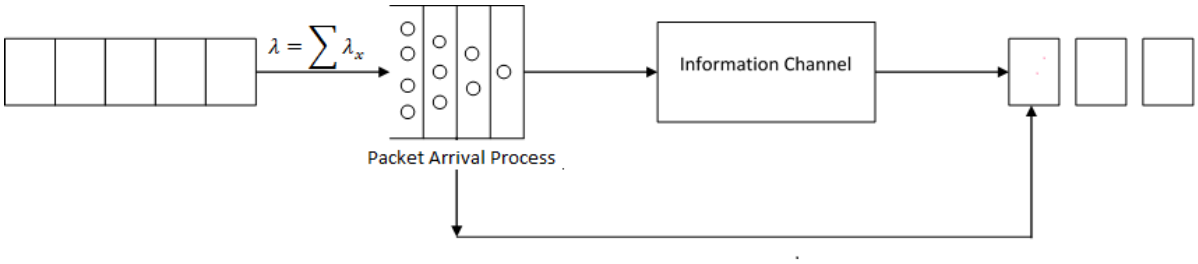

Fig 1: Queue network Bit dropping via flow control

Mathematical Modelling

Define $P_n(t)$ = Probability that a total of n arrivals of packets in the system

Assuming steady state of system (i.e. t→∞) under the proper parametric settings, a set of differential difference equations can be expressed as :

$$0 = -[(\lambda-\delta)+(\mu-\delta)]P_n + (\mu-\delta)P_{n+1} + (\lambda-\delta)\sum_{x=1}^{n} P_{n-k}C_k \text{ for } (n\geq 1) \qquad \ldots 1$$

$$0 = -(\lambda-\delta)P_0 + (\mu-\delta)P_1 \quad \text{for } n = 0 \qquad \ldots 2$$

In general equation 1, the final term arises from the fact that the inclusion of (n-k) within the system, following the introduction of a sample of sizes k, could lead to n number in the system.To solve the above set of equations, proceeding on the basis of Tyagi & Singh (2012) and applying partial generating function (PGF) approach for which we first define

$$P(z) = \sum_{n=0}^{\infty} P_n z^n \quad (|z| \leq 1) \quad \text{and}$$
$$C(z) = \sum_{n=0}^{\infty} C_n z^n \quad (|z| \leq 1)$$

As the g.f of steady state probabilities $\{P_n\}$ and the batch size distribution $\{C_n\}$ respectively.
Multiplying equation (1) by z^n and summing the resultant equations, we find

$$0 = -(\lambda - \delta)\sum_{n=0}^{\infty} P_n z^n - (\mu - \delta)\sum_{n=1}^{\infty} P_n z^n + \frac{(\mu-\delta)}{z}\sum_{n=1}^{\infty} P_n z^n + (\lambda - \delta)\sum_{n=1}^{\infty}\sum_{k=1}^{n} P_{n-k} C_k z^n \quad \ldots 3$$

Where the factor $\sum_{k=1}^{n} P_{n-k} C_k$ is the probability function for the sum of the constant state device size and load size, which is a random variable convolution formulation.
On the basis of property of generating functions, we can show that the g.f. of this sum = product of respective g.f. i.e. mathematically,

$$\sum_{n=1}^{\infty}\sum_{k=1}^{n} P_{n-k} C_k z^n = \sum_{k=1}^{\infty} C_k z^k \sum_{n=k}^{\infty} P_{n-k} z^{n-k} = C(z)\,P(z)$$

Now, equation 2 can be rewritten as

$$0 = -(\lambda - \delta)P(z) - (\mu - \delta)[P(z) - P_0] + \frac{\mu-\delta}{z}[P(z) - P_0] + (\lambda - \delta)\,C(z)\,P(z)$$

On simplification, we get g.f. P(z)

$$P(z) = \frac{(\mu-\delta)(1-z)\,P0}{(\mu-\delta)(1-z)-(\lambda-\delta)z[1-C(z)]} \quad (|z| \leq 1) \quad \ldots 4$$

To find P_0 from P(1) and average number L_s from P'(1), Rewriting (4) i.e. on further simplification,

$$P(z) = \frac{P0}{1-\frac{rz[1-c(z)]}{1-z}} \quad \text{where } r = \frac{(\lambda-\delta)}{(\mu-\delta)}$$
$$= \frac{P0}{1-rz\,\bar{C}(z)} \quad , \ \bar{C}(z) = \frac{1-C(z)}{1-Z}$$

Here, $\bar{C}(z) = \frac{1-C(z)}{1-Z}$ *is the g.f* of complementary batch size probabilities

$$P_r(X > x) = 1 - C_x = \overline{C_x}$$

Since $\frac{1}{1-z}$ is the g.f of 1 and $\frac{C(z)}{1-z}$ is the g.f of commulative probabilities C_x which we can find by noting that

$$\sum_{x=1}^{\infty} C_x Z^x = \sum_{x=1}^{\infty}\left(\sum_{i=1}^{x} C_i\right) z^x = \sum_{i=1}^{\infty}\left(C_i z^i \sum_{x=i}^{\infty} z^{x-i}\right)$$

$= \sum_{i=1}^{\infty} C_i z^i$ (1+z+z^2+----------+∞)

$= (\sum_{i=1}^{\infty} C_i z^i) \frac{1}{1-z}$ [Sum of infinite series of G.P]

After one application of L hospital's rule, we find

$\bar{C}(1) =$ E(X) =Expected value of X and after two applications $\bar{C}\,'(1)$ = E [$\frac{X(X-1)}{2}$]

Using boundary condition that probabilities sum to 1, we have P(1) = 1

∴ 1 = P(1) $= \frac{P_0}{1-r\bar{C}(1)}$ ⇨ $P_0 =$ 1- r $\bar{C}(1)$ = 1- r E(X) = ρ = traffic intensity

And $P^{'}(1) = \frac{r[\bar{C}(1)+C^{-1}(1)]}{1-r\bar{C}(1)}$

Also from equation (2), $P_1 = \frac{(\lambda-\delta)\, P_0}{(\mu-\delta)}$ = r P_0

∵ $\rho < 1$ is the necessary and sufficient condition for stationary state.
Here the batch size be either constant or geometrically distributed. Let us discuss the case when the batch size follows geometrical probability distribution. i.e.

$C_x = (1-\beta)\beta^{x-1}$ $(0 < \beta < 1)$

It follows that $\rho = \frac{(\lambda-\delta)}{(\mu-\delta)} E(X) = rE(X) = \frac{r}{1-\beta}$ $(Here\ E(X) = \frac{1}{1-\beta})$

Now, C(z) = $(1-\beta) \sum_{n=1}^{\infty} \beta^{n-1} z^n$ = (1-β) [z + $\beta z^2 + \beta^2 z^3 + ---\infty$]

∴ C(z) = $\frac{(1-\beta)z}{1-\beta z}$ …5

Using equation 5 in equation 4, we get on simplification,

P(z) = $\frac{(\mu-\delta)(1-\beta z)P_0}{(\mu-\delta)(1-\beta z)-(\lambda-\delta)z}$

Queue characteristics or measure of effectiveness

(a) Applying the condition that P(1) = 1 , we get probability that system is empty or there is no information or packet in the system
P(0) = $1 - \frac{(\lambda-\delta)}{(\mu-\delta)(1-\beta)} = 1 - \rho$, where $\rho = \frac{(\lambda-\delta)}{(\mu-\delta)(1-\beta)}$

(b) The average length of queue in the system is (waiting and execution)
L_s = E (N) = $\sum_{n=O}^{\infty} n\, P_n$, N = no of customer in the system in steady state

$= \frac{\rho}{(1-\beta)(1-\rho)}$ where $\rho = \frac{\lambda-\delta}{(1-\beta)(\mu-\delta)}$

(c) The expected length of queue
$L_q = L_s - (1 - P_0) = L_s - \rho$

(d) Using Little's formulae, $L_s = (\lambda-\delta)W_s$ and $L_q = (\lambda-\delta)W_q$
Expected steady state system waiting time $W_s = \frac{L_s}{\lambda-\delta} = \frac{\rho}{(1-\beta)(1-\rho)(\lambda-\delta)}$
and similar W_q .

(e) The variance of number of packets in the system is
Variance = $\frac{\beta\,\rho(1-\rho)+\rho}{(1-\beta)^2(1-\rho)^2}$

Fuzzification of the model

The instability of the natural world has traditionally been found as a result of confusion as ambiguity. Human brain has some specific features by which it can learn, think, generate ideas and reason in vague environment. No doubt, the probability distribution is an exceptional method to capture random variation in data, at the same time these models can also be used in the sutuations depends on stochastic or random process, where happening of events depends upon chance. But when the uncertainty may arise from the probabilistic behavior of physical phenomenon in mechanical or electronic systems, the partial information about the problem or unreliable getting information or inherent imprecision in the language i.e. vague information receipt of information from one source. The fuzzy uncertainty based on the fuzzy set theory play vital role in such situations. Zadeh (1965) introduced theory of fuzzy operations, set theory is an excellent mathematical way to handle the uncertainty arising due to vaguto deal with the random variables through mathematical means. The theory has wider applicability particular in areas of pattern recognition and information processing. Before modelling the fuzzification, we would like to explain some preliminaries of fuzzy set.

Fuzzy set

The membership of components in a set in traditional fuzzy sets can be represented in binary terms, i.e. the bifunctional state wherein an item whether applies to or is not part of the set, that is, and individual object in the given sequence is assigned a value of 1 or 0 by the characterist or membership function of a given sequence. The Fuzzy Set theory is a category of principles of classical set components from different membership functions.

Mathematically, a fuzzy set $\tilde{A}$ in X is defined by
$\tilde{A} = \{ \left(x, \mu_{\tilde{A}}(x)\right) : x \in X, \mu_{\tilde{A}}(x) \in [0,1] \}$, Where x is a member of the crisp set A$\subseteq$ X, and $\mu_{\tilde{A}}(x)$ is a member of the interval [0,1] known as the membership function or characteristics of fucntion. X is collection of objects or space of objects.

α cut

The 'α cut' of a fuzzy set $\tilde{A}$ is crisp set defined by

$A_\alpha = \{x;\ \mu_{\tilde{A}}(x) \geq \alpha, x \epsilon X\} = \{L_{\tilde{A}}(\alpha), U_{\tilde{A}}(\alpha)\}$
Where $L_{\tilde{A}}(\alpha)$ and $U_{\tilde{A}}(\alpha)$ represents the lower and upper bound of α cut of $\tilde{A}$ respectively.

Support of fuzzy set

The support of $\tilde{A}$ is defined as
Supp (A) = { $x \epsilon X$; $\mu_{\tilde{A}}(x) > 0$ }; X being universal set

Triangular membership functions (MF)

A ‘triangular MF’ is specified by three parameters $\{x_1,x_2,x_3\}$as :-

$$\text{Y = triangle}(x;x_1,x_2,x_3) = \begin{cases} \frac{x-x_1}{x_2-x_1}\ ;\ x_1 \leq x \leq x_2 \\ \frac{x_3-x}{x_3-x_2}\ ;\ x_2 \leq x \leq x_3 \\ 0\ ;\ \text{otherwise} \end{cases}$$

The parameters $\{x_1,x_2,x_3\}$ (with $x_1 < x_2 < x_3$) determine x coordinates of three corners of triangular MF.

Trapezoidal MF

A ‘trapezoidal MF’ is specified by four paramters $\{x_1,x_2,x_3,x_4\}$ as

$$\text{Trapezoid}(x;x_1,x_2,x_3,x_4) = \begin{cases} \frac{x-x_1}{x_2-x_1}\ ;\ x_1 \leq x \leq x_2 \\ 1\ ;\ x_2 \leq x \leq x_3 \\ \frac{x_4-x}{x_4-x_3}\ ;\ x_3 \leq x \leq x_4 \\ 0\ ;\ \text{otherwise} \end{cases}$$

The parameters $\{x_1,x_2,x_3,x_4\}$ (with $x_1 < x_2 < x_3 < x_4$) determine x coordinates of four corners of trapezoidal MF.

Interval Number

An interval number A can be defined as :

$$A = [x_1,x_2] = \{x;\ x_1 \leq x \leq x_2\};\ x_1, x_2, x \in \Re$$

Arithmetic operations on intervals

Consider two intervals A = $[x_1, x_2]$ and B = $[y_1, y_2]$ defined on $\Re$, then
Addition A+B = $[x_1 + y_1, x_2 + y_2]$
Subtraction A−B = $[x_1 - y_2, x_2 - y_1]$
Multiplication A.B = $[\min(x_1y_1, x_1y_2, x_2y_1, x_2y_2), \max(x_1y_1, x_1y_2, x_2y_1, x_2y_2)]$
Division A/B = $[x_1, x_2] : [y_1,y_2] = [x_1, x_2]\cdot[\frac{1}{y_2}, \frac{1}{y_1}]$; $0 \notin [y_1, y_2]$

α Level fuzzy interval

For $0 \leq \alpha \leq 1$, the fuzzy set $[a_\alpha, b_\alpha]$ defined on $\Re$ is called an α level fuzzy interval if the membership function of $[a_\alpha, b_\alpha]$ is given by,

$$\mu_\alpha(x) = \begin{cases} \alpha\ ; & a \le x \le \text{ b} \\ 0\ ; & \text{otherwise} \end{cases}$$

Note : If $\bar{A}$ be a fuzzy set on $\Re$ and $0 \le \alpha \le 1$. The α cut A(α) of $\tilde{A}$ consists of points x such that $\mu_A(x) \ge \alpha$

i.e. A(α) = $[A_L(\alpha), A_U(\alpha)]$

Practically in communication network, the parameters such as arrival of packets, service pattern, covariance etc. can be better described in linguistic variables (as very slow, slow, average, fast etc.) instead considering random variables. Hence, fuzzification of the model will be more relevant compared to stochastic model.

Define, $\bar{\lambda}$ → fuzzy arrival rate,
$\bar{\mu}$ → fuzzy service rate,
$\bar{\delta}$ → fuzzy covariance by the following fuzzy sets mathematically as:
$\bar{\lambda} = \{(x\ ,\ \mu_{\bar{\lambda}}\ (x))\ /\ x \in S(\bar{\lambda})\}$
$\bar{\mu} = \{(y\ ,\ \mu_{\bar{\mu}}\ (y))\ /\ y \in S(\bar{\mu})\}$
$\bar{\delta} = \{(z\ ,\ \mu_{\bar{\delta}}\ (z))\ /\ z \in S(\bar{\delta}\ )\}$

Here, $\mu_{\bar{\lambda}}$ (x), $\mu_{\bar{\mu}}$ (y), $\mu_{\bar{\delta}}$ (z) are characteristic functions of the arrival rate $\bar{\lambda}$, service rate $\bar{\mu}$, covariance $\bar{\delta}$ and $S(\bar{\lambda})$, $S(\bar{\mu})$, $S(\bar{\delta})$ are their supports respectively. Now we can desire α cuts to form the characteristic function and design the α cuts of $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\delta}$ in interval form known formulae

$$\lambda(\alpha) = [x_\alpha^L\ ,\ x_\alpha^U] = [\begin{smallmatrix} min \\ x\epsilon X \end{smallmatrix}\{\ x\ /\ \mu_{\bar{\lambda}}\ (x) \ge \alpha\ \}\ ;\ \begin{smallmatrix} max \\ x\epsilon X \end{smallmatrix}\{\ x\ /\ \mu_{\bar{\lambda}}\ (x) \ge \alpha\ \}]$$
$$\mu(\alpha) = [y_\alpha^L\ ,\ y_\alpha^U] = [\begin{smallmatrix} min \\ y\epsilon Y \end{smallmatrix}\{\ y\ /\ \mu_{\bar{\mu}}\ (y) \ge \alpha\ \}\ ;\ \begin{smallmatrix} max \\ y\epsilon Y \end{smallmatrix}\{\ y\ /\ \mu_{\bar{\mu}}\ (y) \ge \alpha\ \}]$$
$$\delta(\alpha) = [z_\alpha^L\ ,\ z_\alpha^U] = [\begin{smallmatrix} min \\ z\epsilon Z \end{smallmatrix}\{\ z\ /\ \mu_{\bar{\delta}}\ (z) \ge \alpha\ \}\ ;\ \begin{smallmatrix} max \\ z\epsilon Z \end{smallmatrix}\{\ x\ /\ \mu_{\bar{\delta}}\ (z) \ge \alpha\ \}]$$

The above intervals are crisp sets rather fuzzy. We can deduce the membership functions $\mu_{\widetilde{L_s}}$, $\mu_{\widetilde{L_q}}$, $\mu_{\widetilde{Var}}$ in terms of maximum and minimum boundary condition through α cuts . To determine optimal value of instead of determining fuzzy boundaries, the defuzzification is considered in the stochastic model through signed distance method or any appropriate defuzzification formulae. The discussion is made clear through numerical example.

Numerical example with solution

To check the validity of the model, a numerical illustration has been presented where the parameters are taken in triangular and trapezoidal fuzzy numbers. α cut is applied to convert the parametric value in interval form. The α cut helps the membership function to represent the interval only one crisp value since the decision maker prefers only optimal values instead of fuzzy boundary. Hence, to minimize these scenarios, the model can also be defuzzified using signed distance as :
(i) For the triangular fuzzy parameter $\tilde{A}$ = $(x_{1,}x_{2,}x_3)$, the α cut of $\tilde{A}$ is $[A_L(\alpha), A_U(\alpha)]$, $\alpha\ \epsilon$ [0,1] where A_L= $x_1\ +\ (x_2\ -\ x_1)\alpha$ and $A_U(\alpha)$ = $x_3\ -(x_3\ -x_2\ )\alpha$. The signed distance of $\tilde{A}$ to $\widetilde{O_1}$ is d $(\widetilde{A}\ ,\widetilde{O_1})$ = $\frac{1}{4}[x_1\ +\ 2x_2\ +x_3\ ]$ (ii) For trapezoidal fuzzy number $\tilde{A}$ = $(x_{1,}x_{2,}x_3, x_4\ )$, we can apply the graded mean integration presentation given by Chen and Hseilr (1999) by the formula as :

$\rho(\tilde{A}) = \int_0^1 \frac{\frac{h}{2}[(x_1 + x_4)+h(x_2 - x_1 - x_4 + x_3)]\, dh}{\int_0^1 h\, dh} = \frac{x_1+2x_2+2x_3+x_4}{6}$

Consider fuzzy arrival rate of bulk packets in a modern communication system are $\tilde{\lambda}$ = [3,4,5], the fuzzy service rate $\tilde{\mu}$ = [7,8,9] and fuzzy covariance rate $\tilde{\delta}$ = [0.03,0.05,0.07] while geometric distribution parameter $\beta = 0.2$. Our objective is to find queue characteristics i.e

(i) Mean queue length of the system
(ii) Measure of flactuation i.e variance of system
Solution : given $\tilde{\lambda}$ = [3,4,5] ; $\tilde{\mu}$ = [7,8,9] ; $\tilde{\delta}$ = [0.03,0.05,0.07] ; $\beta = 0.2$

Applying arithmetic operations on interval and α cut, we get

$$\tilde{\lambda}(\alpha) = [\lambda_\alpha^L, \lambda_\alpha^U] = [\alpha + 3 ,\ 5 - \alpha]$$
$$\tilde{\mu}(\alpha) = [\mu_\alpha^L, \mu_\alpha^U] = [\alpha + 7 ,\ 9 - \alpha]$$
$$\tilde{\delta}(\alpha) = [\delta_\alpha^L, \delta_\alpha^U] = [0.02\alpha + 0.03 ,\ 0.07 - 0.02\alpha]$$

Now,

$$\tilde{\lambda}(\alpha) - \tilde{\delta}(\alpha) = [(\alpha + 3) - (0.07 - 0.02\alpha) , (5 - \alpha) - (0.02\alpha + 0.03)]$$
$$= [1.02\ \alpha + 2.93 , 4.97 - 1.02\ \alpha]$$
$$\tilde{\mu}(\alpha) - \tilde{\delta}(\alpha) = [(\alpha + 7) - (0.07 - 0.02\alpha) , (9 - \alpha) - (0.02\alpha + 0.03)]$$
$$= [1.02\ \alpha + 6.93 , 8.97 - 1.02\ \alpha]$$
$$\therefore\ \tilde{\rho}(\alpha) = [\rho_\alpha^L, \rho_\alpha^U] = [\frac{\lambda_\alpha^L - \delta_\alpha^L}{\mu_\alpha^L - \delta_\alpha^L} \times \frac{1}{(1-\beta)} , \frac{\lambda_\alpha^U - \delta_\alpha^U}{\mu_\alpha^U - \delta_\alpha^U} \times \frac{1}{(1-\beta)}]$$
$$= [\frac{1.02\ \alpha + 2.93}{8.97 - 1.02\ \alpha} \times \frac{1}{0.8} ,\ \frac{4.97 - 1.02\ \alpha}{1.02\ \alpha + 6.93} \times \frac{1}{0.8}]$$
$$= [\frac{1.02\ \alpha + 2.93}{7.176 - 0.816\ \alpha} ,\ \frac{4.97 - 1.02\ \alpha}{0.816\ \alpha + 5.544}]$$

Mean queue length of system

$$\therefore\ \tilde{L_s} = [(L_s)_\alpha^L, (L_s)_\alpha^U] = [\frac{\rho_\alpha^L}{(1-\beta)} \times \frac{1}{(1-\rho_\alpha^L)} , \frac{\rho_\alpha^U}{(1-\beta)} \times \frac{1}{(1-\rho_\alpha^U)}]$$
$$= [\frac{1.02\ \alpha + 2.93}{0.8(4.246 - 1.836\ \alpha)} ,\ \frac{4.97 - 1.02\ \alpha}{0.8(1.836\ \alpha + 0.574)}]$$
$$= [\frac{1.02\ \alpha + 2.93}{3.3968 - 1.4688\ \alpha} ,\ \frac{4.97 - 1.02\ \alpha}{1.4688\ \alpha + 0.4592}]$$

Now, we take different values of α starting from 0, 0.1, 0.2, 0.3--------,1 etc. The system's mean fuzzy queue length is illustrated in the table below for a triangular fuzzy system .

Table 1: Average queue length for triangular system

α	$(L_s)_\alpha^L$	$(L_s)_\alpha^U$
0	0.8625	10.8231
0.1	1.0035	8.0330
0.2	1.0099	6.3301
0.3	1.0946	5.1833
0.4	1.1882	4.3584

0.5	1.2920	3.7365
0.6	1.4080	3.2512
0.7	1.5384	2.8615
0.8	1.6860	2.5419
0.9	1.8546	2.2749
1.0	2.0487	2.0487

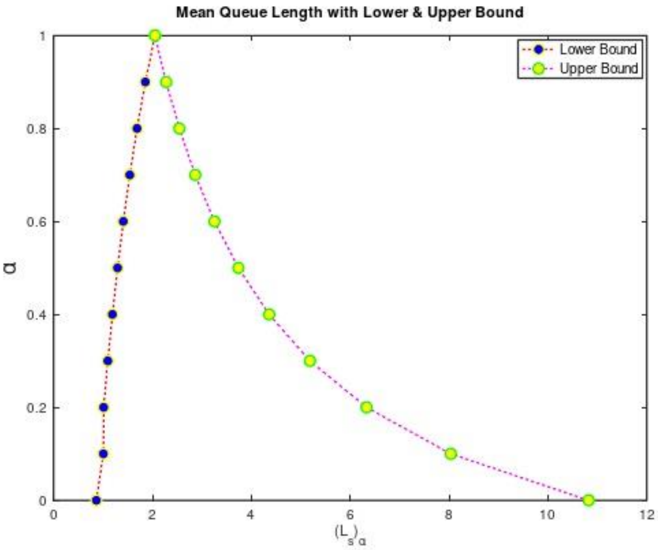

Fig 1: Mean Queue length with lower and upper bound

Similar, we can frame the table for mean waiting line i.e L_q.

Variance

Using α cuts and the arithmetic operations on intervals, the measure of fluctuation i.e variance can be calculated as :

$$\widetilde{\mathrm{Var}} = \mathrm{Var}(\alpha) = [\mathrm{Var}_\alpha^L, \mathrm{Var}_\alpha^U]$$

$$= [\frac{\beta\rho_\alpha^L(1-\rho_\alpha^L)+\rho_\alpha^L}{(1-\beta)^2(1-\rho_\alpha^L)^2}, \frac{\beta\rho_\alpha^U(1-\rho_\alpha^U)+\rho_\alpha^U}{(1-\beta)^2(1-\rho_\alpha^U)^2}]$$

$$= [\ \frac{0.2\times\frac{1.02\,\alpha+2.93}{7.176-0.816\,\alpha}\times\left(1-\frac{1.02\,\alpha+2.93}{7.176-0.816\,\alpha}\right)+\frac{1.02\,\alpha+2.93}{7.176-0.816\,\alpha}}{0.64\times(1-\frac{1.02\,\alpha+2.93}{7.176-0.816\,\alpha})^2},$$

$$\frac{0.2\times\frac{4.97-1.02\,\alpha}{0.816\,\alpha+5.544}\times\left(1-\frac{4.97-1.02\,\alpha}{0.816\,\alpha+5.544}\right)+\frac{4.97-1.02\,\alpha}{0.816\,\alpha+5.544}}{0.64\times(1-\frac{4.97-1.02\,\alpha}{0.816\,\alpha+5.544})^2}\ ]$$

$$= [\ \frac{\frac{0.204\,\alpha+0.586}{7.176-0.816\,\alpha}\left(\frac{4.246-1.836\alpha}{7.176-0.816\,\alpha}\right)+\frac{1.02\,\alpha+2.93}{7.176-0.816\,\alpha}}{0.64\times(1-\frac{1.02\,\alpha+2.93}{7.176-0.816\,\alpha})^2}, \frac{\frac{0.994-0.204\alpha}{0.816\,\alpha+5.544}\left(\frac{0.574+1.836\,\alpha}{0.816\,\alpha+5.544}\right)+\frac{4.97-1.02\,\alpha}{0.816\,\alpha+5.544}}{0.64\times(1-\frac{4.97-1.02\,\alpha}{0.816\,\alpha+5.544})^2}]$$

on solving, we get

$$= [\frac{-1.2068\alpha^2+4.7190\alpha+23.5137}{2.1573\alpha^2-9.9784\alpha+11.5382}, \frac{-1.2068\alpha^2+0.1086\alpha+28.1241}{2.1573\alpha^2+1.3489\alpha+0.2108}]$$

Now, we take different values of α starting from 0, 0.1, 0.2, 0.3--------,1 etc. the measure of fluctuation i.e variance can be depicted in the following table for a triangular fuzzy system .

Table 2: Variance for triangular system

Z	$\mathrm{Var}_{\alpha}^{L}$	$\mathrm{Var}_{\alpha}^{U}$
0	2.0379	133.4160
0.1	2.0735	76.8175
0.2	2.5355	49.5810
0.3	2.8402	34.6485
0.4	3.1941	25.5381
0.5	3.6075	19.5694
0.6	4.0947	15.4476
0.7	4.6744	12.4813
0.8	5.3719	10.2747
0.9	6.2212	8.5884
1.0	7.2706	7.2706

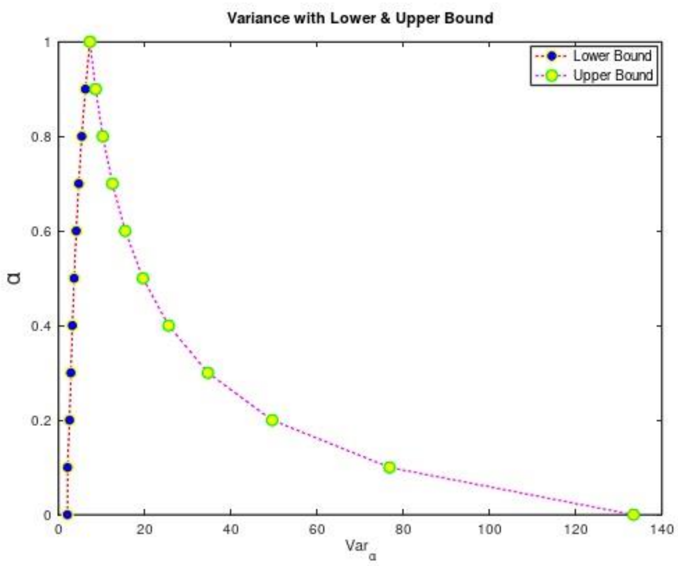

Fig 2: Variance with lower and upper bound

Fuzzified with trapezoidal fuzzy parameters

Taking the parameters in trapezoidal fuzzy numbers represented by $\tilde{\lambda}$ = [4,5,6,7] ; $\tilde{\mu}$ =

$$[10,11,12,13] ; \tilde{\delta} = [0.02,0.03,0.04,0.05] ; \beta = 0.2$$

Our objective is to find L_s and variance.
Then, Applying α cut and arithmetic operations on interval, we get

$$\tilde{\lambda}(\alpha) = [\lambda_{\alpha}^{L}, \lambda_{\alpha}^{U}] = [\alpha + 4, 7 - \alpha]$$

$$\tilde{\mu}(\alpha) = [\mu_\alpha^L, \mu_\alpha^U] = [\alpha + 10,\ 13 - \alpha]$$
$$\tilde{\delta}(\alpha) = [\delta_\alpha^L, \delta_\alpha^U] = [0.01\alpha + 0.02\ ,\ 0.05 - 0.01\alpha]$$

Now,

$$\tilde{\lambda}(\alpha) - \tilde{\delta}(\alpha) = [(\alpha + 4) - (0.05 - 0.01\alpha)\ , (7 - \alpha) - (0.01\alpha + 0.02)]$$
$$= [1.01\ \alpha + 3.95\ ,\ 6.98\ -\ 1.01\ \alpha]$$
$$\tilde{\mu}(\alpha) - \tilde{\delta}(\alpha) = [(\alpha + 10) - (0.05 - 0.01\alpha)\ , (13 - \alpha) - (0.01\alpha + 0.02)]$$
$$= [9.95+1.01\ \alpha\ ,\ 12.98 - 1.01\ \alpha]$$
$$\therefore\ \tilde{\rho}(\alpha) = [\rho_\alpha^L, \rho_\alpha^U] = [\frac{\widetilde{\lambda_\alpha^L} - \widetilde{\delta_\alpha^L}}{\widetilde{\mu_\alpha^L} - \widetilde{\delta_\alpha^L}} \times \frac{1}{(1-\beta)}\ , \frac{\widetilde{\lambda_\alpha^U} - \widetilde{\delta_\alpha^U}}{\widetilde{\mu_\alpha^U} - \widetilde{\delta_\alpha^U}} \times \frac{1}{(1-\beta)}]$$
$$= [\frac{1.01\ \alpha + 3.95}{12.98 - 1.01\ \alpha} \times \frac{1}{0.8}\ ,\ \frac{6.98 - 1.01\ \alpha}{9.95+1.01\ \alpha} \times \frac{1}{0.8}]$$
$$= [\frac{1.01\ \alpha + 3.95}{10.384 - 0.808\ \alpha}, \frac{6.98 - 1.01\ \alpha}{7.960+0.808\alpha}] \qquad \ldots 6$$

Average queue length of system i.e L_s

$\widetilde{L_s}$ is given by,

$$\widetilde{L_s} = L_s(\alpha) = [(L_s)_\alpha^L, (L_s)_\alpha^U] = [\frac{\rho_\alpha^L}{(1-\beta)} \times \frac{1}{(1-\rho_\alpha^L)}\ , \frac{\rho_\alpha^U}{(1-\beta)} \times \frac{1}{(1-\rho_\alpha^U)}]$$

using result from equation 6 and on solving, we get

$$= [\frac{1.01\ \alpha + 3.95}{5.1472 - 1.4544\ \alpha}\ ,\ \frac{6.98 - 1.01\ \alpha}{1.4544\ \alpha + 0.784}] \qquad \ldots 7$$

Putting different values of α in 7, we can depict the values of $(L_s)_\alpha^L$ and $(L_s)_\alpha^U$ in tabular form as :

Table 3: Average queue length for trapezoidal system

α	$(L_s)_\alpha^L$	$(L_s)_\alpha^U$
0	0.7674	8.9030
0.1	0.8099	7.4015
0.2	0.8549	6.3062
0.4	0.9536	4.8151
0.6	1.0658	3.8476
0.8	1.1943	3.1691
1.0	1.3441	2.6670

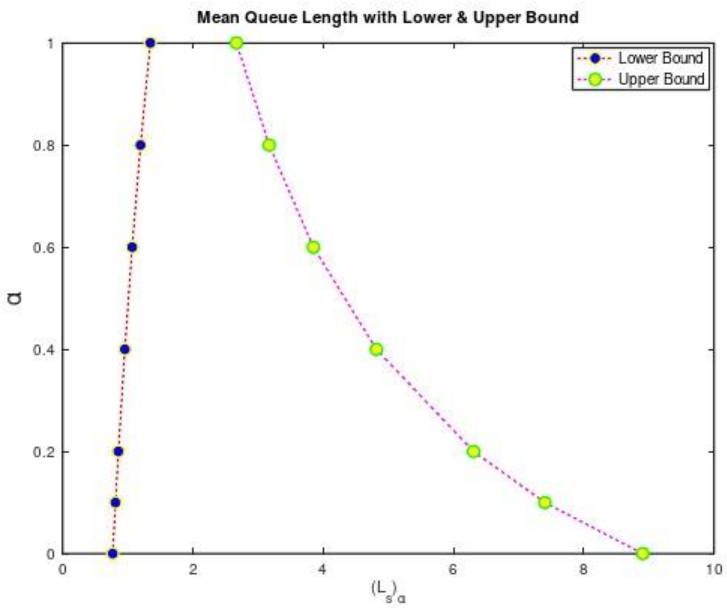

Fig 3: Mean Queue length with lower and upper bound

Variance for trapezoidal fuzzy parameters

$$\widetilde{\mathrm{Var}} = \mathrm{Var}(\alpha) = [\mathrm{Var}_\alpha^L, \mathrm{Var}_\alpha^U]$$
$$= [\frac{\beta\rho_\alpha^L(1-\rho_\alpha^L)+\rho_\alpha^L}{(1-\beta)^2(1-\rho_\alpha^L)^2}, \frac{\beta\rho_\alpha^U(1-\rho_\alpha^U)+\rho_\alpha^U}{(1-\beta)^2(1-\rho_\alpha^U)^2}]$$

Putting the value of β = 0.2 and value of ρ_α^L and ρ_α^U from (6) ,

$$= [\ \frac{0.2 \times \frac{1.01\,\alpha + 3.95}{10.384 - 0.808\,\alpha} \times \left(1 - \frac{1.01\,\alpha + 3.95}{10.384 - 0.808\,\alpha}\right) + \frac{1.01\,\alpha + 3.95}{10.384 - 0.808\,\alpha}}{0.64 \times (1 - \frac{1.01\,\alpha + 3.95}{10.384 - 0.808\,\alpha})^2},$$
$$\frac{0.2 \times \frac{6.98 - 1.01\alpha}{7.960 + 0.808\alpha} \times \left(1 - \frac{6.98 - 1.01\alpha}{7.960 + 0.808\alpha}\right) + \frac{6.98 - 1.01\alpha}{7.960 + 0.808\alpha}}{0.64 \times (1 - \frac{6.98 - 1.01\alpha}{7.960 + 0.808\alpha})^2}\ ]$$

on solving, we get

$$= [\frac{-1.1832\alpha^2+7.1566\alpha+46.0996}{2.1152\alpha^2-14.9721\alpha+26.4936}, \frac{-1.1832\alpha^2-0.0598\alpha+56.9288}{2.1152\alpha^2+2.280\alpha+0.6146}]$$

Putting different values of α starting from α = 0,0.1,0.2,0.4,0.6,0.8,1.0 etc. , we get

Table 4: Variance for trapezoidal system

α	Var_α^L	Var_α^U
0	1.7400	92.6273
0.1	1.8708	65.8919
0.2	2.0134	49.2291
0.4	2.3400	30.4104
0.6	2.7346	20.5783
0.8	3.2179	14.7993
1.0	3.8185	11.1153

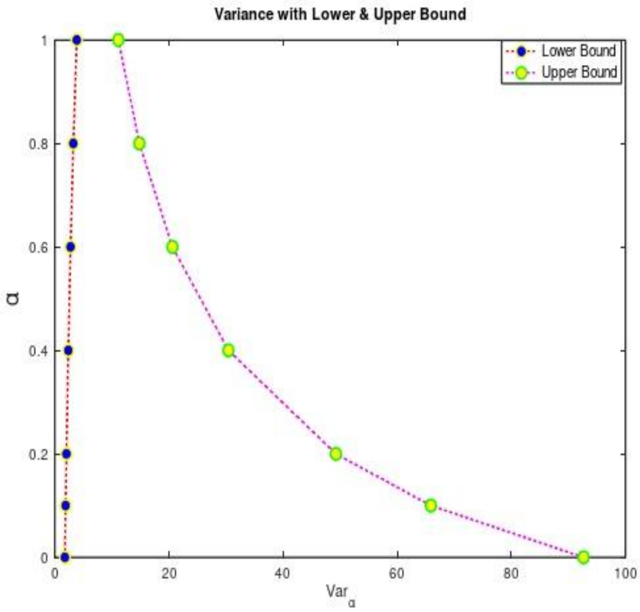

Fig 4: Variance with lower and upper bound

Result & Conclusion

From numerical illustration , it is clear that if the input and output parameters are fuzzy in nature, the queue characteristic in system also represent fuzziness behaviour, demonstrating complete knowledge conservation. It was observed that value of α increases as the lower boundary of queue length increases whereas the upper boundary of the reduces, which shows sensitivity of the models is reduces. The triangular membership function based fuzzy parameters observed that when α =1, the lower and upper boundary of average queue length and variance of the system have almost identical values (as given in table no. 1 & 2) showing least uncertainty and proving the validity of the model. The results have also been represented graphically as in fig. no. 1 & 2. For trapezoidal membership function based fuzzy parameters the value of α cuts give the respective variation of average queue length and variance in the model. From the trapezoidal fuzzy table clear the most acceptable range of average queue length of the system fall within the range of 1.3441 and 2.6670 while the value can't lie outside the range of below the range of 0.7674 and 8.9030. Similar explanation can be made for variance or fluctuation of the system also in which the variance can't lie outside the range of 1.17400 and 92.6273.

If in the proposed model, the input, output parameters and covariance are represented by crisp set rather than fuzzy, the model resemble to some extent along with the studies performed by Singh (2012). The results found from there study is compared with the results of Arti & Singh (2012) but differs in the sense that we have applied α cut technique for defuzzification. The alpha cut approach analyse the system in better way with more clear arguments and logic. The approach is more generalised and wider than the earlier authors work in this direction.

Future Research

In our proposed and generalized model, we have assumed the input / output parameters and covariance rate to be triangular and trapezoidal fuzzy parameters. The approach is not necessarily to be confined only to these parametric values only , it may further can be extended to the case of pentagonal fuzzy parameters or can be extended to the case of fuzzy batch size or variable batch size which requires further research. Moreover we may convert the various queue characteristics into linguistic variables such as small, average, big, very big etc.

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