

Results for the Groups $SL(2,3^4)$ and $SL(2,3^6)$

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Abstract

The character table of rational representations and the induced characters table study to gain the Artin indicator for the groups $SL(2,3^4)$ and $SL(2,3^6)$ in this work.

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1. Introduction

The group of all matrices of determinant 1 is $SL(n, F)$, [1] and [2], searchers in [3] define the representation of the group.

In this work we compute the Artin indicator for the groups $SL(2, 3^4)$ and $SL(2, 3^6)$.

2. The Concepts

The relationships that we extracted above represent a complex analysis of all the components of technologies

Theorem 2.1: [1] $|SL(2, q^n)| = q^n (q^{2n} - 1)$.

Definition 2.4: [4] Let H be a cyclic subgroup of a group G , and ϕ be a class function of H . Then

$$\phi \uparrow^G(g) = \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \phi(x_i)$$

The character induced from the principal character of cyclic subgroups of G is Artin character.

Definition 2.5: [5] The character induced from the principal character of a cyclic subgroups of G is called Artin character.

Definition 2.6: [5] Let G be a finite group and let χ be any rational valued character on G . The smallest positive number n such that,

$$n\chi = \sum_c a_c \phi_c$$

where $a_c \in \mathbb{Z}$ and ϕ_c is Artin character, is called the Artin exponent of G and denoted by $A(G)$.

3. The Results

Authors in [4] and [7-8] study the character table of rational representations for the group $\mathcal{SL}(2, p)$ we apply that idea and compute the character table of rational representations for the groups $\mathcal{SL}(2, 3^4)$ and $\mathcal{SL}(2, 3^6)$. Also we apply the same idea in [4] and [6] to compute the Artin indicator for the same groups.

3.1. The results for the group $\mathcal{SL}(2, 3^4)$

The character table of rational representations for the group $\mathcal{SL}(2, 3^4)$ is

C_g	1	z	$c=d$	$zc=zd$	a^1	a^2	a^4	a^5	a^8	a^{10}	a^{16}	a^{20}	b^1	b^2
$ C_g $	1	1	3280	3280	6642	6642	6642	6642	6642	6642	6642	6642	6480	6480
$ C_G(g) $	531360	531360	162	162	80	80	80	80	80	80	80	80	82	82
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ	81	81	0	0	1	1	1	1	1	1	1	1	-1	-1
χ_1	2624	-2624	32	-32	0	0	0	0	256	0	-256	0	0	0
χ_2	656	656	8	8	0	0	64	0	-64	0	-128	-256	0	0
χ_4	328	328	4	4	0	8	-8	0	-16	-32	0	32	0	0
χ_5	656	-656	8	-8	0	0	0	-32	-64	-64	64	64	0	0
χ_8	164	164	2	2	-2	-4	-8	8	8	4	8	8	0	0
χ_{10}	164	164	2	2	0	0	-8	-8	8	8	8	8	0	0
χ_{16}	164	164	2	2	-2	-4	8	8	8	8	8	8	0	0
χ_{20}	82	82	1	1	0	-2	2	2	2	2	2	2	0	0
θ_1	3200	-3200	-40	40	0	0	0	0	0	0	0	0	-40	40
θ_2	1600	1600	-20	-20	0	0	0	0	0	0	0	0	20	0
ξ	82	82	1	1	-2	2	2	-2	2	2	2	2	0	0
η	160	-160	-1	1	0	0	0	0	0	0	0	0	4	-4

The Artin character table for the group $\mathcal{SL}(2, 3^4)$ is

C_g	1	z	$c=d$	$zc=zd$	a^1	a^2	a^4	a^5	a^8	a^{10}	a^{16}	a^{20}	b^1	b^2
$ C_g $	1	1	3280	3280	6642	6642	6642	6642	6642	6642	6642	6642	6480	6480
$ C_G(g) $	531360	531360	162	162	80	80	80	80	80	80	80	80	82	82
Φ_1	531360	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	132840	132840	0	0	0	0	0	0	0	0	0	0	0	0
Φ_3	6560	0	2	0	0	0	0	0	0	0	0	0	0	0
Φ_4	6560	9840	2	2	0	0	0	0	0	0	0	0	0	0
Φ_5	6642	13284	0	0	2	0	0	0	0	0	0	0	0	0
Φ_6	13284	26568	0	0	0	4	0	0	0	0	0	0	0	0
Φ_7	26568	0	0	0	0	0	8	0	0	0	0	0	0	0
Φ_8	88560	177120	0	0	0	0	0	10	0	0	0	0	0	0
Φ_9	53136	0	0	0	0	0	0	0	16	0	0	0	0	0
Φ_{10}	66420	0	0	0	0	0	0	0	0	20	0	0	0	0
Φ_{11}	106272	0	0	0	0	0	0	0	0	0	32	0	0	0
Φ_{12}	132840	0	0	0	0	0	0	0	0	0	0	40	0	0
Φ_{13}	6480	12960	0	0	0	0	0	0	0	0	0	0	2	0
Φ_{14}	12960	0	0	0	0	0	0	0	0	0	0	0	0	2

Hence, we written the rational valued characters in the first tables as a linear combination of induced characters in the second table

$$\begin{aligned}
 1 &= \frac{1}{2}\Phi_{14} + \frac{1}{2}\Phi_{13} + \frac{1}{40}\Phi_{12} + \frac{1}{32}\Phi_{11} + \frac{1}{20}\Phi_{10} + \frac{1}{16}\Phi_9 + \frac{1}{10}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{1}{2}\Phi_4 - \\
 &\quad \frac{42395}{132840}\Phi_2 - \frac{2707}{531360}\Phi_1 \\
 \Psi &= -\frac{1}{2}\Phi_{14} - \frac{1}{2}\Phi_{13} + \frac{1}{40}\Phi_{12} + \frac{1}{32}\Phi_{11} + \frac{1}{20}\Phi_{10} + \frac{1}{16}\Phi_9 + \frac{1}{10}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 - \\
 &\quad \frac{24435}{132840}\Phi_2 + \frac{2133}{531360}\Phi_1 \\
 \chi_1 &= -8\Phi_{11} + 16\Phi_9 - 16\Phi_4 + 32\Phi_3 + \frac{154816}{132840}\Phi_2 - \frac{257152}{531360}\Phi_1 \\
 \chi_2 &= -6.4\Phi_{12} - 4\Phi_{11} - 4\Phi_9 + 8\Phi_7 + 4\Phi_4 - \frac{38704}{132840}\Phi_2 + \frac{1288384}{531360}\Phi_1 \\
 \chi_4 &= 0.8\Phi_{12} - 1.6\Phi_{10} - \Phi_9 - \Phi_7 + 2\Phi_6 - \frac{72488}{132840}\Phi_2 - \frac{112832}{531360}\Phi_1 \\
 \chi_5 &= 1.6\Phi_{12} + 2\Phi_{11} - 3.2\Phi_{10} - 4\Phi_9 - 3.2\Phi_8 - 4\Phi_4 + 8\Phi_3 + \frac{605488}{132840}\Phi_2 - \frac{347680}{531360}\Phi_1 \\
 \chi_8 &= 0.2\Phi_{12} + 0.25\Phi_{11} + 0.2\Phi_{10} + 0.5\Phi_9 + 0.8\Phi_8 - \Phi_7 - \Phi_6 - \Phi_5 + \Phi_4 - \frac{111520}{132840}\Phi_2 - \frac{112218}{531360}\Phi_1 \\
 \chi_{10} &= 0.2\Phi_{12} + 0.25\Phi_{11} + 0.4\Phi_{10} + 0.05\Phi_9 - 0.8\Phi_8 - \Phi_7 + \Phi_4 + \frac{132020}{132840}\Phi_2 - \frac{133402}{531360}\Phi_1 \\
 \chi_{16} &= 0.2\Phi_{12} + 0.25\Phi_{11} + 0.4\Phi_{10} + 0.05\Phi_9 + 0.8\Phi_8 + \Phi_7 - \Phi_6 - \Phi_5 + \Phi_4 - \frac{111520}{132840}\Phi_2 - \frac{78638}{531360}\Phi_1 \\
 \chi_{20} &= 0.05\Phi_{12} + 0.0625\Phi_{11} + 0.1\Phi_{10} + 0.125\Phi_9 + 0.2\Phi_8 + 0.25\Phi_7 - 0.5\Phi_6 + 0.5\Phi_4 - \frac{26978}{132840}\Phi_2 - \\
 &\quad \frac{20500}{531360}\Phi_1 \\
 \theta_1 &= 20\Phi_{14} - 20\Phi_{13} + 20\Phi_4 - 40\Phi_3 + \frac{59200}{132840}\Phi_2 - \frac{54400}{531360}\Phi_1 \\
 \theta_2 &= 10\Phi_{13} - 10\Phi_4 - \frac{29600}{132840}\Phi_2 + \frac{32000}{531360}\Phi_1 \\
 \zeta &= 0.05\Phi_{12} + 0.0625\Phi_{11} + 0.1\Phi_{10} + 0.125\Phi_9 - 0.2\Phi_8 + 0.25\Phi_7 + 0.5\Phi_6 - \Phi_5 + 0.5\Phi_4 - \\
 &\quad \frac{40262}{132840}\Phi_2 + \frac{21566}{531360}\Phi_1 \\
 \eta &= -2\Phi_{14} + 2\Phi_{13} + 0.5\Phi_4 + \Phi_3 - \frac{31000}{132840}\Phi_2 + \frac{33680}{531360}\Phi_1
 \end{aligned}$$

Therefore $\mathcal{A}(\mathcal{SL}(2, 3^4)) = 531360\chi_1$

3.2. The results for the group $\mathcal{SL}(2, 3^6)$

The character table of rational representations for the group $\mathcal{SL}(2, 3^6)$ is

C_g	1	z	$c=d$	$zc=zd$	a	a^2	a^4	a^7	a^8	a^{13}	a^{14}	a^{28}
$ C_g $	1	1	265720	265720	532170	532170	532170	532170	532170	532170	532170	532170
$ C_G(g) $	387419760	387419760	1458	1458	728	728	728	728	728	728	728	728
1_G	1	1	1	1	1	1	1	1	1	1	1	1
ψ	729	729	0	0	1	1	1	1	1	1	1	1
χ_1	210240	-210240	288	-288	0	0	-1152	0	1152	0	0	0
χ_2	52560	52560	72	72	0	-144	144	0	0	0	864	1728
χ_4	26280	26280	36	36	-36	36	0	216	216	432	-216	-432
χ_7	35040	-35040	48	-48	0	0	192	288	-192	0	-384	0
χ_8	26280	26280	36	36	36	0	216	-216	-648	-432	-432	0
χ_{13}	17520	-17520	24	-24	0	0	96	0	-96	0	0	-576
χ_{14}	8760	8760	12	12	0	24	-24	-48	-48	0	0	-288
χ_{26}	4380	4380	6	6	0	12	-12	0	0	-72	-72	72
χ_{28}	4380	4380	6	6	6	-6	-12	0	-36	-72	-72	72
χ_{52}	2190	2190	3	3	3	-3	0	-18	-18	18	18	18
χ_{56}	4380	4380	6	6	-6	-12	-36	-72	-72	72	72	72
χ_{91}	2920	-2920	4	-4	0	0	-16	16	16	16	16	16
χ_{104}	2190	2190	3	3	-3	0	-18	18	18	18	18	18
χ_{182}	730	730	1	1	0	-2	2	2	2	2	2	2
θ_1	209664	-209664	-288	288	0	0	0	0	0	0	0	0
θ_2	104832	104832	-144	-144	0	0	0	0	0	0	0	0
θ_5	52416	-52416	-72	72	0	0	0	0	0	0	0	0
θ_{10}	26208	26208	-36	-36	0	0	0	0	0	0	0	0

θ_{73}	2912	-2912	-4	4	0	0	0	0	0	0	0	0
θ_{146}	1456	1456	-2	-2	0	0	0	0	0	0	0	0
ξ	730	730	1	1	-2	2	2	-2	2	-2	2	2
η	1456	-1456	-1	1	0	0	0	0	0	0	0	0

C_g	a^{28}	a^{52}	a^{56}	a^{91}	a^{104}	a^{182}	b^1	b^2	b^5	b^{10}	b^{73}	b^{146}
$ C_g $	532170	532170	532170	532170	532170	532170	530712	530712	530712	530712	530712	530712
$ C_G(g) $	728	728	728	728	728	728	730	730	730	730	730	730
1_G	1	1	1	1	1	1	1	1	1	1	1	1
ψ	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_1	6912	13824	-6912	0	-13824	0	0	0	0	0	0	0
χ_2	-864	-1738	-1738	0	0	-10368	0	0	0	0	0	0
χ_4	-432	0	-1296	-2592	-2592	2592	0	0	0	0	0	0
χ_7	0	-2304	-2304	2304	2304	2304	0	0	0	0	0	0
χ_8	-1296	-2592	-2592	2592	2592	2592	0	0	0	0	0	0
χ_{13}	-576	576	576	576	576	576	0	0	0	0	0	0
χ_{14}	-288	288	288	288	288	288	0	0	0	0	0	0
χ_{26}	72	72	72	72	72	72	0	0	0	0	0	0
χ_{28}	72	72	72	72	72	72	0	0	0	0	0	0
χ_{52}	18	18	18	18	18	18	0	0	0	0	0	0
χ_{56}	72	72	72	72	72	72	0	0	0	0	0	0
χ_{91}	16	16	16	16	16	16	0	0	0	0	0	0
χ_{104}	18	18	18	18	18	18	0	0	0	0	0	0
χ_{182}	2	2	2	2	2	2	0	0	0	0	0	0
θ_1	0	0	0	0	0	0	-288	-288	-1152	-1152	-20736	20736
θ_2	0	0	0	0	0	0	-144	576	576	1152	10368	20736
θ_5	0	0	0	0	0	0	-72	72	648	-648	5184	-5184
θ_{10}	0	0	0	0	0	0	36	72	-324	432	-2592	-2592
θ_{73}	0	0	0	0	0	0	-4	4	16	-16	-16	-16
θ_{146}	0	0	0	0	0	0	2	4	-8	-8	-8	-8
ξ	2	2	2	-2	2	2	0	0	0	0	0	0
η	0	0	0	0	0	0	4	-4	4	-4	4	-4

The Artincharacter table for the group $\mathcal{SL}(2, 3^6)$ is

C_g	1	z	$c=d$	$zc=zd$	a	a^2	a^4	a^7	a^8	a^{13}	a^{14}	a^{26}
$ C_g $	1	1	265720	265720	532170	532170	532170	532170	532170	532170	532170	532170
$ C_G(g) $	387419760	387419760	1458	1458	728	728	728	728	728	728	728	728
Φ_1	387419760	0	0	0	0	0	0	0	0	0	0	0
Φ_2	64569960	64569960	0	0	0	0	0	0	0	0	0	0
Φ_3	531440	0	2	0	0	0	0	0	0	0	0	0
Φ_4	531440	797160	2	2	0	0	0	0	0	0	0	0
Φ_5	532170	1064340	0	0	2	0	0	0	0	0	0	0
Φ_6	1064340	2128680	0	0	0	4	0	0	0	0	0	0
Φ_7	2128680	0	0	0	0	0	8	0	0	0	0	0
Φ_8	3725190	7450380	0	0	0	0	14	0	0	0	0	0
Φ_9	4257360	0	0	0	0	0	0	16	0	0	0	0
Φ_{10}	6918210	13836420	0	0	0	0	0	0	26	0	0	0
Φ_{11}	7450380	0	0	0	0	0	0	0	0	28	0	0
Φ_{12}	13836420	0	0	0	0	0	0	0	0	0	52	0
Φ_{13}	14900760	0	0	0	0	0	0	0	0	0	0	0
Φ_{14}	27672840	0	0	0	0	0	0	0	0	0	0	0
Φ_{15}	29801520	0	0	0	0	0	0	0	0	0	0	0
Φ_{16}	48427470	96854940	0	0	0	0	0	0	0	0	0	0
Φ_{17}	55345680	0	0	0	0	0	0	0	0	0	0	0
Φ_{18}	96854940	0	0	0	0	0	0	0	0	0	0	0
Φ_{19}	1061424	2122848	0	0	0	0	0	0	0	0	0	0
Φ_{20}	2122848	0	0	0	0	0	0	0	0	0	0	0
Φ_{21}	5307120	10614240	0	0	0	0	0	0	0	0	0	0
Φ_{22}	10614240	0	0	0	0	0	0	0	0	0	0	0
Φ_{23}	77483952	154967904	0	0	0	0	0	0	0	0	0	0
Φ_{24}	154967904	0	0	0	0	0	0	0	0	0	0	0

C_g	a^{38}	a^{52}	a^{56}	a^{91}	a^{104}	a^{182}	b^1	b^2	b^5	b^{10}	b^{73}	b^{146}
$ C_g $	532170	532170	532170	532170	532170	532170	530712	530712	530712	530712	530712	530712
$ C_G(q) $	728	728	728	728	728	728	730	730	730	730	730	730
Φ_1	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	0	0	0	0	0	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0	0	0	0	0	0
Φ_9	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{10}	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{11}	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{12}	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{13}	56	0	0	0	0	0	0	0	0	0	0	0
Φ_{14}	0	104	0	0	0	0	0	0	0	0	0	0
Φ_{15}	0	0	112	0	0	0	0	0	0	0	0	0
Φ_{16}	0	0	0	182	0	0	0	0	0	0	0	0
Φ_{17}	0	0	0	0	208	0	0	0	0	0	0	0
Φ_{18}	0	0	0	0	0	364	0	0	0	0	0	0
Φ_{19}	0	0	0	0	0	0	2	0	0	0	0	0
Φ_{20}	0	0	0	0	0	0	0	2	0	0	0	0
Φ_{21}	0	0	0	0	0	0	0	0	10	0	0	0
Φ_{22}	0	0	0	0	0	0	0	0	0	20	0	0
Φ_{23}	0	0	0	0	0	0	0	0	0	0	146	0
Φ_{24}	0	0	0	0	0	0	0	0	0	0	0	292

Hence, we written the rational valued characters in the first tables as a linear combination of induced characters in the second table

$$\begin{aligned}
 1 &= \frac{1}{292} \Phi_{24} + \frac{1}{146} \Phi_{23} + \frac{1}{20} \Phi_{22} + \frac{1}{10} \Phi_{21} + \frac{1}{2} \Phi_{20} + \frac{1}{2} \Phi_{19} + \frac{1}{364} \Phi_{18} + \frac{1}{208} \Phi_{17} + \frac{1}{182} \Phi_{16} + \frac{1}{112} \Phi_{15} + \\
 &\quad \frac{1}{104} \Phi_{14} + \frac{1}{56} \Phi_{13} + \frac{1}{52} \Phi_{12} + \frac{1}{28} \Phi_{11} + \frac{1}{26} \Phi_{10} + \frac{1}{16} \Phi_9 + \frac{1}{14} \Phi_8 + \frac{1}{8} \Phi_7 + \frac{1}{4} \Phi_6 + \frac{1}{2} \Phi_5 + \frac{1}{2} \Phi_4 - \\
 &\quad \frac{0.0966966838}{64569960} \Phi_2 + \frac{0.3200308497}{387419760} \Phi_1 \\
 \Psi &= -\frac{1}{292} \Phi_{24} - \frac{1}{146} \Phi_{23} - \frac{1}{20} \Phi_{22} - \frac{1}{10} \Phi_{21} - \frac{1}{2} \Phi_{20} - \frac{1}{2} \Phi_{19} + \frac{1}{364} \Phi_{18} + \frac{1}{208} \Phi_{17} + \frac{1}{182} \Phi_{16} + \\
 &\quad \frac{1}{112} \Phi_{15} + \frac{1}{104} \Phi_{14} + \frac{1}{56} \Phi_{13} + \frac{1}{52} \Phi_{12} + \frac{1}{28} \Phi_{11} + \frac{1}{26} \Phi_{10} + \frac{1}{16} \Phi_9 + \frac{1}{14} \Phi_8 + \frac{1}{8} \Phi_7 + \frac{1}{4} \Phi_6 + \frac{1}{2} \Phi_5 - \\
 &\quad \frac{8241.764477189857}{64569960} \Phi_2 - \frac{1.1236073802}{387419760} \Phi_1 \\
 \chi_1 &= -66.46154 \Phi_{17} - \frac{61.71429}{15.51786} \Phi_{15} - 132.923077 \Phi_{14} + 123.42857 \Phi_{13} + 72 \Phi_9 - 144 \Phi_7 - 144 \Phi_4 + 288 \Phi_3 + \\
 &\quad \frac{114580800}{64569960} \Phi_2 - \frac{18386930802050}{387419760} \Phi_1 \\
 \chi_2 &= -28.48352 \Phi_{18} - \frac{15.51786}{36 \Phi_6 + 36 \Phi_4 +} \Phi_{15} - 16.71154 \Phi_{14} - 15.42857 \Phi_{13} + 33.23077 \Phi_{12} + 30.85714 \Phi_{11} + 18 \Phi_7 - \\
 &\quad \frac{47987280}{64569960} \Phi_2 + \frac{8.6823907941}{387419760} \Phi_1 \\
 \chi_4 &= 7.12088 \Phi_{18} - 12.46154 \Phi_{17} - 14.24176 \Phi_{16} - 11.57143 \Phi_{15} - 7.71429 \Phi_{13} - 8.30769 \Phi_{12} - 7.71429 \Phi_{11} + 16.61538 \Phi_{10} + 13.5 \Phi_9 + 15.42858 \Phi_8 + 9 \Phi_6 - 18 \Phi_5 + 18 \Phi_4 - \\
 &\quad \frac{26.9251145625}{64569960} \Phi_2 + \frac{7.1480607026}{387419760} \Phi_1 \\
 \chi_7 &= 6.32967 \Phi_{18} + 11.7692 \Phi_{17} + 12.65934 \Phi_{16} - 20.57143 \Phi_{15} - 22.15385 \Phi_{14} - 13.71429 \Phi_{11} - 12 \Phi_9 + 20.57143 \Phi_8 + 24 \Phi_7 - 24 \Phi_4 + 48 \Phi_3 - \\
 &\quad \frac{21.0668829094}{64569960} \Phi_2 + \frac{1.6918333266}{387419760} \Phi_1 \\
 \chi_8 &= 7.12088 \Phi_{18} + 12.46154 \Phi_{17} + 14.24176 \Phi_{16} - 23.14296 \Phi_{15} - 24.92308 \Phi_{14} - 23.14286 \Phi_{13} - 15.42857 \Phi_{11} - 16.61538 \Phi_{10} - 40.5 \Phi_9 - 15.42857 \Phi_8 + 27 + 18 \Phi_5 + 18 \Phi_4 - \\
 &\quad \frac{20.1009395625}{64569960} \Phi_2 + \frac{3.1524636323}{387419760} \Phi_1 \\
 \chi_{13} &= 1.58242 \Phi_{18} + 2.76923 \Phi_{17} + 3.16484 \Phi_{16} + 5.14286 \Phi_{15} + 5.53846 \Phi_{14} - 10.28571 \Phi_{13} - 11.07692 \Phi_{12} - 6 \Phi_9 + 12 \Phi_7 - 12 \Phi_4 - 24 \Phi_3 + \\
 &\quad \frac{3171120}{64569960} \Phi_2 - \frac{0.8158105289}{387419760} \Phi_1 \\
 \chi_{14} &= 0.79121 \Phi_{18} + 1.38462 \Phi_{17} + 1.58242 \Phi_{16} + 2.57143 \Phi_{15} + 2.76923 \Phi_{14} - 5.14286 \Phi_{13} - 5.53846 \Phi_{12} - 3 \Phi_9 - 3.42857 \Phi_8 - 3 \Phi_7 + 6 \Phi_6 + 6 \Phi_4 - \\
 &\quad \frac{132495245}{64569960} \Phi_2 - \frac{75038410}{387419760} \Phi_1
 \end{aligned}$$

$$\begin{aligned}
& \chi_{26}=0.19780 \quad \Phi_{18}+0.34615\Phi_{17}+0.39560 \quad \Phi_{16}+0.64286 \quad \Phi_{15}+0.692308 \\
& \Phi_{14}-1.28571\Phi_{13}+1.38462\Phi_{12}-2.57143\Phi_{11}-2.76923\Phi_{10}-1.5\Phi_7+3\Phi_6+3 \\
& \Phi_4-\frac{0.135863874}{64569960}\Phi_2-\frac{1.1127652965}{387419760}\Phi_1 \\
& \chi_{28}=0.19780\Phi_{18}+0.34615\Phi_{17}+0.39560\Phi_{16}+0.64286 \quad \Phi_{15}+0.692308 \quad \Phi_{14}- \\
& 1.28571\Phi_{13}+1.38462\Phi_{12}-2.57143\Phi_{11}-2.76923\Phi_{10}-2.25 \quad \Phi_9-1.5\Phi_7+1.5\Phi_6+3\Phi_5+3 \\
& \Phi_4+\frac{1.1498372249}{64569960}\Phi_2-\frac{0.4100273676}{387419760}\Phi_1 \\
& \chi_{52}=0.49451 \quad \Phi_{18}+0.08654\Phi_{17}+0.09890 \quad \Phi_{16}+0.16071 \quad \Phi_{15}+0.17308 \\
& \Phi_{14}+0.32143\Phi_{13}+0.34615\Phi_{12}+0.64286\Phi_{11}+0.69231\Phi_{10}-1.125 \quad \Phi_9-1.28571\Phi_8- \\
& 0.75\Phi_6+1.5\Phi_5+1.5\Phi_4-\frac{0.1668355908}{64569960}\Phi_2-\frac{0.1609208234}{387419760}\Phi_1 \\
& \chi_{56}=0.19780 \quad \Phi_{18}+0.34615\Phi_{17}+0.39560 \quad \Phi_{16}+0.64286 \quad \Phi_{15}+0.692308 \\
& \Phi_{14}-1.28571\Phi_{13}+1.38462\Phi_{12}+2.57143\Phi_{11}+2.76923\Phi_{10}-4.5 \quad \Phi_9-5.14286\Phi_8-4.5\Phi_7-3\Phi_6- \\
& 3\Phi_5+3\Phi_4-\frac{0.4820170608}{64569960}\Phi_2-\frac{0.0844818731}{387419760}\Phi_1 \\
& \chi_{91}=0.04396 \quad \Phi_{18}+0.07692\Phi_{17}+0.08791 \quad \Phi_{16}+0.14286 \quad \Phi_{15}+0.15385 \\
& \Phi_{14}+0.28571\Phi_{13}+0.30769\Phi_{12}+0.57143\Phi_{11}+0.61538\Phi_{10}+\Phi_9+1.4286\Phi_8-2\Phi_7-2\Phi_4+4\Phi_3- \\
& \frac{0.3709544686}{64569960}\Phi_2-\frac{0.0508008205}{387419760}\Phi_1 \\
& \chi_{104}=0.49451 \quad \Phi_{18}+0.08654\Phi_{17}+0.09890 \quad \Phi_{16}+0.16071 \quad \Phi_{15}+0.17308 \\
& \Phi_{14}+0.32143\Phi_{13}+0.34615\Phi_{12}+0.64286\Phi_{11}+0.69231\Phi_{10}-1.125 \\
& \Phi_9+1.28571\Phi_8-2.25\Phi_7-1.5\Phi_5+1.5\Phi_4-\frac{0.1421103161}{64569960}\Phi_2-\frac{0.1753438179}{387419760}\Phi_1 \\
& \chi_{182}=0.00549\Phi_{18}+0.00962\Phi_{17}+0.01099\Phi_{16}+0.01786\Phi_{15}+0.01923\Phi_{14}+0.03571\Phi_{13}+0.03846 \\
& \Phi_{12}+0.7143\Phi_{11}+0.7692\Phi_{10}+0.125\Phi_9+0.14286\Phi_8+0.25\Phi_7-0.5\Phi_5+0.5\Phi_4-\frac{0.0391297208}{64569960}\Phi_2- \\
& \frac{0.0092718584}{387419760}\Phi_1 \\
& \theta_1=71.01369\Phi_{24}-142.02739\Phi_{23}-57.6\Phi_{22}-115.2\Phi_{21}-144\Phi_{20}-144\Phi_{19}+144\Phi_4-28\Phi_3+ \\
& \frac{345.7201583351}{64569960}\Phi_2-\frac{53.4348626927}{387419760}\Phi_1 \\
& \theta_2=71.01369\Phi_{24}+71.01369\Phi_{23}+57.6\Phi_{22}+57.6\Phi_{21}+288\Phi_{20}-72\Phi_{19}-144\Phi_4- \\
& \frac{175.7548245439}{64569960}\Phi_2-\frac{16.8699012443}{387419760}\Phi_1 \\
& \theta_5= \\
& -17.75342\Phi_{24}+35.50685\Phi_{23}-32.4\Phi_{22}+648\Phi_{21}+36\Phi_{20}-36\Phi_{19}-72\Phi_3-\frac{192.9213613597}{64569960}\Phi_2- \\
& \frac{40.3373349903}{387419760}\Phi_1 \\
& \theta_{10}= \\
& -8.87672\Phi_{24}-17.75342\Phi_{23}+21.6\Phi_{22}-32.4\Phi_{21}+36\Phi_{20}+18\Phi_{19}-36\Phi_4+\frac{47.7873049051}{64569960}\Phi_2- \\
& \frac{8.0788241774}{387419760}\Phi_1 \\
& \theta_{73}=-0.05479\Phi_{24}-0.10959\Phi_{23}-0.8\Phi_{22}+1.6\Phi_{21}+2\Phi_{20}-2\Phi_{19}-2\Phi_4-4\Phi_3- \\
& \frac{0.0411048327}{64569960}\Phi_2-\frac{0.0014069516}{387419760}\Phi_1 \\
& \theta_{146}=-0.02739\Phi_{24}-0.05479\Phi_{23}-0.4\Phi_{22}-0.8\Phi_{21}+2\Phi_{20}+\Phi_{19}-2\Phi_4+\frac{2.548400442}{64569960}\Phi_2- \\
& \frac{0.0095650435}{387419760}\Phi_1 \\
& \zeta=0.00549\Phi_{18}+0.00962\Phi_{17}- \\
& 0.01099\Phi_{16}+0.01786\Phi_{15}+0.01923\Phi_{14}+0.03571\Phi_{13}+0.03846\Phi_{12}+0.07142\Phi_{11}-0.07692\Phi_{10}+0. \\
& 125\Phi_9-1014286\Phi_8+0.25\Phi_7+0.5\Phi_6-\Phi_5+\Phi_4+\frac{0.0371173299}{64569960}\Phi_2-\frac{0.014421012}{387419760}\Phi_1
\end{aligned}$$

$$\eta = -0.01369\Phi_{24} + 0.02739\Phi_{23} + 0.2\Phi_{22} + 0.4\Phi_{21} - 2\Phi_{20} + 2\Phi_{19} + 0.5\Phi_4 - \Phi_3 - \frac{0.203438238}{64569960}\Phi_2 + \frac{0.0279008248}{387419760}\Phi_1$$

Therefore $\mathcal{A}(\mathcal{SL}(2, 3^6)) = 387419760\chi_1$

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