

# Artin Indicator for the Groups $SL(2,5^3)$ and $SL(2,5^5)$

Mohammed Salah Aldeen Zidan<sup>#1</sup>, Niran Sabah Jasim<sup>\*2</sup>

<sup>#1</sup>Ministry of Education, Directorate General of Education karkh 1, Baghdad, Iraq

<sup>\*2</sup> University of Baghdad, College of Education for Pure Science Ibn Al-Haitham, Department of Mathematics, Baghdad, Iraq

<sup>#1</sup>mohammed.salahaldeen1203a@ihcoedu.uobaghdad.edu.iq, <sup>\*2</sup>niraan.s.j@ihcoedu.uobaghdad.edu.iq

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## Abstract

In this work the Artin indicator for the groups  $SL(2,5^3)$  and  $SL(2,5^5)$  from the character table of rational representations and the induced characters table.

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## 1. Introduction

The group of all matrices of determinant 1 is  $\mathcal{SL}(n, F)$ , [1] and [2], searchers in [3] define the representation of the group. In this work we compute the Artin indicator for the groups  $\mathcal{SL}(2, 5^3)$  and  $\mathcal{SL}(2, 5^5)$ .

In this work we compute the Artin indicator for the groups  $\mathcal{SL}(2, 3^4)$  and  $\mathcal{SL}(2, 3^6)$ .

## 2. The Concepts

The relationships that we extracted above represent a complex analysis of all the components of technologies Theorem 2.1: [1]

$$|\mathcal{SL}(2, q^n)| = q^n (q^{2n} - 1).$$

**Definition 2.4:** [4] Let  $H$  be a cyclic subgroup of a group  $G$ , and  $\phi$  be a class function of  $H$ . Then

$$\phi \uparrow^G(g) = \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \phi(x_i)$$

The character induced from the principal character of cyclic subgroups of  $G$  is Artin character.

**Definition 2.5:** [5] The character induced from the principal character of a cyclic subgroups of  $G$  is called Artin character.

**Definition 2.6:** [5] Let  $G$  be a finite group and let  $\chi$  be any rational valued character on  $G$ . The smallest positive number  $n$  such that,

$$n\chi = \sum_c a_c \phi_c$$

where  $a_c \in \mathbb{Z}$  and  $\phi_c$  is Artin character, is called the Artin exponent of  $G$  and denoted by  $A(G)$ .

### 3. The Results

Authors in [4] and [7-8] study the character table of rational representations for the group  $\mathcal{SL}(2, p)$  we apply that idea and compute the character table of rational representations for the groups  $\mathcal{SL}(2, 3^4)$  and  $\mathcal{SL}(2, 3^6)$ . Also we apply the same idea in [4] and [6] to compute the Artin indicator for the same groups.

#### 3.1. The results for the group $\mathcal{SL}(2, 3^4)$

The character table of rational representations for the group  $\mathcal{SL}(2, 3^4)$  is

$C_g$	1	$z$	$c=d$	$zc=zd$	$a$	$a^2$	$a^4$	$a^{31}$
$ C_g $	1	1	7812	7812	15750	15750	15750	15750
$ C_G(g) $	1953000	1953000	250	250	124	124	124	124
$1_G$	1	1	1	1	1	1	1	1
$\psi$	125	125	0	0	1	1	1	1
$\chi_1$	7560	-7560	60	-60	0	4	-4	0
$\chi_2$	1890	1890	15	15	1	-1	0	-30
$\chi_4$	1890	1890	15	15	-1	0	0	30
$\chi_{31}$	252	-252	2	-2	0	-4	4	4
$\theta_1$	4464	-4464	-36	36	0	0	0	0
$\theta_2$	2232	2232	-81	-81	0	0	0	0
$\theta_3$	1488	-1488	-12	12	0	0	0	0
$\theta_6$	744	744	-6	-6	0	0	0	0
$\theta_7$	744	-744	-6	6	0	0	0	0
$\theta_9$	744	-744	-6	6	0	0	0	0
$\theta_{14}$	372	372	-3	-3	0	0	0	0
$\theta_{18}$	372	372	-3	-3	0	0	0	0
$\theta_{21}$	248	-248	-2	2	0	0	0	0
$\theta_{42}$	124	124	-1	-1	0	0	0	0
$\xi_5$	126	126	1	1	-2	2	2	-2
$\eta$	248	-248	-1	1	0	0	0	0

$C_g$	$b$	$b^2$	$b^3$	$b^6$	$b^7$	$b^9$	$b^{14}$	$b^{18}$	$b^{21}$	$b^{42}$
$ C_g $	15500	15500	15500	15500	15500	15500	15500	15500	15500	15500
$ C_G(g) $	126	126	126	126	126	126	126	126	126	126
$1_G$	1	1	1	1	1	1	1	1	1	1
$\psi$	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$\chi_1$	0	0	0	0	0	0	0	0	0	0
$\chi_2$	0	0	0	0	0	0	0	0	0	0
$\chi_4$	0	0	0	0	0	0	0	0	0	0
$\chi_{31}$	0	0	0	0	0	0	0	0	0	0
$\theta_1$	0	0	6	-6	0	-12	0	12	-36	36
$\theta_2$	0	0	-3	-9	0	6	9	0	18	36
$\theta_3$	2	-2	-4	4	-12	6	12	24	24	-24
$\theta_6$	-1	-3	2	0	6	12	12	-12	-12	-12
$\theta_7$	0	0	-6	6	6	12	-12	-12	-12	-12
$\theta_9$	-2	2	6	12	12	12	-12	-12	-12	-12
$\theta_{14}$	0	0	3	6	-6	-6	-6	-6	-6	-6
$\theta_{18}$	1	0	6	-6	-6	-6	-6	-6	-6	-6
$\theta_{21}$	-2	2	4	-4	-4	-4	-4	-4	-4	-4
$\theta_{42}$	1	2	-2	-2	-2	-2	-2	-2	-2	-2

$\xi$	0	0	0	0	0	0	0	0	0	0
$\eta$	4	-4	4	-4	4	4	-4	-4	4	-4

The Artin character table for the group  $\mathcal{SL}(2, 5^3)$  is

$C_g$	1	$z$	$c=d$	$zc=zd$	$a$	$a^2$	$a^4$	$a^{31}$
$ C_g $	1	1	7812	7812	15750	15750	15750	15750
$ C_G(g) $	1953000	1953000	250	250	124	124	124	124
$\Phi_1$	1953000	0	0	0	0	0	0	0
$\Phi_2$	390600	390600	0	0	0	0	0	0
$\Phi_3$	15624	0	2	0	0	0	0	0
$\Phi_4$	15624	7812	3	3	0	0	0	0
$\Phi_5$	15750	31500	0	0	2	0	0	0
$\Phi_6$	31500	63000	0	0	0	4	0	0
$\Phi_7$	63000	0	0	0	0	0	8	0
$\Phi_8$	488250	976500	0	0	0	0	0	62
$\Phi_9$	31000	62000	0	0	0	0	0	0
$\Phi_{10}$	62000	0	0	0	0	0	0	0
$\Phi_{11}$	93000	186000	0	0	0	0	0	0
$\Phi_{12}$	186000	0	0	0	0	0	0	0
$\Phi_{13}$	217000	434000	0	0	0	0	0	0
$\Phi_{14}$	279000	558000	0	0	0	0	0	0
$\Phi_{15}$	434000	0	0	0	0	0	0	0
$\Phi_{16}$	558000	0	0	0	0	0	0	0
$\Phi_{17}$	651000	1302000	0	0	0	0	0	0
$\Phi_{18}$	1302000	0	0	0	0	0	0	0

$C_g$	$b$	$b^2$	$b^3$	$b^6$	$b^7$	$b^9$	$b^{14}$	$b^{18}$	$b^{21}$	$b^{42}$
$ C_g $	15500	15500	15500	15500	15500	15500	15500	15500	15500	15500
$ C_G(g) $	126	126	126	126	126	126	126	126	126	126
$\Phi_1$	0	0	0	0	0	0	0	0	0	0
$\Phi_2$	0	0	0	0	0	0	0	0	0	0
$\Phi_3$	0	0	0	0	0	0	0	0	0	0
$\Phi_4$	0	0	0	0	0	0	0	0	0	0
$\Phi_5$	0	0	0	0	0	0	0	0	0	0
$\Phi_6$	0	0	0	0	0	0	0	0	0	0
$\Phi_7$	0	0	0	0	0	0	0	0	0	0
$\Phi_8$	0	0	0	0	0	0	0	0	0	0
$\Phi_9$	2	0	0	0	0	0	0	0	0	0
$\Phi_{10}$	0	2	0	0	0	0	0	0	0	0
$\Phi_{11}$	0	0	6	0	0	0	0	0	0	0
$\Phi_{12}$	0	0	0	12	0	0	0	0	0	0
$\Phi_{13}$	0	0	0	0	14	0	0	0	0	0
$\Phi_{14}$	0	0	0	0	0	18	0	0	0	0
$\Phi_{15}$	0	0	0	0	0	0	28	0	0	0
$\Phi_{16}$	0	0	0	0	0	0	0	36	0	0
$\Phi_{17}$	0	0	0	0	0	0	0	0	42	0
$\Phi_{18}$	0	0	0	0	0	0	0	0	0	84

Hence, we written the rational valued characters in the first tables as a linear combination of induced characters in the second table

$$\begin{aligned}
 l &= \frac{1}{84}\Phi_{18} + \frac{1}{42}\Phi_{17} + \frac{1}{36}\Phi_{16} + \frac{1}{28}\Phi_{15} + \frac{1}{18}\Phi_{14} + \frac{1}{14}\Phi_{13} + \frac{1}{14}\Phi_{12} + \frac{1}{12}\Phi_{11} + \frac{1}{6}\Phi_{10} + \frac{1}{2}\Phi_9 + \frac{1}{2}\Phi_8 + \frac{1}{62}\Phi_7 + \frac{1}{8}\Phi_6 + \frac{1}{4}\Phi_5 + \frac{1}{2}\Phi_4 + \frac{1}{3}\Phi_3 \\
 &\quad - \frac{20453}{390600}\Phi_2 + \frac{5646}{1953000}\Phi_1 \\
 \Psi &= -\frac{1}{84}\Phi_{18} - \frac{1}{42}\Phi_{17} - \frac{1}{36}\Phi_{16} - \frac{1}{28}\Phi_{15} - \frac{1}{18}\Phi_{14} - \frac{1}{14}\Phi_{13} - \frac{1}{12}\Phi_{12} - \frac{1}{6}\Phi_{11} - \frac{1}{2}\Phi_{10} - \frac{1}{2}\Phi_9 + \frac{1}{62}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + \frac{107875}{390600} \\
 &\quad \Phi_2 + \frac{247000}{1953000}\Phi_1 \\
 \chi_1 &= -\frac{1}{2}\Phi_7 + \Phi_6 - 20\Phi_4 + 60\Phi_3 + \frac{85680}{390600}\Phi_2 - \frac{703080}{1953000}\Phi_1 \\
 \chi_2 &= -0.48387\Phi_8 - \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 + 5\Phi_4 - 1.1145161290066\Phi_2 - 0.3048387096671\Phi_1 \\
 \chi_4 &= 0.48387\Phi_8 - \frac{1}{2}\Phi_5 + 5\Phi_4 - 1.2645135816692\Phi_2 - 0.4088702163338\Phi_1 \\
 \chi_{31} &= 0.06452\Phi_8 + \frac{1}{2}\Phi_7 - \Phi_6 - 0.66667\Phi_4 + 2\Phi_3 + 0.012683681618\Phi_2 - 0.241308714183\Phi_1 \\
 \theta_1 &= 0.42857\Phi_{18} - 0.85714\Phi_{17} + \frac{1}{3}\Phi_{16} - 0.66667\Phi_{14} - \frac{1}{2}\Phi_{12} + \Phi_{11} + 12\Phi_4 - 36\Phi_3 + 3.3129\Phi_2 + 0.8586713108038 \\
 &\quad \Phi_1 \\
 \theta_2 &= 0.42857\Phi_{18} + 0.42857\Phi_{17} + 0.32143\Phi_{15} + \frac{1}{3}\Phi_{14} - 0.75\Phi_{12} - \frac{1}{2}\Phi_{11} - 6\Phi_3 - 0.3757453661034\Phi_2 \\
 &\quad - 0.1757018381976\Phi_1 \\
 \theta_3 &= -0.28571\Phi_{18} + 0.57143\Phi_{17} + 0.66667\Phi_{16} + 0.42857\Phi_{15} + \frac{1}{3}\Phi_{14} - 0.85714\Phi_{13} + \frac{1}{3}\Phi_{12} - 0.66667\Phi_{11} - \Phi_{10} + \Phi_9 + \\
 &\quad 4\Phi_4 - 12\Phi_3 - 0.8774626984126\Phi_2 - 0.2530801894521\Phi_1 \\
 \theta_6 &= -\frac{1}{7}\Phi_{18} - 0.28571\Phi_{17} \\
 &\quad - \frac{1}{3}\Phi_{16} + 0.42857\Phi_{15} + 0.66667\Phi_{14} + 0.42857\Phi_{13} + \frac{1}{3}\Phi_{11} - 1.5\Phi_{10} - \frac{1}{2}\Phi_9 - 2\Phi_3 - 0.1166841269841\Phi_2 + 0.074199 \\
 &\quad 8617511\Phi_1 \\
 \theta_7 &= -\frac{1}{7}\Phi_{18} - 0.28571\Phi_{17} - \frac{1}{3}\Phi_{16} - 0.42857\Phi_{15} + 0.66667\Phi_{14} + 0.42857\Phi_{13} + \frac{1}{2}\Phi_{12} - \Phi_{11} + 2\Phi_4 - 6\Phi_3 - 0.04122 \\
 &\quad 2222222\Phi_2 + 0.2559452585765\Phi_1 \\
 \theta_9 &= -\frac{1}{7}\Phi_{18} - 0.28571\Phi_{17} - \frac{1}{3}\Phi_{16} - 0.42857\Phi_{15} + 0.66667\Phi_{14} + 0.85714\Phi_{13} + \Phi_{12} + \Phi_{11} + \Phi_{10} - \Phi_9 + 2\Phi_4 - 6\Phi_3 - 1. \\
 &\quad 3517618023553\Phi_2 - 0.3076094828469\Phi_1 \\
 \theta_{14} &= \frac{1}{14}\Phi_{18} - \frac{1}{7}\Phi_{17} - \frac{1}{6}\Phi_{16} - 0.21429\Phi_{15} - \frac{1}{3}\Phi_{14} - 0.42857\Phi_{13} + \frac{1}{2}\Phi_{12} + \frac{1}{2}\Phi_{11} - \Phi_4 - 1.5\Phi_3 + 1.2114269841269\Phi_2 \\
 &\quad + 0.448751176472\Phi_1 \\
 \theta_{18} &= -\frac{1}{14}\Phi_{18} - \frac{1}{7}\Phi_{17} - \frac{1}{6}\Phi_{16} - 0.21429\Phi_{15} - \frac{1}{3}\Phi_{14} - 0.42857\Phi_{13} - \frac{1}{2}\Phi_{12} + \Phi_{11} + \frac{1}{2}\Phi_9 - \Phi_4 - 1.5\Phi_3 + 0.893966666666 \\
 &\quad 66\Phi_2 + 0.448751176472\Phi_1 \\
 \theta_{21} &= -\frac{1}{21}\Phi_{18} - 0.9524\Phi_{17} - \frac{1}{9}\Phi_{16} - \\
 &\quad \frac{1}{7}\Phi_{15} - 0.22222\Phi_{14} - 0.28571\Phi_{13} - \frac{1}{3}\Phi_{12} + 0.66667\Phi_{11} + \Phi_{10} - \Phi_9 + 0.66667\Phi_4 - 2\Phi_3 + 0.779679298259\Phi_2 + 1.52 \\
 &\quad 77120286123\Phi_1 \\
 \theta_{42} &= -\frac{1}{42}\Phi_{18} - \frac{1}{21}\Phi_{17} - \frac{1}{18}\Phi_{16} - \frac{1}{14}\Phi_{15} - \frac{1}{9}\Phi_{14} - \frac{1}{7}\Phi_{13} - \frac{1}{6}\Phi_{12} - \frac{1}{3}\Phi_{11} + \Phi_{10} + \frac{1}{2}\Phi_9 - \frac{1}{3}\Phi_4 + 0.5625396825396\Phi_2 + 0.20 \\
 &\quad 25396825396\Phi_1 \\
 \zeta &= -\frac{1}{31}\Phi_8 + \frac{1}{4}\Phi_7 + \frac{1}{2}\Phi_6 - \Phi_5 + \frac{1}{3}\Phi_4 + 0.743010752688\Phi_2 + 0.0122580645161\Phi_1
 \end{aligned}$$

$$\eta = -\frac{1}{21}\Phi_{18} + 0.9524\Phi_{17} - \frac{1}{9}\Phi_{16} - \frac{1}{7}\Phi_{15} + 0.22222\Phi_{14} + 0.28571\Phi_{13} - \frac{1}{3}\Phi_{12} + 0.66667\Phi_{11} - 2\Phi_{10} + 2\Phi_9 + \frac{1}{3}\Phi_4 - \Phi_3 - 4.4518031746031\Phi_2 - 1.1388742857142\Phi_1$$

Therefore  $\mathcal{A}(\mathcal{SL}(2, 5^3)) = 1953000\chi_1$

### 3.2. The results for the group $\mathcal{SL}(2, 3^6)$

The character table of rational representations for the group  $\mathcal{SL}(2, 3^6)$  is

$C_g$	1	$z$	$c=d$	$zc=zd$	$a$	$a^2$	$a^4$	$a^7$	$a^8$	$a^{13}$	$a^{14}$	$a^{26}$
$ C_g $	1	1	265720	265720	532170	532170	532170	532170	532170	532170	532170	532170
$ C_G(g) $	387419760	387419760	1458	1458	728	728	728	728	728	728	728	728
$I_G$	1	1	1	1	1	1	1	1	1	1	1	1
$\psi$	729	729	0	0	1	1	1	1	1	1	1	1
$\chi_1$	210240	-210240	288	-288	0	0	-1152	0	1152	0	0	0
$\chi_2$	52560	52560	72	72	0	-144	144	0	0	0	864	1728
$\chi_4$	26280	26280	36	36	-36	36	0	216	216	432	-216	-432
$\chi_7$	35040	-35040	48	-48	0	0	192	288	-192	0	-384	0
$\chi_8$	26280	26280	36	36	36	0	216	-216	-648	-432	-432	0
$\chi_{13}$	17520	-17520	24	-24	0	0	96	0	-96	0	0	-576
$\chi_{14}$	8760	8760	12	12	0	24	-24	-48	-48	0	0	-288
$\chi_{26}$	4380	4380	6	6	0	12	-12	0	0	-72	-72	72
$\chi_{28}$	4380	4380	6	6	6	-6	-12	0	-36	-72	-72	72
$\chi_{52}$	2190	2190	3	3	3	-3	0	-18	-18	18	18	18
$\chi_{56}$	4380	4380	6	6	-6	-12	-36	-72	-72	72	72	72
$\chi_{91}$	2920	-2920	4	-4	0	0	-16	16	16	16	16	16
$\chi_{104}$	2190	2190	3	3	-3	0	-18	18	18	18	18	18
$\chi_{182}$	730	730	1	1	0	-2	2	2	2	2	2	2
$\theta_1$	209664	-209664	-288	288	0	0	0	0	0	0	0	0
$\theta_2$	104832	104832	-144	-144	0	0	0	0	0	0	0	0
$\theta_5$	52416	-52416	-72	72	0	0	0	0	0	0	0	0
$\theta_{10}$	26208	26208	-36	-36	0	0	0	0	0	0	0	0
$\theta_{73}$	2912	-2912	-4	4	0	0	0	0	0	0	0	0
$\theta_{146}$	1456	1456	-2	-2	0	0	0	0	0	0	0	0
$\xi$	730	730	1	1	-2	2	2	-2	2	-2	2	2
$\eta$	1456	-1456	-1	1	0	0	0	0	0	0	0	0

$C_g$	$a^{28}$	$a^{52}$	$a^{56}$	$a^{91}$	$a^{104}$	$a^{182}$	$b^1$	$b^2$	$b^5$	$b^{10}$	$b^{73}$	$b^{146}$
$ C_g $	532170	532170	532170	532170	532170	532170	530712	530712	530712	530712	530712	530712
$ C_G(g) $	728	728	728	728	728	728	730	730	730	730	730	730
$I_G$	1	1	1	1	1	1	1	1	1	1	1	1
$\psi$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
$\chi_1$	6912	13824	-6912	0	-13824	0	0	0	0	0	0	0
$\chi_2$	-864	-1738	-1738	0	0	-10368	0	0	0	0	0	0
$\chi_4$	-432	0	-1296	-2592	-2592	2592	0	0	0	0	0	0
$\chi_7$	0	-2304	-2304	2304	2304	2304	0	0	0	0	0	0
$\chi_8$	-1296	-2592	-2592	2592	2592	2592	0	0	0	0	0	0
$\chi_{13}$	-576	576	576	576	576	576	0	0	0	0	0	0
$\chi_{14}$	-288	288	288	288	288	288	0	0	0	0	0	0
$\chi_{26}$	72	72	72	72	72	72	0	0	0	0	0	0
$\chi_{28}$	72	72	72	72	72	72	0	0	0	0	0	0
$\chi_{52}$	18	18	18	18	18	18	0	0	0	0	0	0
$\chi_{56}$	72	72	72	72	72	72	0	0	0	0	0	0
$\chi_{91}$	16	16	16	16	16	16	0	0	0	0	0	0
$\chi_{104}$	18	18	18	18	18	18	0	0	0	0	0	0
$\chi_{182}$	2	2	2	2	2	2	0	0	0	0	0	0
$\theta_1$	0	0	0	0	0	0	-288	-288	-1152	-1152	-20736	20736
$\theta_2$	0	0	0	0	0	0	-144	576	576	1152	10368	20736
$\theta_5$	0	0	0	0	0	0	-72	72	648	-648	5184	-5184
$\theta_{10}$	0	0	0	0	0	0	36	72	-324	432	-2592	-2592
$\theta_{73}$	0	0	0	0	0	0	-4	4	16	-16	-16	-16
$\theta_{146}$	0	0	0	0	0	0	2	4	-8	-8	-8	-8
$\xi$	2	2	2	-2	2	2	0	0	0	0	0	0

$\eta$	0	0	0	0	0	0	4	-4	4	-4	4	-4
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The Artin character table for the group  $\mathcal{SL}(2, 3^6)$  is

$C$	1	$z$	$c=d$	$zc=zd$	$a$	$a^2$	$a^4$	$a^7$	$a^8$	$a^{13}$	$a^{14}$	$a^{26}$
$ C_g $	1	1	265720	265720	532170	532170	532170	532170	532170	532170	532170	532170
$ C_C(g) $	387419760	387419760	1458	1458	728	728	728	728	728	728	728	728
$\Phi_1$	387419760	0	0	0	0	0	0	0	0	0	0	0
$\Phi_2$	64569960	64569960	0	0	0	0	0	0	0	0	0	0
$\Phi_3$	531440	0	2	0	0	0	0	0	0	0	0	0
$\Phi_4$	531440	797160	2	2	0	0	0	0	0	0	0	0
$\Phi_5$	532170	1064340	0	0	2	0	0	0	0	0	0	0
$\Phi_6$	1064340	2128680	0	0	0	4	0	0	0	0	0	0
$\Phi_7$	2128680	0	0	0	0	0	8	0	0	0	0	0
$\Phi_8$	3725190	7450380	0	0	0	0	0	14	0	0	0	0
$\Phi_9$	4257360	0	0	0	0	0	0	0	16	0	0	0
$\Phi_{10}$	6918210	13836420	0	0	0	0	0	0	0	26	0	0
$\Phi_{11}$	7450380	0	0	0	0	0	0	0	0	0	28	0
$\Phi_{12}$	13836420	0	0	0	0	0	0	0	0	0	0	52
$\Phi_{13}$	14900760	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{14}$	27672840	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{15}$	29801520	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{16}$	48427470	96854940	0	0	0	0	0	0	0	0	0	0
$\Phi_{17}$	55345680	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{18}$	96854940	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{19}$	1061424	2122848	0	0	0	0	0	0	0	0	0	0
$\Phi_{20}$	2122848	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{21}$	5307120	10614240	0	0	0	0	0	0	0	0	0	0
$\Phi_{22}$	10641240	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{23}$	77483952	154967904	0	0	0	0	0	0	0	0	0	0
$\Phi_{24}$	154967904	0	0	0	0	0	0	0	0	0	0	0

$g$	$a^{28}$	$a^{52}$	$a^{56}$	$a^{91}$	$a^{104}$	$a^{182}$	$b^1$	$b^2$	$b^5$	$b^{10}$	$b^{73}$	$b^{146}$
$ C_g $	532170	532170	532170	532170	532170	532170	530712	530712	530712	530712	530712	530712
$ C_C(g) $	728	728	728	728	728	728	730	730	730	730	730	730
$\Phi_1$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_2$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_3$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_4$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_5$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_6$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_7$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_8$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_9$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{10}$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{11}$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{12}$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{13}$	56	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{14}$	0	104	0	0	0	0	0	0	0	0	0	0
$\Phi_{15}$	0	0	112	0	0	0	0	0	0	0	0	0
$\Phi_{16}$	0	0	0	182	0	0	0	0	0	0	0	0
$\Phi_{17}$	0	0	0	0	208	0	0	0	0	0	0	0
$\Phi_{18}$	0	0	0	0	0	364	0	0	0	0	0	0
$\Phi_{19}$	0	0	0	0	0	0	2	0	0	0	0	0
$\Phi_{20}$	0	0	0	0	0	0	0	2	0	0	0	0
$\Phi_{21}$	0	0	0	0	0	0	0	0	10	0	0	0
$\Phi_{22}$	0	0	0	0	0	0	0	0	0	20	0	0
$\Phi_{23}$	0	0	0	0	0	0	0	0	0	0	146	0
$\Phi_{24}$	0	0	0	0	0	0	0	0	0	0	0	292

Hence, we written the rational valued characters in the first tables as a linear combination of induced characters in the second table

$$1 = \frac{1}{292} \Phi_{24} + \frac{1}{146} \Phi_{23} + \frac{1}{20} \Phi_{22} + \frac{1}{10} \Phi_{21} + \frac{1}{2} \Phi_{20} + \frac{1}{2} \Phi_{19} + \frac{1}{364} \Phi_{18} + \frac{1}{208} \Phi_{17} + \frac{1}{182} \Phi_{16} + \frac{1}{112} \Phi_{15} + \frac{1}{104} \Phi_{14} + \frac{1}{56} \Phi_{13} + \frac{1}{52} \Phi_{12} + \frac{1}{28} \Phi_{11} + \frac{1}{26} \Phi_{10} + \frac{1}{16} \Phi_9 + \frac{1}{14} \Phi_8 + \frac{1}{8} \Phi_7 + \frac{1}{4} \Phi_6 + \frac{1}{2} \Phi_5 + \frac{1}{2} \Phi_4 - \frac{0.0966966838}{64569960} \Phi_2 + \frac{0.3200308497}{387419760} \Phi_1$$

$$\Psi = -\frac{1}{292}\Phi_{24} - \frac{1}{146}\Phi_{23} - \frac{1}{20}\Phi_{22} - \frac{1}{10}\Phi_{21} - \frac{1}{2}\Phi_{20} - \frac{1}{2}\Phi_{19} + \frac{1}{364}\Phi_{18} + \frac{1}{208}\Phi_{17} + \frac{1}{182}\Phi_{16} + \frac{1}{112}\Phi_{15} +$$

$$\frac{1}{104}\Phi_{14} + \frac{1}{56}\Phi_{13} + \frac{1}{52}\Phi_{12} + \frac{1}{28}\Phi_{11} + \frac{1}{26}\Phi_{10} + \frac{1}{16}\Phi_9 + \frac{1}{14}\Phi_8 + \frac{1}{8}\Phi_7 + \frac{1}{4}\Phi_6 + \frac{1}{2}\Phi_5 -$$

$$\frac{8241.764477189857}{64569960}\Phi_2 - \frac{1.1236073802}{387419760}\Phi_1$$

$$\chi_1 = -66.46154\Phi_{17} - 61.71429\Phi_{15} - 132.923077\Phi_{14} + 123.42857\Phi_{13} + 72\Phi_9 - 144\Phi_7 - 144\Phi_4 + 288\Phi_3 + \frac{114580800}{64569960}\Phi_2 - \frac{18386930802050}{387419760}\Phi_1$$

$$\chi_2 = -28.48352\Phi_{18} - 15.51786\Phi_{15} - 16.71154\Phi_{14} - 15.42857\Phi_{13} + 33.23077\Phi_{12} + 30.85714\Phi_{11} + 18\Phi_7 - 36\Phi_6 + 36\Phi_4 + \frac{47987280}{64569960}\Phi_2 +$$

$$\frac{8.6823907941}{387419760}\Phi_1$$

$$\chi_4 = 7.12088\Phi_{18} - 12.46154\Phi_{17} - 14.24176\Phi_{16} - 11.57143\Phi_{15} - 7.71429\Phi_{13} - 8.30769\Phi_{12} - 7.71429\Phi_{11} + 16.61538\Phi_{10} + 13.5\Phi_9 + 15.42858\Phi_8 + 9\Phi_6 - 18\Phi_5 + 18\Phi_4 -$$

$$\frac{26.9251145625}{64569960}\Phi_2 + \frac{7.1480607026}{387419760}\Phi_1$$

$$\chi_7 = 6.32967\Phi_{18} + 11.7692\Phi_{17} + 12.65934\Phi_{16} - 20.57143\Phi_{15} - 22.15385\Phi_{14} - 13.71429\Phi_{11} - 12\Phi_9 + 20.57143\Phi_8 + 24\Phi_7 - 24\Phi_4 + 48\Phi_3 - \frac{21.0668829094}{64569960}\Phi_2 + \frac{1.6918333266}{387419760}\Phi_1$$

$$\chi_8 = 7.12088\Phi_{18} + 12.46154\Phi_{17} + 14.24176\Phi_{16} - 23.14296\Phi_{15} - 24.92308\Phi_{14} - 23.14286\Phi_{13} - 15.42857\Phi_{11} - 16.61538\Phi_{10} - 40.5\Phi_9 - 15.42857\Phi_8 + 27$$

$$+ 18\Phi_5 + 18\Phi_4 - \frac{20.1009395625}{64569960}\Phi_2 + \frac{3.1524636323}{387419760}\Phi_1$$

$$\chi_{13} = 1.58242\Phi_{18} + 2.76923\Phi_{17} + 3.16484\Phi_{16} + 5.14286\Phi_{15} + 5.53846\Phi_{14} - 10.28571\Phi_{13} - 11.07692\Phi_{12} - 6\Phi_9 + 12\Phi_7 - 12\Phi_4 - 24\Phi_3 + \frac{3171120}{64569960}\Phi_2 - \frac{0.8158105289}{387419760}\Phi_1$$

$$\chi_{14} = 0.79121\Phi_{18} + 1.38462\Phi_{17} + 1.58242\Phi_{16} + 2.57143\Phi_{15} + 2.76923\Phi_{14} - 5.14286\Phi_{13} - 5.53846\Phi_{12} - 3\Phi_9 - 3.42857\Phi_8 - 3\Phi_7 + 6\Phi_6 + 6\Phi_4 - \frac{132495245}{64569960}\Phi_2 - \frac{75038410}{387419760}\Phi_1$$

$$\chi_{26} = 0.19780\Phi_{18} + 0.34615\Phi_{17} + 0.39560\Phi_{16} + 0.64286\Phi_{15} + 0.692308\Phi_{14} - 1.28571\Phi_{13} + 1.38462\Phi_{12} - 2.57143\Phi_{11} - 2.76923\Phi_{10} - 1.5\Phi_7 + 3\Phi_6 + 3\Phi_4 - \frac{0.135863874}{64569960}\Phi_2 -$$

$$\frac{1.1127652965}{387419760}\Phi_1$$

$$\chi_{28} = 0.19780\Phi_{18} + 0.34615\Phi_{17} + 0.39560\Phi_{16} + 0.64286\Phi_{15} + 0.692308\Phi_{14} - 1.28571\Phi_{13} + 1.38462\Phi_{12} - 2.57143\Phi_{11} - 2.76923\Phi_{10} - 2.25\Phi_9 - 1.5\Phi_7 + 1.5\Phi_6 + 3\Phi_5 + 3\Phi_4 +$$

$$\frac{1.1498372249}{64569960}\Phi_2 - \frac{0.4100273676}{387419760}\Phi_1$$

$$\chi_{52} = 0.49451\Phi_{18} + 0.08654\Phi_{17} + 0.09890\Phi_{16} + 0.16071\Phi_{15} + 0.17308\Phi_{14} + 0.32143\Phi_{13} + 0.34615\Phi_{12} + 0.64286\Phi_{11} + 0.69231\Phi_{10} - 1.125\Phi_9 - 1.28571\Phi_8 -$$

$$0.75\Phi_6 + 1.5\Phi_5 + 1.5\Phi_4 - \frac{0.1668355908}{64569960}\Phi_2 - \frac{0.1609208234}{387419760}\Phi_1$$

$$\chi_{56} = 0.19780\Phi_{18} + 0.34615\Phi_{17} + 0.39560\Phi_{16} + 0.64286\Phi_{15} + 0.692308\Phi_{14} - 1.28571\Phi_{13} + 1.38462\Phi_{12} + 2.57143\Phi_{11} + 2.76923\Phi_{10} - 4.5\Phi_9 - 5.14286\Phi_8 - 4.5\Phi_7 - 3\Phi_6 -$$

$$3\Phi_5 + 3\Phi_4 - \frac{0.4820170608}{64569960}\Phi_2 - \frac{0.0844818731}{387419760}\Phi_1$$

$$\chi_{91} = 0.04396\Phi_{18} + 0.07692\Phi_{17} + 0.08791\Phi_{16} + 0.14286\Phi_{15} + 0.15385\Phi_{14} + 0.28571\Phi_{13} + 0.30769\Phi_{12} + 0.57143\Phi_{11} + 0.61538\Phi_{10} + \Phi_9 + 1.4286\Phi_8 - 2\Phi_7 - 2\Phi_4 + 4\Phi_3 -$$

$$\frac{0.3709544686}{64569960}\Phi_2 - \frac{0.0508008205}{387419760}\Phi_1$$

$$\begin{aligned} \chi_{104} &= 0.49451 & \Phi_{18} + 0.08654\Phi_{17} + 0.09890 & & \Phi_{16} + 0.16071 & & \Phi_{15} + 0.17308 \\ \Phi_{14} & + 0.32143\Phi_{13} + 0.34615\Phi_{12} + 0.64286\Phi_{11} + 0.69231\Phi_{10} - 1.125 \\ \Phi_9 & + 1.28571\Phi_8 - 2.25\Phi_7 - 1.5\Phi_5 + 1.5\Phi_4 - \frac{0.1421103161}{64569960}\Phi_2 - \frac{0.1753438179}{387419760}\Phi_1 \\ \chi_{182} & = 0.00549\Phi_{18} + 0.00962\Phi_{17} + 0.01099\Phi_{16} + 0.01786\Phi_{15} + 0.01923\Phi_{14} + 0.03571\Phi_{13} + 0.03846\Phi_{12} + 0.7 \\ & 143\Phi_{11} + 0.7692\Phi_{10} + 0.125\Phi_9 + 0.14286\Phi_8 + 0.25\Phi_7 - 0.5\Phi_5 + 0.5\Phi_4 - \frac{0.0391297208}{64569960}\Phi_2 - \frac{0.0092718584}{387419760}\Phi_1 \\ \theta_1 & = 71.01369\Phi_{24} - 142.02739\Phi_{23} - 57.6\Phi_{22} - 115.2\Phi_{21} - 144\Phi_{20} - 144\Phi_{19} + 144\Phi_4 - 28\Phi_3 + \\ & \frac{345.7201583351}{64569960}\Phi_2 - \frac{53.4348626927}{387419760}\Phi_1 \\ \theta_2 & = 71.01369\Phi_{24} + 71.01369\Phi_{23} + 57.6\Phi_{22} + 57.6\Phi_{21} + 288\Phi_{20} - 72\Phi_{19} - 144\Phi_4 - \\ & \frac{175.7548245439}{64569960}\Phi_2 - \frac{16.8699012443}{387419760}\Phi_1 \\ \theta_5 & = -17.75342\Phi_{24} + 35.50685\Phi_{23} - 32.4\Phi_{22} + 648\Phi_{21} + 36\Phi_{20} - 36\Phi_{19} - 72\Phi_3 - \frac{192.9213613597}{64569960}\Phi_2 - \\ & \frac{40.3373349903}{387419760}\Phi_1 \\ \theta_{10} & = -8.87672\Phi_{24} - 17.75342\Phi_{23} + 21.6\Phi_{22} - 32.4\Phi_{21} + 36\Phi_{20} + 18\Phi_{19} - 36\Phi_4 + \frac{47.7873049051}{64569960}\Phi_2 - \\ & \frac{8.0788241774}{387419760}\Phi_1 \\ \theta_{73} & = -0.05479\Phi_{24} - 0.10959\Phi_{23} - 0.8\Phi_{22} + 1.6\Phi_{21} + 2\Phi_{20} - 2\Phi_{19} - 2\Phi_4 - 4\Phi_3 - \frac{0.0411048327}{64569960}\Phi_2 - \\ & \frac{0.0014069516}{387419760}\Phi_1 \\ \theta_{146} & = -0.02739\Phi_{24} - 0.05479\Phi_{23} - 0.4\Phi_{22} - 0.8\Phi_{21} + 2\Phi_{20} + \Phi_{19} - 2\Phi_4 + \frac{2.548400442}{64569960}\Phi_2 - \\ & \frac{0.0095650435}{387419760}\Phi_1 \\ \zeta & = 0.00549\Phi_{18} + 0.00962\Phi_{17} - \\ & 0.01099\Phi_{16} + 0.01786\Phi_{15} + 0.01923\Phi_{14} + 0.03571\Phi_{13} + 0.03846\Phi_{12} + 0.07142\Phi_{11} - 0.07692\Phi_{10} + 0.125 \\ & \Phi_9 - 1014286\Phi_8 + 0.25\Phi_7 + 0.5\Phi_6 - \Phi_5 + \Phi_4 + \frac{0.0371173299}{64569960}\Phi_2 - \frac{0.014421012}{387419760}\Phi_1 \\ \eta & = -0.01369\Phi_{24} + 0.02739\Phi_{23} + 0.2\Phi_{22} + 0.4\Phi_{21} - 2\Phi_{20} + 2\Phi_{19} + 0.5\Phi_4 - \Phi_3 - \frac{0.203438238}{64569960}\Phi_2 + \\ & \frac{0.0279008248}{387419760}\Phi_1 \end{aligned}$$

Therefore  $\mathcal{A}(\mathcal{SL}(2, 3^6)) = 387419760\chi_1$

#### 4. References

- [1] Mohamed.S K., (2006), On Rational-Valued Characters of Certain Types of Permutation Group, Ibn Al-Haitham Journal For Pure and Applied Sciences, Vol.19(4), pp.99-108.
- [2] Saad O. B., (2019), Investigating Particular Representations for Matrix Lie Groups  $SO(3)$  and  $SL(2, \mathbb{C})$ , Iraqi Journal of Science, Vol. 60, No. 4, pp: 856-858, <https://doi.org/10.24996/ij.s.2019.60.4.19>.
- [3] Taghreed H. M., Khawla A. Al-Z. Niran S. J., (2016), On the Representations of M-Groups, Baghdad Science Journal, Vol.13, No.2, pp. 394-401, <https://doi.org/10.21123/bsj.2016.13.2.0394>.
- [4] Niran S. J., Hadeel H. L. & Rana N. M., (2021), Computations for the special linear group  $(2, 49)$ , Vol. 24, No. 6, pp. 1677-1683, Journal of Interdisciplinary Mathematics, <https://doi.org/10.1080/09720502.2021.1892273>.
- [5] Sigler L.E., Algebra, Springer-Verlage, Berlin, 1976.



- [6] Dunya M. H., Ahmed K. M., Intidhar Z. M., (2021), Score for some Groups  $SUT(2,p)$ , Int. J. Nonlinear Anal. Appl., Vol.12, No. 2, pp.1-15.
- [7] Noor Alhuda S. S. andNiran S. J., 2022, Periodical split for the groups  $PSL(2,31)$  and  $PSL(2,37)$ , Journal of Discrete Mathematical Sciences and Cryptography, Vol. 25, No. 2, pp. 605-608 <https://doi.org/10.1080/09720529.2021.1982490>.
- [8] Sherouk A. K. and Niran S. J., (2022), Calculationfor the groups $SL(2,U)$ ,  $U=31$  and  $37$ , Journal of Discrete Mathematical Sciences and Cryptography, Vol. 25, No. 2, pp. 609-613, <https://doi.org/10.1080/09720529.2021.1972614>.