Construction the Linear Codes in Projective Plane of Order Sixteen

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Abstract

The major aim of this research is to introduce the maximum value of size of complete (n;r)-arc to be existence in the projective plane of order sixteen PG(2,16) and then study the relationship between coding theory and a finite projective plane, so apply the results of complete $(n;r)$ −arc; $r = 4, \dots ,17$ to coding theory.

Keywords: Projective Plane and coding theory .

1.Introduction. The aim of coding theory is to develop methods that enable the recipient of message to detect or even correct that occur while transmitting data. Many aspects of coding theory can be directly translated into geometry problems. A linear $[n; k, n-r]_q$-code is an n-dimensional subspace of the k-dimensional vector space $V(k,q)$ with non zero vectors weight at least d. An important problem in coding theory is that to optimise one of the parameters k, n, d for given value of the other two and fixed q.The subject of arcs is not only interesting in its purely geometrical setting. An (n;r)-arcs have applications in coding theory, where they can be interpreted as a linear $[n; n-r]_q$-code. So every $[n; 3, n-r]_q$-code is equivalent to $(n;r) - arc\ in\ PG(2,q)$ containing at least r collinear points. In [5] R. Hill studied a the fundamentals of coding theory. In [7] Al-Zangana has been studied the geometry of the plane of order nineteen and its application to error -correcting codes. For more details see [11],[3]. The aim of this research is to construct the complete $(n;r)$ −arcs where $r = 4, \dots, 17$, and construct the all codes corresponding to these arcs. All calculations are by Gap program[4].

2.The projective plane PG(2,16).

In PG(2,16) there are 273 points and lines, 17 points on each line and 17 lines passage through each point. Take $H(X) = X^2 + X + \omega^2$ in $F_{16}[X]$, where

$$F_{16} = \{0,1,\omega,\omega^2,\omega^3,\omega^4,\omega^5,\omega^6,\omega^7,\omega^8,\omega^9,\omega^{10},\omega^{11},\omega^{12},\omega^{13},\omega^{14}\}$$

a polynomial is primitive in F_{16}.The companion matrix of H is $T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \omega^7 & 1 & 0 \end{bmatrix}$.The points of PG(2,16) are generated by T as follows: $P_i = P[1,0,0]T^i; \quad i = 0, \dots 272$.

To find the lines in PG(2,16) :Let l_0 contains of 17 points such that the third coordinate of it is equal to zero. Then the points $P_i = i$ and the lines l_i in $PG(2,16)$ can be represented by:

$l_{0=}$\{0,1,3,7,15,31,63,90,116,127,136,181,194,204,233,138,255\}. Moreover, $l_i = l_0 T^i; \quad i = 0, \dots 272$.

3.Some definitions and basic properties

Definition 3.1.[6] An $(n;r)$-arc $\mathcal{K}$in PG(2,q) is a set of n points,, satisfies that every line meet it in less than or equal r points, that is $|K \cap l| \leq r \; for \; all \; l \in PG(2,16)$.

Definition 3.2[9]**:**The points out of arc K which passes through it i $bisecant$ of K is called a point of index i.The number of these points is denoted by c_i.So which represents the number of the points not on $bisecant$ of K.

Lemma 3.1: For a $(n,r) - arc \; K$,the following equations hold:

$$\sum_{i=0}^{n^`} c_i = q^2 + q + 1 - n \qquad (1.1)$$

$$\sum_{i=0}^{n^`} ic_i = K(k-1)(q-1)/2 \qquad (1.2)$$

$$\sum_{i=0}^{n^`} i(i-1)c_i = K(K-1)(K-2)(K-3)/8 \qquad (13)$$

$$such\ that\ n^{`} = \left\lfloor \frac{1}{2} n \right\rfloor$$

Proof . See[1].

Definition 3.3.[1] An linear $[n,k,d]_q$ -code C over a finite field is subspace of dimension k of the $n-$ dimensional vector space $V(n,q) = F_q^n$ such that any two distinct vectors in C differ in at least of d places. The elements of the code are called codewords. Also the parameters n,k and d are called length, dimension, and minimum distance of C.

Definition 3.4.[5] For any two code words the minimum distance (Hamming distance) between c_1 and, c_2 is denoted by $d(c_1, c_{2)}$ and it is defined to be the number of positions in which the corresponding coordinates differ. The minimum distance of C is $d(C) = \min\{d(c_1,c_2);\ c_1,c_2 \in C, c_1 \neq c_2\}$.

Definition 3.5.[5] The weight $w(x)$ of $x \in V(n,q)$ is $w(x) = d(x,0)$; that is, $w(x)$ is the number of non-zero elements in x .

Lemma 3.1. $d(c_1,c_2) = w(c_1 - c_2)$ for $c_1, c_2 \in C$.

Proof. See [1].

Definition 3.6.[5] Let C be a linear $[n,k,d]_q$ -code and A_i be the number of codewords of weight i in a code C, the list A_i for $0 \leq i \leq n$ is called the weight distribution of C.

Definition 3.7.[1] Two linear codes C_1 and C_2 in $V(n,q)$ are equivalent if C_1 can be obtained from C_2 by permuting coordinates and by multiplying coordinates by non-zero elements of F_q.

Definition 3.8.[1] A generator matrix of a linear $[n,k,d]_q$ -code C is $k \times n$ matrix over the finite field F_q whose rows from a basis of C; it is denoted by G.

Definition 3.9.[1] For a linear $[n,k,d]_q$ -code C over the finite field F_q, the singleton bound that $d(C) \leq n - k + 1$.

Definition 3.10.[1] A linear$[n,k,d]_q$ -code C over a finite field is said to be maximum distance separable (MDS) code if d satisfies the following bound :

$$d(C) = n - k + 1.$$

And if $d = n - k$, then the code is called almost maximum distance separable (AMDS).

Theorem 3.1.[5] There exists a projective $[n, K, d]_q$-code if and only if there exists a $(n, n - d)$-arc .

Definition 3.11 .[1] For $x_0 \in F_q^n$ and $r \in Z, r \geq 0$, the ball of centre x_0 and radius r is

$$S(x_0, r) = \{x \in F_q^n; d(x_0, x) \leq r\}$$

Definition 3.12.[1] The covering radius of linear$[n, k, d]_q$ code C is the smallest $\mu = \mu(C)$ such that $\cup_{x \in C} S(x, \mu) = F_q^n$.

4. The construction of complete arc of higher degree in projective plane of order sixteen

Theorem 4.1. In projective plane PG (2,16) there exists :

I. A complete (36;4)-arc .

II. A complete (46;5)-arc

III. A complete (57;6)-arc

IV. A complete (72;7)-arc

V. A complete (83;8)-arc

VI. A complete (100;9)-arc

VII. A complete (118;10)-arc

VIII. A complete (132;11)-arc

IX. A complete (154;12)-arc

X. A complete (175;13)-arc

XI. A complete (192,14)-arc

XII. A complete (213 ;15)-arc

XIII. A complete (234;16)-arc

XIV. A complete (273;17)-arc

Proof:

I. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 4$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,4)-arc as follows:

$[c_0, \ldots, c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;4)-arc to (24;4)-arc to construct (25;4)-arc, the values of parameters c_i of (25;4)-arc $; i = 0, \ldots 12$ are respectively, $[c_0, \ldots, c_{12}] = [183,65,0,0,0,0,0,0,0,0,0,0,0]$;

Then by adding the first point of the c_0 of (25;4)-arc to (25;4)-arc to construct (26;4)-arc, the values of parameters c_i of (26;4)-arc $; i = 0, \ldots 13$ are respectively, $[c_0, \ldots, c_{13}] = [180,65,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then continue in the same way until we get (36;4)-arc which is complete since $c_0 = 0$.

II. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 5$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,5)-arc as follows:

$[c_0, \ldots, c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;5)-arc to (24;5)-arc to construct (25;5)-arc, the values of parameters c_i of (25;5)-arc $; i = 0, \ldots 12$ are respectively, $[c_0, \ldots, c_{12}] = [148,0,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then by adding the first point of the c_0 of (25;5)-arc to (25;5)-arc to construct (26;5)-arc, the values of parameters c_i of (26;5)-arc $; i = 0, \ldots 13$ are respectively, $[c_0, \ldots, c_{13}] = [146,0,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then continue in the same way until we get (46;5)-arc which is complete since $c_0 = 0$.

III. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 6$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,6)-arc as follows:

$[c_0, \ldots, c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;6)-arc to (24;6)-arc to construct (25;6)-arc, the values of parameters c_i of (25;6)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;6)-arc to (25;6)-arc to construct (26;6)-arc, the values of parameters c_i of (26;6)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (57;6)-arc which is complete since $c_0 = 0$.

V. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 7$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,7)-arc as follows:

$[c_0, \dots, c_{12}] =$ [249,0,0,0,0,0,0,0,0,0,0,0,0]

Then by adding the first point of the c_0 of (24;7)-arc to (24;7)-arc to construct (25;7)-arc, the values of parameters c_i of (25;7)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;7)-arc to (25;7)-arc to construct (26;7)-arc, the values of parameters c_i of (26;7)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (72;7)-arc which is complete since $c_0 = 0$.

IV. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 8$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,8)-arc as follows:

$[c_0, \dots, c_{12}] =$ [249,0,0,0,0,0,0,0,0,0,0,0,0]

Then by adding the first point of the c_0 of (24;8)-arc to (24;8)-arc to construct (25;8)-arc, the values of parameters c_i of (25;8)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;8)-arc to (25;8)-arc to construct (26;8)-arc, the values of parameters c_i of (26;8)-arc $; i = 0, \ldots 13$ are respectively, $[c_0, \ldots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (83;8)-arc which is complete since $c_0 = 0$.

V. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 9$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,9)-arc as follows:

$[c_0, \ldots, c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;9)-arc to (24;9)-arc to construct (25;9)-arc, the values of parameters c_i of (25;9)-arc $; i = 0, \ldots 12$ are respectively, $[c_0, \ldots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;9)-arc to (25;9)-arc to construct (26;9)-arc, the values of parameters c_i of (26;9)-arc $; i = 0, \ldots 13$ are respectively, $[c_0, \ldots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (100;9)-arc which is complete since $c_0 = 0$.

VI. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 10$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,10)-arc as follows:

$[c_0, \ldots, c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;10)-arc to (24;10)-arc to construct (25;10)-arc, the values of parameters c_i of (25;10)-arc $; i = 0, \ldots 12$ are respectively, $[c_0, \ldots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;10)-arc to (25;10)-arc to construct (26;10)-arc, the values of parameters c_i of (26;10)-arc $; i = 0, \ldots 13$ are respectively, $[c_0, \ldots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (118;10)-arc which is complete since $c_0 = 0$.

VII. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 11$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,11)-arc as follows:

$[c_0 , \dots , c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;11)-arc to (24;11)-arc to construct (25;11)-arc, the values of parameters c_i of (25;11)-arc ; $i = 0, \dots 12$ are respectively, $[c_0 , \dots , c_{12}] = [148,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then by adding the first point of the c_0 of (25;11)-arc to (25;11)-arc to construct (26;11)-arc, the values of parameters c_i of (26;11)-arc ; $i = 0, \dots 13$ are respectively, $[c_0 , \dots , c_{13}] = [146,0,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then continue in the same way until we get (132;11)-arc which is complete since $c_0 = 0$.

VIII. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 12$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,12)-arc as follows:

$[c_0 , \dots , c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;12)-arc to (24;5)-arc to construct (25;12)-arc, the values of parameters c_i of (25;12)-arc ; $i = 0, \dots 12$ are respectively, $[c_0 , \dots , c_{12}] = [148,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then by adding the first point of the c_0 of (25;12)-arc to (25;12)-arc to construct (26;12)-arc, the values of parameters c_i of (26;12)-arc ; $i = 0, \dots 13$ are respectively, $[c_0 , \dots , c_{13}] = [146,0,0,0,0,0,0,0,0,0,0,0,0,0]$;

Then continue in the same way until we get (154;12)-arc which is complete since $c_0 = 0$.

IX. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 13$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,13)-arc as follows:

$[c_0 , \dots , c_{12}] = [249,0,0,0,0,0,0,0,0,0,0,0,0]$

Then by adding the first point of the c_0 of (24;13)-arc to (24;13)-arc to construct (25;13)-arc, the values of parameters c_i of (25;13)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;13)-arc to (25;13)-arc to construct (26;13)-arc, the values of parameters c_i of (26;13)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (175;13)-arc which is complete since $c_0 = 0$.

X. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 14$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,5)-arc as follows:

$[c_0, \dots, c_{12}] =$ [249,0,0,0,0,0,0,0,0,0,0,0,0]

Then by adding the first point of the c_0 of (24;14)-arc to (24;14)-arc to construct (25;14)-arc, the values of parameters c_i of (25;14)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;14)-arc to (25;14)-arc to construct (26;14)-arc, the values of parameters c_i of (26;14)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (192;14)-arc which is complete since $c_0 = 0$.

XI. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 15$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,15)-arc as follows:

$[c_0, \dots, c_{12}] =$ [249,0,0,0,0,0,0,0,0,0,0,0,0]

Then by adding the first point of the c_0 of (24;15)-arc to (24;15)-arc to construct (25;15)-arc, the values of parameters c_i of (25;15)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;15)-arc to (25;15)-arc to construct (26;15)-arc, the values of parameters c_i of (26;15)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (214;15)-arc which is complete since $c_0 = 0$.

XII. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 16$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,16)-arc as follows:

$[c_0, \dots, c_{12}] =$ [249,0,0,0,0,0,0,0,0,0,0,0,0]

Then by adding the first point of the c_0 of (24;16)-arc to (24;16)-arc to construct (25;16)-arc, the values of parameters c_i of (25;16)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;16)-arc to (25;5)-arc to construct (26;16)-arc, the values of parameters c_i of (26;16)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (234;16)-arc which is complete since $c_0 = 0$.

XIII. we choose the (24;3) -arc and then intersect it with the lines in PG(2,16) when $r = 17$.

The number of the points of c_i which is represents the number of points out of arc K which are passes through them i 2 -secants of (24,17)-arc as follows:

$[c_0, \dots, c_{12}] =$ [249,0,0,0,0,0,0,0,0,0,0,0,0]

Then by adding the first point of the c_0 of (24;17)-arc to (24;17)-arc to construct (25;17)-arc, the values of parameters c_i of (25;17)-arc ; $i = 0, \dots 12$ are respectively, $[c_0, \dots, c_{12}] =$ [148,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then by adding the first point of the c_0 of (25;17)-arc to (25;17)-arc to construct (26;17)-arc, the values of parameters c_i of (26;17)-arc ; $i = 0, \dots 13$ are respectively, $[c_0, \dots, c_{13}] =$ [146,0,0,0,0,0,0,0,0,0,0,0,0,0] ;

Then continue in the same way until we get (273;17)-arc which is complete since $c_0 = 0$.

5 . Constructions of the linear codes in projective plane of order sixteen

The problem of determine the largest size of $(n;r)$ −arcs in PG(2,q) demonstrated an interesting connection with coding theory . This connection is between $(n;r)$ −arcs in PG(2,q) and the $[n,k,d]_q$ codes in coding theory. This link gives the following theorem.

Theorem 5.1. In projective plane PG(2,16) there exists:

I. A projective $[36,3,32]_{16}$-code if there exists a (36,4)-arc .

II. A projective $[46,3,41]_{16}$-code if there exists a (41,5)-arc .

III. A projective $[57,3,51]_{16}$-code if there exists a (57,6)-arc .

IV. A projective $[72,3,65]_{16}$-code if there exists a (72,7)-arc .

V. A projective $[83,3,75]_{16}$-code if there exists a (83,8)-arc .

VI. A projective $[100,3,91]_{16}$-code if there exists a (100,9)-arc .

VII. A projective $[118,3,108]_{16}$-code if there exists a (118,10)-arc .

VIII. A projective $[132,3,121]_{16}$-code if there exists a (132,11)-arc .

IX. A projective $[154,3,142]_{16}$-code if there exists a (154,12)-arc.

X. A projective $[175,3,162]_{16}$-code if there exists a (175,13)-arc .

XI. A projective $[192,3,178]_{16}$-code if there exists a (192,14)-arc .

XII. A projective $[213,3,198]_{16}$-code if there exists a (213,15)-arc .

XIII A projective $[234,3,218]_{16}$-code if there exists a (234,16)-arc .

XIV. A projective $[273,3,256]_{16}$-code if there exists a (273,17)-arc .

Proof.

According to the theorem (3.1) and (4.1) an$(n, n-d)$-arc in $PG(k-1,q)$ is equivalent to a projective $[n,k,d]_q - code$. Now if $q = 16$, $K = 3$ and $n - d = r$; r is degree of arc, then there is an one to one correspondence between $(n;r)$- arc in $PG(k-1,q)$ and a projective $[n,3,n-r]_q - code$.

I. (36;4)-arc give linear $[36,3,32]_{16} - code$ define by the generator matrix $G_{3\times 36}$.

$$G_{3\times 36} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & & w^6 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & \dots & w^5 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 15$, the weight distribution $(A_0, A_1, \dots, A_{36})$=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,960,915,945,705,570). Not that A_0=1, A_{32}=960, A_{33}=915, A_{34}=945, , A_{35}=705, , A_{36}=570, and S= A_0+ A_{32}+ A_{33}+ A_{34}+ A_{35}+ A_{36}=4096=16^3 and the covering radius $\mu = 33$. □

II. (46;5)-arc give linear $[46,3,41]_{16} - code$ define by the generator matrix $G_{3\times 46}$.

$$G_{3\times 46} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & w^2 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \dots & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 20$, the weight distribution $(A_0, A_1, \dots$,46)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,795,750,720, 915,765,150). Not that A_0=1, A_{41}=795, A_{42}=750, A_{43}=720, A_{44}=915, ,A_{45}=765, A_{46}=150 and S= A_0+ A_{41}+ A_{42}+ A_{43}+ A_{44}+ A_{45}+A_{46}=4096=16^3 and the covering radius $\mu = 43$. □

III. (57;6)-arc give linear $[57,3,51]_{16} - code$ define by the generator matrix $G_{3\times 57}$.

$$G_{3\times 57} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & w^6 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \dots & w^7 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 25$, the weight distribution $(A_0, A_1, \dots$,57)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0, 0,0,750,450,765, 930,810,315,75). Not that A_0=1, A_{51}=750, A_{52}=450, A_{53}=765, A_{54}=930, A_{55}=810, A_{56}=315, A_{57}=75 and S= A_0+ A_{51}+ A_{52}+ A_{53}+ A_{54}+ A_{55} + A_{56}+ A_{57} =4096=16^3 and the covering radius $\mu = 53$. □

IV. (72;7)-arc give linear $[72,3,65]_{16} - code$ define by the generator matrix $G_{3\times 72}$.

$$G_{3\times72}=\begin{bmatrix}1&0&0&w^7&1&w^{11}&w^{13}&w^4&w^9&w^4&w^{10}&&w^3\\0&1&0&1&1&w^3&w^7&w^9&w^{13}&w^9&w^8&\dots&w\\0&0&1&0&1&1&1&1&1&1&1&&1\end{bmatrix}$$

With $e=\lfloor(d-1)/2\rfloor=32$, the weight distribution $(A_0,A_1,\dots,72)$=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,870,465,540,735,930,495,60,0).

Not that A_0=1, A_{65}=870, A_{66}=465, A_{67}=540, A_{68}=735, A_{69}=930, A_{70}=495, A_{71}=60 and S= A_0+ A_{65}+ A_{66}+ A_{67}+ A_{68}+ A_{69} + A_{70}+ A_{71} =4096=16^3 and the covering radius $\mu=67$. □

V. (83;8)-arc give linear $[83,3,75]_{16}-code$ define by the generator matrix $G_{3\times83}$.

$$G_{3\times83}=\begin{bmatrix}1&0&0&w^7&1&w^{11}&w^{13}&w^4&w^9&w^4&w^{10}&&w^7\\0&1&0&1&1&w^3&w^7&w^9&w^{13}&w^9&w^8&\dots&w^{10}\\0&0&1&0&1&1&1&1&1&1&1&&1\end{bmatrix}$$

With $e=\lfloor(d-1)/2\rfloor=37$, the weight distribution $(A_0,A_1,\dots,83)$=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,810,285,510,615,1095,615,165,0, 0).

Not that A_0=1, A_{75}=810, A_{76}=285, A_{77}=510 , A_{78}=615 , A_{79}=1095, A_{80}=615, A_{81}=165 and S= A_0+ A_{75}+ A_{76}+ A_{77}+ A_{78}+ A_{79} + A_{80}+ A_{81} =4096=16^3 and the covering radius $\mu=78$. □

VI. (100;9)-arc give linear $[100,3,91]_{16}-code$ define by the generator matrix $G_{3\times100}$.

$$G_{3\times100}=\begin{bmatrix}1&0&0&w^7&1&w^{11}&w^{13}&w^4&w^9&w^4&w^{10}&&w^{11}\\0&1&0&1&1&w^3&w^7&w^9&w^{13}&w^9&w^8&\dots&w^2\\0&0&1&0&1&1&1&1&1&1&1&&1\end{bmatrix}$$

With $e=\lfloor(d-1)/2\rfloor=45$, the weight distribution $(A_0,A_1,\dots,100)$=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 825, 480,390,630,900, 630,225,15,0,0).

Not that A_0=1, A_{91}=825, A_{92}=480, A_{93}=390, A_{94}=630, A_{95}=900, A_{96}=630, A_{97}=225, A_{98}=15 , and S= A_0+ A_{91}+ A_{92}+ A_{93}+ A_{94}+ A_{95} + A_{96}+ A_{97} +A_{98} =4096=16^3 and the covering radius $\mu = 94$. □

VII. (118;10)-arc give linear $[118,3,108]_{16} - code$ define by the generator matrix $G_{3\times 118}$.

$$G_{3\times 118} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & w^5 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \cdots & w^8 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 53$, the weight distribution $(A_0, A_1, \ldots$,118)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0 ,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,915,600,330,540, 810,660,240,0,0,00).

Not that A_0=1, A_{108}=915, A_{109}=600, A_{110}=330, A_{111}=540, A_{112}=810, A_{113}=660, A_{114}=240, and S= A_0+ A_{108}+ A_{109}+ A_{110}+ A_{111}+ A_{112} + A_{113}+ A_{114} =4096=16^3 and the covering radius $\mu = 111$. □

VIII. (132;11)-arc give linear $[132,3,121]_{16} - code$ define by the generator matrix $G_{3\times 132}$.

$$G_{3\times 132} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & 0 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \cdots & w^{13} \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 60$, the weight distribution $(A_0, A_1, \ldots$,132)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0 ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,195,570,660,495, 435,690,720, 300,30,0,0,0).

Not that A_0=1, A_{121}=195, A_{122}=570, A_{123}=660, A_{124}=495, A_{125}=435, A_{126}=690, A_{127}=720, A_{128}=300, A_{129}=30 and S= A_0+ A_{121}+ A_{122}+ A_{123}+ A_{124}+ A_{125} + A_{126}+ A_{127} + A_{128} + A_{129}=4096=16^3 and the covering radius $\mu = 124$. □

IX. (154;12)-arc give linear $[154,3,142]_{16} - code$ define by the generator matrix $G_{3\times 154}$.

$$G_{3\times 154}=\begin{bmatrix}1 & 0 & 0 & w^{7} & 1 & w^{11} & w^{13} & w^{4} & w^{9} & w^{4} & w^{10} & & w^{11}\\ 0 & 1 & 0 & 1 & 1 & w^{3} & w^{7} & w^{9} & w^{13} & w^{9} & w^{8} & \dots & w^{13}\\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1\end{bmatrix}$$

With $e=\lfloor (d-1)/2\rfloor=70$, the weight distribution $(A_0, A_1, \dots$,154)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0 ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0, 0,0,0,0, 825,570,1020,480, 345,585,540, 285,15,0,0,0,0).

Not that A_0=1, A_{142}=825, A_{143}=1020, A_{144}=480, A_{145}=345, A_{146}=585, A_{147}=540, A_{148}=285, A_{149}=15, and S= A_0+ A_{142}+ A_{143}+ A_{144}+ A_{145}+ A_{146} + A_{147}+ A_{148} + A_{149} =4096=16^3 and the covering radius $\mu=144$. □

X. (175;13)-arc give linear $[175,3,162]_{16}-code$ define by the generator matrix $G_{3\times 175}$.

$$G_{3\times 175}=\begin{bmatrix}1 & 0 & 0 & w^{7} & 1 & w^{11} & w^{13} & w^{4} & w^{9} & w^{4} & w^{10} & & w^{14}\\ 0 & 1 & 0 & 1 & 1 & w^{3} & w^{7} & w^{9} & w^{13} & w^{9} & w^{8} & \dots & w^{2}\\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1\end{bmatrix}$$

With $e=\lfloor (d-1)/2\rfloor=80$, the weight distribution $(A_0, A_1, \dots$,175)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0 ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 990,1155,495,285,435,540,195 ,0,0, 0,0,0,0,0).

Not that A_0=1, A_{162}=990, A_{163}=1155, A_{164}=495, A_{165}=285, A_{166}=435, A_{167}=540, A_{168}=195, and S= A_0+ A_{162}+ A_{163}+ A_{164}+ A_{165}+ A_{166} + A_{167}+ A_{168} =4096=16^3 and the covering radius $\mu=166$. □

XI. (192;14)-arc give linear $[192,3,178]_{16}-code$ define by the generator matrix $G_{3\times 192}$.

$$G_{3\times192} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & w^{10} \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \dots & w^3 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 88$, the weight distribution $(A_0, A_1, \dots$,192)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0 ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,915, 1110,780,300,360,435,180,15 ,0,0, 0,0,0,0,0).

Not that A_0=1, A_{178}=915, A_{179}=1110, A_{180}=780, A_{181}=300, A_{182}=360, A_{183}=435, A_{184}=180, A_{185}=15 and S= A_0+ A_{178}+ A_{179}+ A_{180}+ A_{181}+ A_{182} +A_{183}+ A_{184} +A_{185} =4096=16^3 and the covering radius $\mu = 181$. □

XII. (213;15)-arc give linear $[213,3,198]_{16} - code$ define by the generator matrix $G_{3\times213}$.

$$G_{3\times213} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & w \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \dots & w^2 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 98$, the weight distribution $(A_0, A_1, \dots$,213)=(1,0,0,0,0, ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0 ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1050,1305,675,285,405,270,105,0,0, 0,0,0,0, 0,0,0).

Not that A_0=1, A_{198}=1050, A_{199}=1305, A_{200}=675, A_{201}=285, A_{202}=405, A_{203}=270, A_{204}=105, and S= A_0+ A_{197}+ A_{199}+ A_{200}+ A_{201}+ A_{202} +A_{202}+ A_{203} +A_{204} =4096=16^3 and the covering radius $\mu = 200$. □

XIII. (234;16)-arc give linear $[234{,}3{,}218]_{16} - code$ define by the generator matrix $G_{3\times 234}$.

$$G_{3\times 134} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & 1 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \dots & w^7 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 166$, the weight distribution $(A_0, A_1, \dots, 234)$=(1,0,0,0,0,
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 1290,1140,945,
225,345,135,15 ,0,0, 0,0,0,0,0,0,0,0).

Not that A_0=1, A_{218}=1290, A_{219}=1140, A_{220}=945, A_{221}=225, A_{222}=345, A_{223}=135, A_{224}=15, and S= A_0+ A_{218}+ A_{219}+ A_{220}+ A_{221}+ A_{222} + A_{223}+ A_{224} +A_{225} =4096=16^3 and the covering radius $\mu = 220$. □

XIII. (273;17)-arc give linear $[273{,}3{,}256]_{16} - code$ define by the generator matrix $G_{3\times 273}$.

$$G_{3\times 273} = \begin{bmatrix} 1 & 0 & 0 & w^7 & 1 & w^{11} & w^{13} & w^4 & w^9 & w^4 & w^{10} & & 1 \\ 0 & 1 & 0 & 1 & 1 & w^3 & w^7 & w^9 & w^{13} & w^9 & w^8 & \dots & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix}$$

With $e = \lfloor (d-1)/2 \rfloor = 127$, the weight distribution $(A_0, A_1, \dots, 273)$=(1,0,0,0,0,
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,00,4096,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0).

Not that A_0=1, A_{256}=4095, and S= A_0+ A_{256} =4096=16^3 and the covering radius $\mu = 259$. □

Conclusion

we constructed a complete (n,r)-arcs in projective plane with respect to $\boldsymbol{r}$, such that $r = 4,5,..,17$ and find the relation between geometrical objects and linear codes.

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