Analysis of Fully Fuzzy Critical Path in Project Network with a New Representation of Pentagonal Fuzzy Numbers

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Abstract

An alternative notation for Pentagonal Fuzzy Numbers is provided here. The goal is to use the novel representation to determine the best approach to Fully Fuzzy Critical Path (FFCP) problems, and to use that information to compute the total float of the fuzzy set for each activity and its critical path. This is verified by linear programming approach and compared with some more techniques like Range, Mode and Divergence. Numerical example, is illustrated using the newly defined fuzzy number as parameters.

Keywords - Fuzzy Fully Critical Path Problem - Pentagonal Fuzzy Numbers - Fuzzy total float- Fuzzy Linear programming.

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1. INTRODUCTION

Beginning in the early 1960s, Bellman and Zadeh [2] developed the concept of fuzzy decision making issues with maximising choice. The purpose of CPM is to identify the most important tasks on the critical path so that resources may be allocated accordingly, hence shortening the duration of the project.

In fuzzy climate ranking fuzzy numbers is extremely fundamental in dynamic system. In dynamic and fuzzy situations, Jain [8] originally advocated using fuzzy integers to rank items. A portion of these positioning technique have been looked at and checked on by Bortolan and Degani [3] and as of late by Chen [5], Chang [4] utilized in centroid based distance strategies to rank fuzzy numbers in 1998.

Chanas and Radosinski [11] examined how fuzzy math may be used to network design. It was taken into account that the decision makers' risk behaviour record may be used for fuzzy fundamental manner research by M.H. Oladeinde and C.A. Oladeinde [10].

Dubosis and Prade [6] stretched out the fuzzy number-crunching tasks to figure the most recent beginning season of every action in the project network, Hapke and Slowwinski [7] involved fuzzy arithmetic operations in project scheduling, C.T Chen and S.F. Huang [5] using a Fuzzy Logic approach to Project Criticality Assessment.

Over the course of the next several years, many approaches will be handled in order to determine the fuzzy critical path.Gazdik (1983) promoted the use of the fuzzy network of obscure venture to evaluate the duration of an action, and made use of fuzzy algebraic administrators to decide the span of the assignment and its basic course. Mccahon and Lee (1988) proposed a new method for estimating the duration of a project with a degree of uncertainty.

Naution proposes a complete float and a fuzzy critical route for project networks (1994). Based on measured certainty stretch assessments furthermore, an undeniable distance position for (1-α) fuzzy number levels, Lin and Yao (2003) presented a fuzzy CPM. Fuzzy basic way investigation for project organizations was first proposed by Liang and Han (2004) through a computation.

Chen (2007) proposed a method in view of a straight programming definition to look at the basic way in networks with a scaling rule and a fuzzy work term, and this paper builds on that work to promote a fundamental approach to dealing with the critical path problem when the length of the paths involved are fuzzy numbers.

In this paper, another strategy is proposed to track down the ideal answer for the fully fuzzy critical path problems. Likewise, another portrayal of pentagonal fuzzy numbers is proposed. As an example of the proposed technique moreover, to exhibit the advantages of the proposed depiction of pentagonal fuzzy numbers, a mathematical model is tackled by thinking about all terms with these new kinds of fuzzy numbers. The proposed strategy is straightforward and use, making it a reasonable choice for tracking down the fuzzy ideal arrangement of full basic way (FFCP) issues in certifiable circumstances.

2. PRELIMINARIES

This section reviews a few fundamental meanings of fuzzy numbers, pentagonal fuzzy number, math tasks between pentagonal fuzzy numbers and ranking functions.

DEFINITION 2.1

A **fuzzy set** $\widetilde{A}$ where R (the real line) is characterised as a collection of paired elements, $\widetilde{A} = \{x, \mu_{\widetilde{A}}(x)/x \in R\}$ where $\mu_{\widetilde{A}}(x)$termed the fuzzy set's membership function.

DEFINITION 2.2

A problem set $\widetilde{A}^{\sim}$ with **α cut**-offs of components of the universe X whose membership values are greater than the cut-off value forms a set α

That is, $\widetilde{A}_{\alpha} = \{x \in X/\mu_{\widetilde{A}}(x) \ \geq \alpha\}$

DEFINITION 2.3

To create a fuzzy set $\widetilde{A}$any if with at least one point is considered **normal**$x \in R$ with $\mu_{\widetilde{A}}(x) = 1$

DEFINITION 2.4

If for every x,y∈R in R and every in λ∈[0,1], then A is a **convex** fuzzy set $\widetilde{A}$ on R.

$$\mu_{\widetilde{A}}(\lambda_x + (1-\lambda)y) \geq \min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$$

The opposite of a convex function is called a concave function.

DEFINITION 2.5

An exact **fuzzy number**, natural and convex number $\widetilde{A}$ fuzzy real line is assigned.

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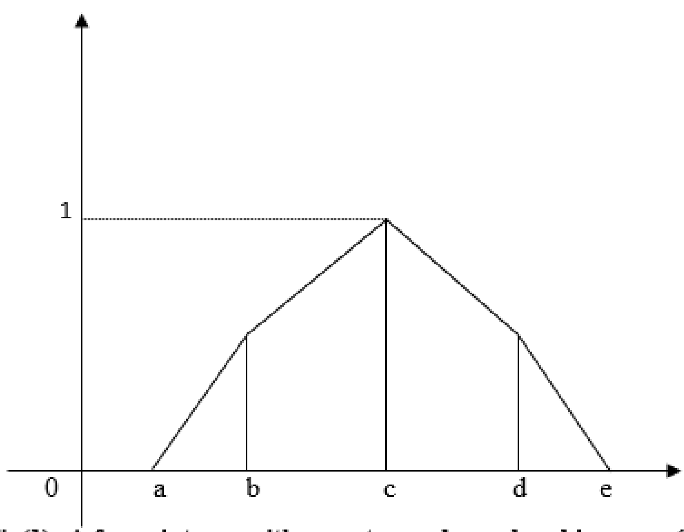
3. PENTAGONAL FUZZY NUMBER

DEFINITION 3.1

Whereas a and b represent the lowest possible qualities, c represents the highest encouraging worth, and d and e represent the highest possible qualities, the five boundaries a, b, c, d, and e define the Pentagonal Fuzzy Number (PFN).

A fuzzy number $\widetilde{\mathrm{A}} = (\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e})$ whose participation capability is known as a fuzzy pentagon

$$\mu_{\widetilde{\mathrm{A}}}(\mathrm{x}) = \begin{cases} 0 & \text{for} \quad \mathrm{x} < a, \\ \dfrac{\mathrm{x}-\mathrm{a}}{\mathrm{b}-\mathrm{a}} & \text{for} \quad \mathrm{a} \le \mathrm{x} \le \mathrm{b} \\ \dfrac{\mathrm{x}-\mathrm{b}}{\mathrm{c}-\mathrm{b}} & \text{for} \quad \mathrm{b} \le \mathrm{x} \le \mathrm{c} \\ 1 & \text{for} \quad \mathrm{x} = \mathrm{c} \\ \dfrac{\mathrm{d}-\mathrm{x}}{\mathrm{d}-\mathrm{c}} & \text{for} \quad \mathrm{c} \le \mathrm{x} \le \mathrm{d} \\ \dfrac{\mathrm{e}-\mathrm{x}}{\mathrm{e}-\mathrm{d}} & \text{for} \quad \mathrm{d} \le \mathrm{x} \le \mathrm{e} \\ 0 & \text{for} \quad \mathrm{x} > e \end{cases}$$



Fig(1): A fuzzy integer with a pentagonal membership curve (PFN)

3.2 CONDITIONAL PENTANGULAR FUZZY NUMBERS

The requirements for a fuzzy pentagonal number $\tilde{\mathrm{A}}$ are as follows.

i. $\mu_{\widetilde{\mathrm{A}}}(\mathrm{x})$ is a continuous function in [0,1]

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ii. $\mu_{\tilde{A}}(x)$is an increasing and continuous function on $[a, b]$ and $[b, c]$.

iii. $\mu_{\tilde{A}}(x)$is a continuous function decreasing with precision $[c, d]$ and $[d, e]$.

3.3 AUTHENTIC OPERATONS ON PENTAGONAL FUZZY NUMBERS (PFN))

3.3.1 ADDITION OF TWO PFNs

$\tilde{A} = (a_1, b_1, c_1, d_1, e_1)$and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2)$be two PFNs, and if we define addition of PFNs to be$\tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2)$

3.3.2 SUBTRACTION OF TWO PFNs

Let$\tilde{A} = (a_1, b_1, c_1, d_1, e_1)$and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2)$ two PFNs, then the difference between them is$\tilde{A} - \tilde{B} = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2, e_1 - e_2)$

3.3.3 MULTIPLICATION OF TWO PFNs

The result of multiplying two PFNs$\tilde{A} = (a_1, b_1, c_1, d_1, e_1)$and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2)$ is given by$\tilde{A}\tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2, e_1 e_2)$

DEFINTION 3.4

A PFN $\tilde{A} = (a, b, c, d, e)$ is said to be **positive**, if $a > 0$.

DEFINTION 3.5

A PFN $\tilde{A} = (a, b, c, d, e)$ is said to be **negative**, if $e < 0$.

DEFINITION 3.6

A PFNs $\tilde{A}$ if all of the items in the PFN are 0 or null, the PFN is said to be **null or zero**. $\tilde{A}(a, b, c, d, e) = 0$. That is, $\tilde{A} = (0, 0, 0, 0, 0)$

DEFINITION 3.7

Two PFNs $\tilde{A} = (a_1, b_1, c_1, d_1, e_1)$and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2)$ are said to be **equal** That is $\tilde{A} = \tilde{B}$ if and only if $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2$

DEFINITION 3.8 (MODE OF A PENTAGONAL FUZZY NUMBER)

If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number then the mode $\tilde{A}$ is given by c.

DEFNITION 3.9 (DIVERGENCE OF A PENTAGONAL FUZZY NUMBER)

If $\tilde{A} = (a, b, c, d, e)$ given a fuzzy pentagonal number, we may write the divergence of A as

$$Div\,\tilde{A} = e - a$$

DEFINITION 3.10 (SPREADS OF A PENTAGONAL FUZZY NUMBER)

If $\tilde{A} = (a, b, c, d, e)$ if A is a fuzzy pentagonal number, then its spreads are provided by left spread and right spread $\tilde{A} = c - b - a$ and right spread $\tilde{A} = e - d - c$

3.8 RANKING FUNCTION

An order function R is defined as R: F(R) → R such that F(R) maps every fuzzy number to a real line and forms a set of fuzzy numbers over the set of real numbers.

Then let (a, b, c, d, e) be a pentagonal fuzzy number

$$\mathbb{R}(a, b, c, d, e) = \frac{a + b + c + d + e}{5}$$

4. NEW REPRESENTATION OF PENTAGONAL FUZZY NUMBER

In this part, we present a new SMS portrayal of pentagonal fuzzy numbers. The proposed approach is utilized to decide the fuzzy ideal arrangement for fuzzy full scale critical path (FFCP) problems, and it is proven that all of its parameters are represented using SMS representation rather than the current representation of pentagonal fuzzy integers.

DEFINITION 4.1

Definition of pentagonal fuzzy number $\tilde{A}$ = (x, y, α, β, γ, δ) A simple pentagonal fuzzy number $\tilde{A} = (a, b, c, d, e), a = x, b = a + \alpha, c = a + \alpha + \beta,$ d = y-γ and e-d =δ

DEFINITION 4.2

A pentagonal fuzzy number $\tilde{A} = (x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ is a fuzzy pentagonal number of zero only if and only if x=0, y=0, α=0, β=0, γ=0, δ =0

DEFINITION 4.3

A pentagonal fuzzy number $\tilde{A} = (x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ is considered positive only when and only if $x > 0$

DEFINITION 4.4

Two pentagonal fuzzy numbers $\tilde{A} = (x_1, y_1, \alpha_1, \beta_1, \gamma_1, X\delta_1)$ and $\tilde{B} = (x_2, y_2, \alpha_2, \beta_2, \gamma_2, \delta_2)$ are said to be equal if and only if$x_1 = x_2, y_1 = y_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2$

4.5 ARITHMETIC OPERATIONS BETWEEN SMS PENTAGONAL FUZZY NUMBERS

Let $\tilde{A} = (a_1, b_1, c_1, d_1, e_1)$and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2)$ be two pentagonal fuzzy numbers and $(x_1, y_1, \alpha_1, \beta_1, \gamma_1, \delta_1)_{SMS}$ and $(x_2, y_2, \alpha_2, \beta_2, \gamma_2, \delta_2)_{SMS}$ be their short message service (SMS) representation, then addition is characterised as,

$$\tilde{A} \oplus \tilde{B} = (x_1 + x_2, y_1 + y_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2, \delta_1 + \delta_2)$$

Image of $\tilde{A}[-\tilde{A}]$ is defined as

$$\sim \tilde{A} = (-y, -x, \delta, \gamma, \beta, \alpha)$$

4.6 RANKING FUNCTION FOR SMS PENTAGONAL FUZZY NUMBER

Pentangular fuzzy number ordering function $\widetilde{A}\ (x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ as

$$\mathbb{R}(x, y, \alpha, \beta, \gamma, \delta) = \frac{a + b + c + d + e}{5} \quad where\ \tilde{A} = (a, b, c, d, e)$$

$$= \frac{x + y + x + \alpha + x + \alpha + \beta + y - \delta}{5}$$

$$= \frac{3x + 2y + 2\alpha + \beta - \delta}{5}$$

(or)

$$= \frac{x + y + x + \alpha + y - \delta - \gamma - y - \delta}{5}$$

$$= \frac{2x + 3y + \alpha - 2\delta - \gamma}{5}$$

5. FUZZY CRITICAL PATH ANALYSIS

In this study, we use a new pentagon-based integer representation to extend the fuzzy basic way technique. Consider a venture framework, where action periods are addressed by fuzzy numbers; This is known as the strange web.

Let E˜S_i and L˜S_i mean the hour of the past occasion and the hour of the following vague event of occasion I. Amounts, for example, beginning fuzzy occasion time E˜S_i, most recent fluffy time L˜S_i and floats T are each of the five fuzzy numbers because the functions defining these and other times in terms of fuzzy activity periods are convex and natural and their limb functions. They are part of continuous knowledge.

5.2 ALGORITHM FOR PROPOSED METHOD

To find the critical path with $(x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ representation of pentagonal fuzzy numbers

Step 1: Calculate Earliest start using

$\tilde{E}_j = max[\tilde{E}_i \oplus \tilde{t}_{ij}]$, where $\tilde{E}_1 = 0$ and $\tilde{E}_f = E_S \oplus t_{ij}$

Step 2: Calculate latest finish using

$\tilde{L}_i = min[\tilde{L}_j \oplus (-\tilde{t}_{ij})]$, where $\tilde{L}_n = \tilde{E}_n$

Step 3: The sum of the floats for each endeavour may be determined by

$$\tilde{T}_{ij} = \tilde{L}_j \oplus (-\tilde{E}_i) \oplus (-\tilde{t}_{ij})$$

Step 4: Investigate every conceivable route and determine the total slack fuzzy time it would take to complete it.

With the help of a ranking function, we can demystify the complete float of every movement and recognize the basic way where the sum of all floats is zero.

5.3 NUMERICAL EXAMPLE

Take a network of 10 activities in a project as an example.

Activity	Duration
1-2	(1,3,5,6,7)
1-3	(2,3,4,5,7)
2-3	(1,2,3,4,7)
2-4	(2,3,4,5,6)
3-5	(3,4,5,6,8)
4-5	(2,4,6,7,8)
4-7	(2,5,6,8,9)
5-6	(1,3,5,7,8)
5-7	(1,2,3,4,6)
6-7	(2,3,4,5,6)

The network diagram of the given data is,

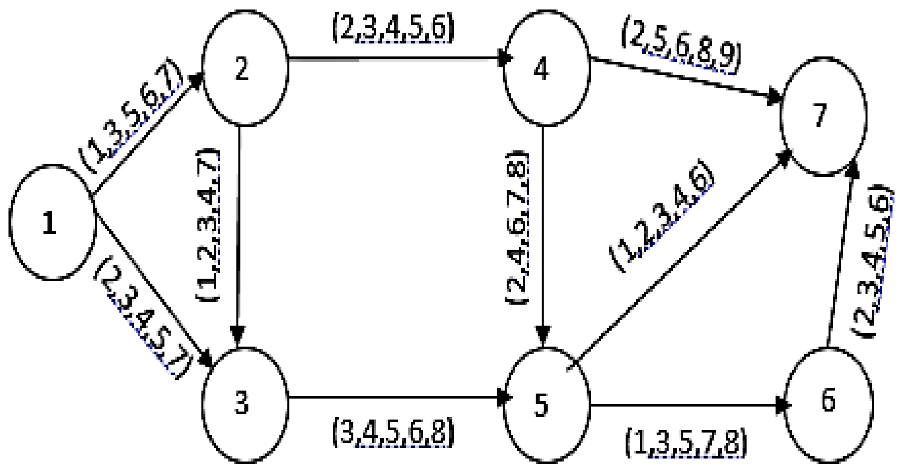

Fig 2: (a, b, c, d, e) representation

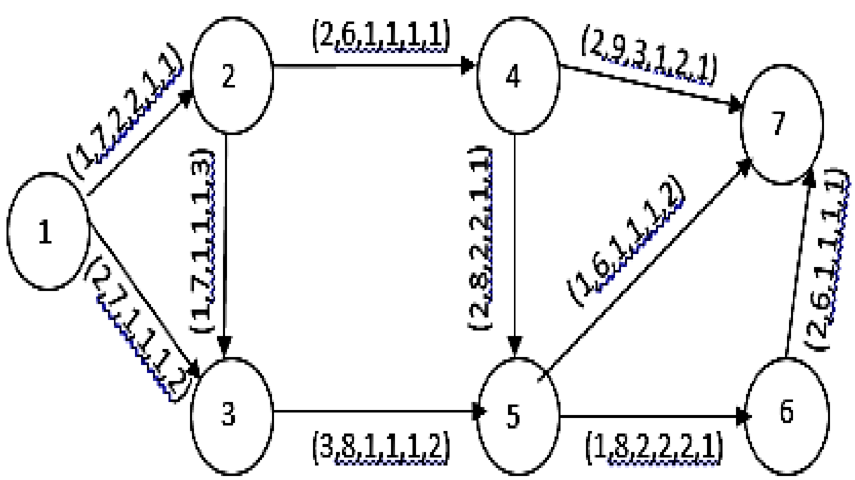

Fig 3: $(x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ representation

When determining when to begin, we choose the earliest possible moment. So the earliest start of 1-2 and 2-3 are $(0, 0, 0, 0, 0, 0)_{SMS}$, using the given algorithm the following table shows EST, LFT and TF of all the activities.

Activity	Duration	Earliest start	Earliest Finish	Latest Finish	Total Float	Rank
1-2	1,7,2,2,1,1	0,0,0,0,0,0	1,7,2,2,1,1	- 20,28,12,13,12,11	- 27,27,13,14,14,13	0
1-3	2,7,1,1,1,2	0,0,0,0,0,0	2,7,1,1,1,2	- 14,29,12,12,10,9	- 21,27,14,13,11,10	4.4
2-3	1,7,1,1,1,3	1,7,2,2,1,1	2,14,3,3,2,4	- 14,29,12,12,10,9	- 28,27,16,14,13,12	0.8
2-4	2,6,1,1,1,1	1,7,2,2,1,1	3,13,3,3,2,2	- 14,30,11,12,11,10	- 27,27,13,14,14,13	0
3-5	3,8,1,1,1,2	2,14,3,3,2,4	5,22,4,4,3,6	- 6,32,10,11,9,8	- 28,27,16,14,13,12	0.8
4-5	2,8,2,8,1,1	3,13,3,3,2,2	5,21,5,5,3,3	- 6,32,10,11,9,8	- 27,27,13,14,14,13	0
4-7	2,9,3,1,2,1	3,13,3,3,2,2	5,24,6,4,4,3	8,35,8,8,6,5	- 14,30,11,12,10,11	8.2
5-6	1,8,2,2,2,1	5,21,5,5,3,3	6,29,7,7,5,4	2,33,9,9,7,6	- 27,27,13,14,14,13	0
5-7	1,6,1,1,1,2	5,21,5,5,3,3	6,27,6,6,4,5	8,35,8,8,6,5	- 19,29,13,12,12,11	5.6
6-7	2,6,1,1,1,1	6,29,7,7,5,4	8,35,8,8,6,5	8,35,8,8,6,5	- 27,27,13,14,14,13	0

Table 1 : Computation of Total Floats for all the activities

Hence the critical path of the network is found to be 1-2-4-5-6-7.

6. LINEAR PROGRAMMING APPROACH AND COMPARISION USING PROPOSED PENTAGONAL FUZZY NUMBER

DEFINITION 6.1

MODE IN SMS REPRESENTATION

If $\tilde{A} = (a, b, c, d, e)$ isa pentagonal fuzzy number and $(x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ in it's SMS representation, then mode $\tilde{A}$ is given by

$$Mode\ \tilde{A} = x + \alpha + \beta\ \ (or)\ y - \delta - \gamma$$

DEFINITION 6.2

DIVERGENCE IN SMS REPRESENTATION

If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number and $(x, y, \alpha, \beta, \gamma, \delta)_{SMS}$ If an abbreviation for short message service, then the divergence of A is $Div\ \tilde{A} = y - x$

DEFINITION 6.3

SPREADS IN SMS REPRESENTATION

If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number, and the left and right spreads of A are represented by the SMS representation and the left spread $\tilde{A} = \alpha + \beta$ and right spread$\tilde{A} = \gamma + \delta$

6.4 FORMULATE THE LINEAR PROGRAMMING OF THE FUZZY CRITICAL PATH METHOD

In this context, we'll use the notation G=(N,A) to refer to a directed and linked network consisting of n nodes and a collection of i,j activities in a project. That is $T_{ij} = (i, j)$ where all $(i, j) \in A$, The linear programming problem is an effective method for determining the critical pathways and overall length times of project networks. Finding the longest route from the beginning to the end of the project will reveal which path is the crucial one, since the CPM issue is the inverse of the shortest path problem. If the longest route is taken into account, the overall time required for the project's network is calculated. Allow us to expect that a unit stream enters the errand network close to the beginning and exits close to the end. The CPM for n nodes is expressed as subject to,

$$Max\ z = \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij} x_{ij}$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{Ki}, i = 2, \ldots, n-1$$

$$\sum_{k=1}^{n} x_{kn} = 1$$

$$x_{ij} \geq 0 \quad (i, j) \in A$$

6.5 NEW APPROACH FOR DETERMINING THE CRITICAL ROUTE IN A PROJECT NETWORK

Step (1):

It is recommended that the critical route be posed as a linear programming problem.

Step (2)

Solve a linear programming issue. Once the solution has been found, if it is unique, the critical path may be calculated.

Step (3):

It is possible to determine the critical route, where the amount of the floats of the exercises along that way is zero, if there are several possible solutions.

Step (4):

The answer is found if and only if there is a single, unbroken critical route.

Step (5):

If not, then rank the possible pathways such that the one with the highest mode is the crucial path. This mode may be calculated by finding the mode of the total float.

Step (6):

If all the pathways have the same mode, then the one with the greatest divergence is the crucial one.

EXAMPLE 6.7

Consider the network given in Numerical Example 5.2

The given problem is mathematically formulated as,

$$Max[(1,7,2,2,1,1)x_{12}\oplus(2,7,1,1,1,2)x_{13}\oplus(1,7,1,1,1,3)x_{23}\oplus(2,6,1,1,1,1)x_{24} \oplus(3,8,1,1,1,2)x_{35}\oplus(2,8,2,2,1,1)x_{45}\oplus(2,9,3,1,2,1)x_{47}\oplus(1,8,2,2,2,1)x_{56}\oplus(1,6,1,1,1,2)x_{57} \oplus(2,6,1,1,1,1)x_{67}]$$

subject to

$$x_{12}+x_{13}=1$$

$$x_{12}=x_{23}+x_{24}$$

$$x_{13}+x_{23}=x_{35}$$

$$x_{24}=x_{45}+x_{47}$$

$$x_{35}+x_{45}=x_{56}+x_{57}$$

$$x_{56}=x_{67}$$

$$x_{47}+x_{57}+x_{67}=1$$

$$x_{12}, x_{13}, x_{23}, x_{24}, x_{35}, x_{45}, x_{47}, x_{56}, x_{57}, x_{67} \geq 0$$

The linear programming formulation will be based on the ranking function.

$Max\ [4.4x_{12} \oplus 4.2x_{13} \oplus 3.4x_{23} \oplus 4x_{24} \oplus 5.2x_{35} \oplus 5.4x_{45} \oplus 6x_{47} \oplus 4.8x_{56} \oplus 3.2x_{57} \oplus 4x_{67}]$

subject to

$x_{12} + x_{13} = 1$

$x_{12} - x_{23} - x_{24} = 0$

$x_{13} + x_{23} - x_{35} = 0$

$x_{24} - x_{45} - x_{47} = 0$

$x_{35} + x_{45} - x_{56} - x_{57} = 0$

$x_{56} - x_{67} = 0$

$-x_{47} - x_{57} - x_{67} = -1$

$$x_{12}, x_{13}, x_{23}, x_{24}, x_{35}, x_{45}, x_{47}, x_{56}, x_{57}, x_{67} \geq 0$$

On solving CLP using TORA software, it can be found that the optimum solution is

Max Z= 22.6

when $x_{12} = x_{24} = x_{45} = x_{56} = x_{67} = 1$

and$x_{13} = x_{23} = x_{35} = x_{47} = x_{57} = 0$

The fuzzy critical path, when employing this best option, is $\mathbf{1 - 2 - 4 - 5 - 6 - 7.}$

The other alternative paths obtained are

i. $1 - 2 - 4 - 7$
ii. $1 - 2 - 4 - 5 - 7$
iii. $1 - 2 - 4 - 5 - 6 - 7$
iv. $1 - 2 - 3 - 5 - 7$
v. $1 - 2 - 3 - 5 - 6 - 7$
vi. $1 - 3 - 5 - 7$
vii. $1 - 3 - 5 - 6 - 7$

It is possible to determine the beginning, ending, and total floats for each route by consulting table 1.

Use the following table to quickly and easily compare the total float, rank, mode, and divergence of all the pathways.

Paths	**Total Float**	**Rank**	**Mode**	**Divergence**
$1-2-4-7$	$(-68, 84,37,40,48,37)$	10.25	9	152
$1-2-4-5-7$	$(-100,110,52,54,54,50)$	5.6	6	210
$1-2-4-5-6-7$	$(-135,135,65,70,70,65)$	0	0	270
$1-2-3-5-7$	$(-102,110,58,54,52,48)$	7.2	10	212
$1-2-3-5-6-7$	$(-137,135,71,70,68,63)$	1.6	4	272
$1-3-5-7$	$(-68,83,43,39,36,33)$	10.8	14	151
$1-3-5-6-7$	$(-103,108,56,55,52,48)$	5.2	8	211

Table 2 : Comparing table

Hence, it is found and verified that $\mathbf{1-2-4-5-6-7}$ is the specified network's critical path.

7. CONCLUSION

A novel form of pentagonal fuzzy numbers termed SMS representation of pentagonal fuzzy number is presented. Using a linear programming method, it is shown that the suggested representation of pentagonal fuzzy numbers is superior to the current form while trying to solve completely fuzzy critical route issues.

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