Irregular Bipolar Fuzzy Labeling Graph

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Abstract
In this paper a new concept of bipolar fuzzy labeling graph is introduced. Irregular, neighbourly irregular and highly irregular graphs are defined, based on the open and closed neighbourhood degrees. And some of the properties of these graphs are discussed.Also the concept of semiregular bipolar fuzzy labeling graph is defined by investigating some of its salient properties.

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1.INTRODUCTION

Zadeh proposed the concept of a fuzzy subset of a set in 1965. Signal processing, multiagent systems, medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, robotics, computer networks, expert systems, artificial intelligence, pattern recognition, decision making, and automata theory have all embraced the theory of fuzzy sets since then.

Zhang proposed the idea of bipolar fuzzy sets as a generalisation of fuzzy sets in 1994. Fuzzy sets with a membership degree range of [-1, 1] are called bipolar fuzzy sets.In 1987[5] Bhattacharyadiscussed some properties of fuzzy graphs.

Akram first established the concept of a bipolar fuzzy graph in 2011 [1] and defined various operations on it. The applicability of the bipolar fuzzy graph has now been extended to all fields.

The irregular bipolar fuzzy graph was introduced by Samanta and pal [7]. [4] Basheer Ahamed et al. proposed the notion of closed neighbourhood degree and its extension in fuzzy graphs. Thangaraj et al [8,9,10] discussed some concepts of fuzzy network and fuzzy critical path.

In crisp graph, a bijection $:V \cup E \rightarrow N$ that assigns to each vertex and/or edge if $G = (V, E)$, a unique natural number is called a labeling. In 2012 [6]Gani et al., introduced a new concept of fuzzy labeling on "properties of fuzzy labeling graph".

In this paper a new concept of bipolar fuzzy labeling graph is defined. We also came up with terms like irregular bipolar fuzzy labeling graph, highly irregular bipolar fuzzy labeling graph, completely irregular bipolar fuzzy labeling graph, and semi regular bipolar fuzzy labeling graph. Some of its properties were discussed. The open neighbourhood degree of a vertex v is denoted by deg(v), while the closed neighbourhood degree is denoted by deg[v] throughout this work.

2.PRELIMINARIES

Definition 2.1[1]:

A bipolar fuzzy graph with an underlying set V is defined as the pair $G = (A, B)$ where $A = (m_A{}^+, m_A{}^-)$ is a bipolar fuzzy set on V and $B = (m_B{}^+, m_B{}^-)$ is a bipolar fuzzy set on $E \subseteq V \times V$ in such a way that $m_B{}^+(x, y) \leq \min\{m_A{}^+(x), m_A{}^+(y)\}$ and $m_B{}^-(x, y) \geq \max\{m_A{}^-(x), m_A{}^-(y)\}$ for all $(x, y) \in E$ .The bipolar fuzzy vertex set is A of V, B is the bipolar fuzzy edge set of E.

Definition 2.2[1]:

Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m_1{}^+, m_1{}^-)$ and $B = (m_2{}^+, m_2{}^-)$ be two bipolar fuzzy sets on a non-empty finite set V and $E \subseteq V \times V$ respectively. where $O^+(G) = \sum_{u \in V} m_1{}^+(u)$ and $O^-(G) = \sum_{u \in V} m_1{}^-(u)$ are used to denoted $O(G)$ and determine the order of $O(G) = (O^+(G), O^-(G))$

Definition 2.3[1]:

Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m_1{}^+, m_1{}^-)$ and $B = (m_2{}^+, m_2{}^-)$ being two bipolar fuzzy sets on a non-empty finite set V and $E \subseteq V \times V$ respectively. Where, determines the size of the G is by $S(G) = (S^+(G), S^-(G))$ where $S^+(G) = \sum_{(u,v) \in E, u \neq v} m_2{}^+(u, v), S^-(G) = \sum_{(u,v) \in E, u \neq v} m_2{}^-(u, v).$

Definition 2.4[2]:

Let G be a bipolar fuzzy graph. The (open) neighbourhood degree of a vertex x in G is defined by $\deg(x) = (\deg_\mu(x), \deg_\gamma(x))$ where $\deg_\mu(x) = \sum_{y \in N(x)} \mu_A{}^P(y)$ and

$\deg_{\gamma}(x) = \sum_{y \in N(x)} \mu_A{}^N(y)$. The graph is said to be regular bipolar fuzzy graph if every vertex has the same open neighbourhood degree.

Definition 2.5[2]:

Let $G = (V, E, \mu, \rho)$ be a bipolar fuzzy graph. if $\rho^P(vw) = \min\{\mu^P(v), \mu^P(w)\}$ and for all $(xy \in E)$, an edge is referred to as effective.

Definition 2.6[1]:

Let $G = (A, B)$ be a bipolar fuzzy graph, with $A = (m_1{}^+, m_1{}^-)$ and $B = (m_2{}^+, m_2{}^-)$ being two bipolar fuzzy sets on a non-empty finite set V and $E \subseteq V \times V$ respectively. If a vertex with unique degrees is close to another vertex, the graph is said to be irregular bipolar fuzzy graph.

Definition 2.7[2]:

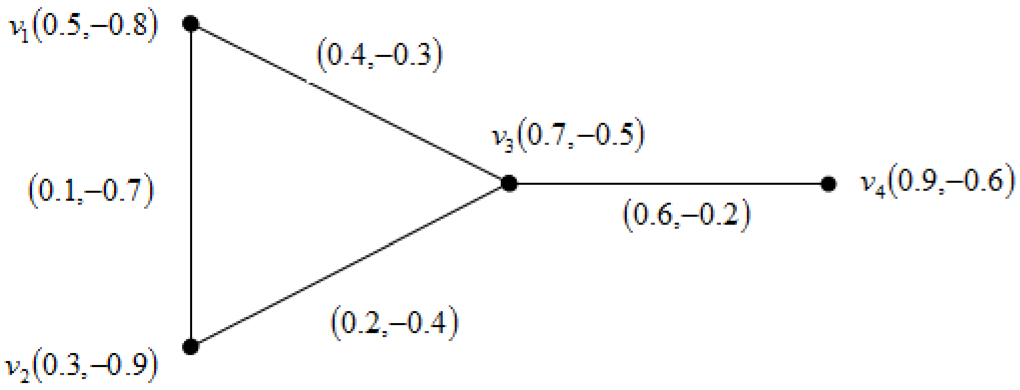
Let G be a bipolar fuzzy graph .The degree of a vertex's closed neighbourhood is defined as $\deg[x] = (\deg_{\mu}[x], \deg_{\gamma}[x])$ where $\deg_{\mu}[x] = \sum_{y \in N(x)} \mu_A{}^P(v) + \mu_A{}^P(x)$ and $\deg_{\gamma}[x] = \sum_{y \in N(x)} \mu_A{}^N(v) + \mu_A{}^N(x)$. The graph is said to be semi regular bipolar fuzzy graph if every vertex has the same closed neighbourgood degree.

3.Irregular Bipolar Fuzzy Labeling graph

Definition 3.1:

A bipolar fuzzy labeling graph is defined as the pair $G = (A, B)$, where $A = (\sigma_A{}^P, \sigma_A{}^N)$ is a bipolar set in V and $B = (\mu_B{}^P, \mu_B{}^N)$ is a bipolar fuzzy set in $E \subseteq V \times V$ such that $\sigma_A{}^P, \mu_B{}^P, \sigma_A{}^N, \mu_B{}^N$ are bijective and $\mu_B{}^P(x, y) \le \min\{\sigma_A{}^P(x), \sigma_A{}^P(y)\}$ and $\mu_B{}^N(x, y) \ge \max\{\sigma_A{}^N(x), \sigma_A{}^N(y)\}$ for all $(x, y) \in E$

Example:3.2

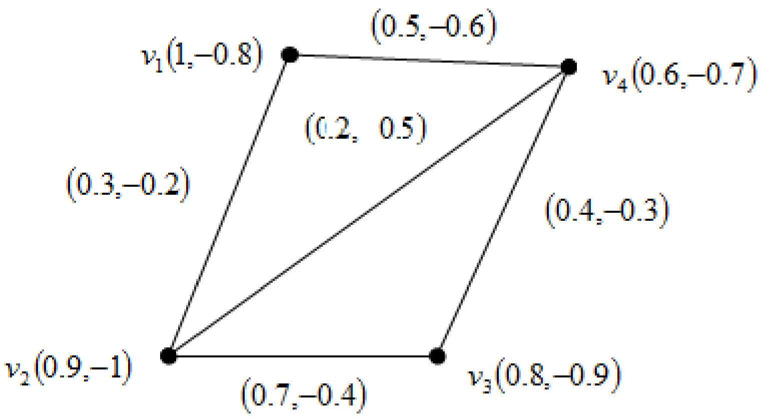


Bipolar Fuzzy Labeling Graph

Definition 3.3:

Let G be a bipolar fuzzy labeling graph .If a vertex which is adjacent to another vertex with distinct open neighbourhood degrees. The graph G is said to be irregular bipolar fuzzy labeling graph.

Example: 3.4



Irregular Bipolar Fuzzy Labeling Graph

Consider the vertex v_3 whose adjacent vertices are v_2, v_4 and whose open neighbourhood degrees are $(2.4,-2.4),(2.7,-2.7)$.They are distinct. Therefore it is a irregular bipolar fuzzy labeling graph.

Proposition 3.5: Every bipolar fuzzy labeling graph is irregular.

Proof:

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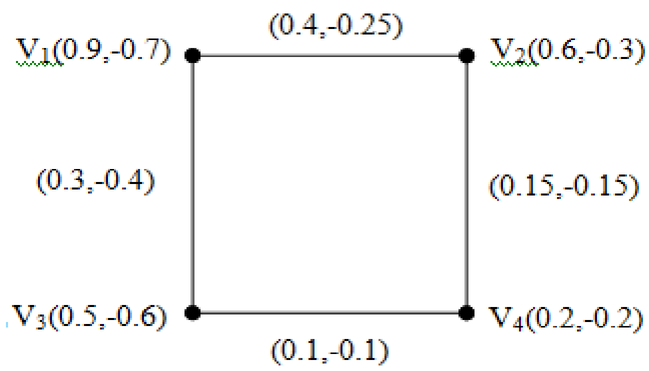
Let G be bipolar fuzzy labeling graph, suppose we assume that G is not irregular (i.e) G is regular, then all the vertices of G will have the same open neighborhood degree, which is not possible, since $\mu_A^P, \mu_B^N, \sigma_A^P, \sigma_B^N$ are bijective.

Note 3.6: The above preposition is not true, If G^* is cycle with n=4.

Example 3.7:

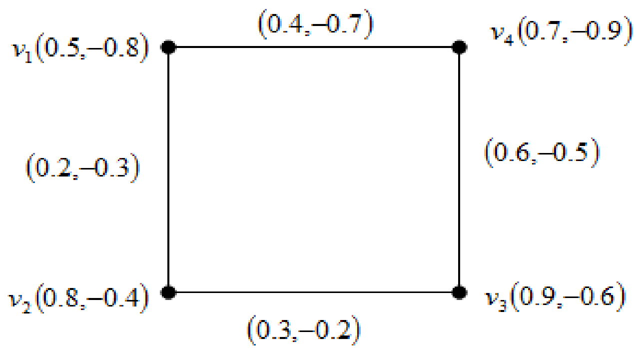


Here deg(v_1) = (1.1,-0.9) , deg(v_2) = (1.1,-0.9) , deg(v_3) = (1.1, -0.9), deg(v_4) = (1.1,-0.9)

Definition 3.8:

Consider a bipolar fuzzy labeling graph that is connected. If every two adjacent vertices of G have a distinct open neighbourhood degree, the graph is called neighbourly irregular bipolar fuzzy labeling graph.

Example 3.9:



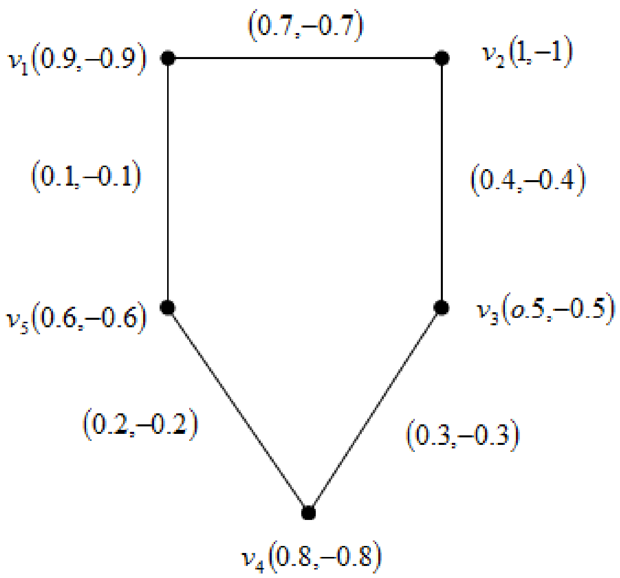
Neighbourly Irregular Bipolar Fuzzy Labeling graph

Consider the adjacent vertices v_1, v_4 and whose open neighbourhood degrees are $(1.5,-1.5),(1.4,-1.4)$.They are distinct.similarly the other adjacent vertices also has distinct neighbourhood degree.

Definition 3.10:

Let G be a connected bipolar fuzzy labeling graph and if every vertex of G is adjacent to vertices with distinct open neighbourhood degrees, then G is called highly irregular bipolar fuzzy labeling graph

Example 3.11:



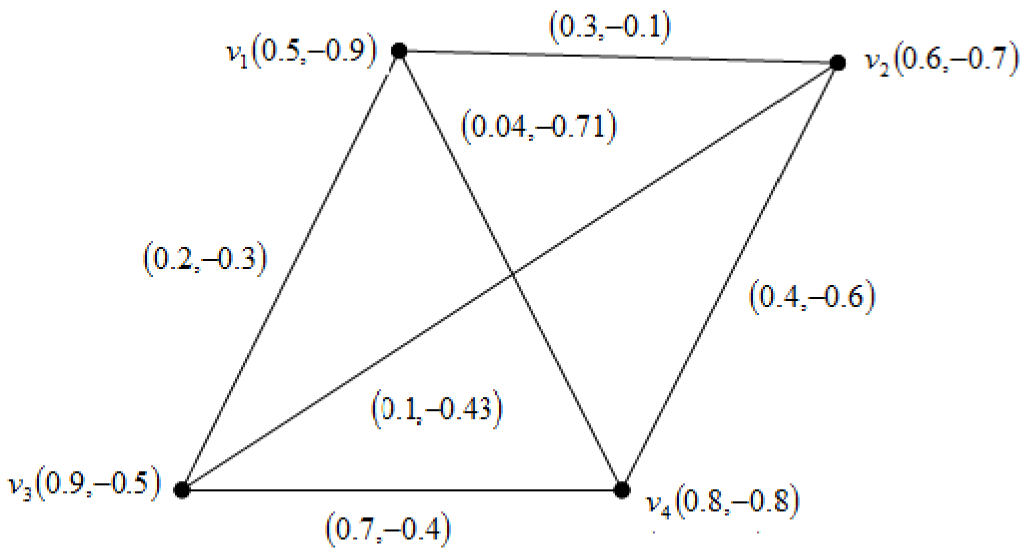
Highly Irregular Bipolar Fuzzy Labeling Graph

Neighbourly irregular bipolar fuzzy labeling graph need not be highly irregular bipolar fuzzy labeling graph.

Definition 3.12:

The bipolar fuzzy labeling graph G is said to be completely highly irregular if every vertex of G has distinct open neighbourhood degree.

Example 3.13:



Completely Highly Irregular Bipolar Fuzzy Labeling Graph

Preposition 3.14: G is completely highly irregular if G is a bipolar fuzzy labeling graph G^* being cycle.

Proof:

Let G be a bipolar fuzzy labeling graph such that G^* is a cycle with n=3, say v_1, v_2, v_3 be the vertices of G, then $\deg(v_1) = (\deg_\alpha(v_1), \deg_\beta(v_1))$,

where $\deg_\alpha(v_1) = \sigma_A^P(v_2) + \sigma_A^P(v_3)$, $\deg_\beta(v_1) = \sigma_A^N(v_2) + \sigma_A^N(v_3)$

Simillarly, $\deg(v_2) = (\sigma_A^P(v_1) + \sigma_A^P(v_3),\ \sigma_A^N(v_1) + \sigma_A^N(v_3))$

$\deg(v_3) = (\sigma_A^P(v_1) + \sigma_A^P(v_2),\ \sigma_A^N(v_1) + \sigma_A^N(v_2))$

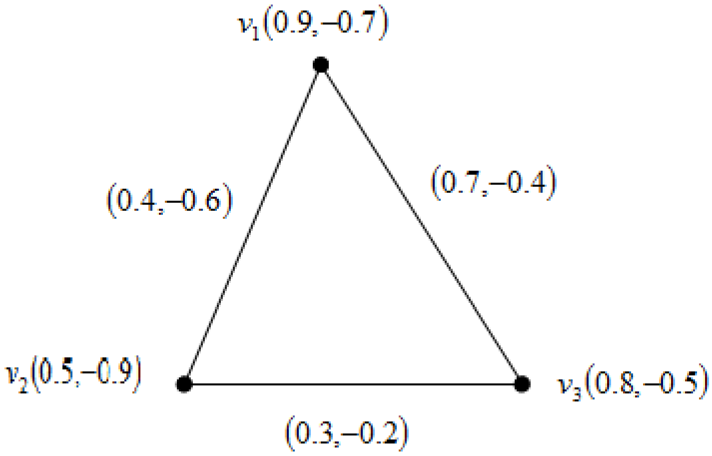
Since σ_A^P, σ_A^N are bijective, from (1),(2),(3) Clearly G is completely highly irregular.

4.Semiregular Bipolar Fuzzy Labeling Graph

Definition 4.1:

A bipolar fuzzy labeling graph G is said to be semi regular if every vertex of G has same closed neighbourhood degree.

Example4.2 :



Semiregular Bipolar Fuzzy Labeling Graph

Preposition 4.3: If G is a bipolar fuzzy labeling graph such that G^* is complete, then G is semi regular.

Proof:

Consider such a graph G with 4 vertices say u, v, w, x, then

deg [u] =(deg$_\alpha$[u], deg$_\beta$[v]), where deg $_\alpha$[u]=$\sum_{v\in N(u)} \sigma_A^P(v) + \sigma_A^P(u)$ and

deg $_\beta$[u] $\sum_{v\in N(u)} \sigma_A^N(v) + \sigma_A^N(u)$

Simillary, deg[v]=($\sum_{W\in N(u)} \sigma_A^P(w) + \sigma_A^P(v)$,$\sum_{W\in N(u)} \sigma_A^N(w) + \sigma_A^N(v)$)

deg[w]= ($\sum_{X\in N(w)} \sigma_A^P(x) + \sigma_A^P(w)$,$\sum_{X\in N(w)} \sigma_A^N(x) + \sigma_A^N(w)$)

deg[x]= ($\sum_{U\in N(x)} \sigma_A^P(u) + \sigma_A^P(x)$,$\sum_{U\in N(x)} \sigma_A^N(u) + \sigma_A^N(x)$)

Since, the underlying graph is complete, it is clear that deg[u] = deg[v] = deg[w] = deg[x].

Observations:

1. If G is bipolar fuzzy labeling graph then
(i) $O(G) \leq S(G)$

(ii) $O(G)+S(G) > \sum deg(v_i^P)$, $O(G)+S(G) < \sum deg(v_i^N)$

2. If G is bipolar fuzzy labeling graph and G is semi regular then deg[v_i]=O(G)

3. If G is not a semi regular graph then O(G) >deg[v_i]

4. Let G be bipolar fuzzy labeling graph such that G^* is a star then O(G) is equal to the membership value of the maximum degree vertex of the graph.

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