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Estimating Mean Using Auxiliary Information Under Measurement Errors

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Abstract
The present paper concerns through the estimating of population mean with auxiliary information in measurements errors. This is vital since it is common in everyday situations for data to be wrong and contain measurement errors for a variety of reasons. An estimator for approximating a finite population mean is proposed in the presence of measurement errors. Up to first order of approximation, the proposed estimator's bias and mean squared error (MSE) expressions are obtained. The proposed estimator is more efficient than the existing estimator of population mean under measurement error, based on a theoretical comparison of their relative performance between the two types of estimators. To support the theoretical conclusions, an empirical study based on real data is conducted.

Keywords: Auxiliary information, Bias, Measurement errors, Mean Squared Error.

1. Introduction

In the past few years, Statisticians have focused on the challenge of approximation of parameters in the presence of measurement errors. The characteristics of estimators based on survey sample data often assume that the observations are accurate measures. But in many circumstances this requirement is not met and data collected may have measurement errors due to non-response, reporting, and computing problems etc. The results that were introduced to be used in the absence of measurement errors are invalidated by these measurement errors. Statistical inferences based on observed data are nevertheless valid when measurement errors are low enough to be ignored or tolerated. On the other hand, the results might not be correct if they are not considerably small. Some of the most typical sources of measurement errors in survey data are reviewed by Cochran (1968), Paul et al (1991), and Shalabh (1997). By Singh and Karpe (2009), Kumar et al. (2011), Misra and Yadav (2015), Misra et al (2017).

Let $U = U_1, U_2, \dots, U_N$ be a finite population of N different and identifiable units with Y being the study variable and X being the auxiliary variable taking the value Y_i and X_i for the unit i of the population U respectively. Further, let a set of n paired observations are obtained through simple random sampling without replacement procedure of both the characteristics X and Y. Further for a simple random sampling of size n , let (x_i, y_i) be the observed values instead of true values (X_i, Y_i) for the x_i, $(i = 1,2, \dots n)$ sampling unit in the sample as $u_i = y_i - Y_i$ and $v_i = x_i - X_i$ where u_i and v_i are associated measurement errors which are stochastic in nature with mean zero and variances $\sigma_u^2 \, and \, \sigma_v^2$ respectively. Further, let $u_i' \;\; and \; v_i's$ are uncorrelated while

$X_i's\ and\ Y_i's$ are correlated. Let the population mean, of $X\ and\ Y$ characteristics be $\mu_X\ and\ \mu_y$ population variances of (X, Y) are σ_X^2 and σ_Y^2 respectively and ρ is the population correlation coefficient between $X\ and\ Y$.

Let $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ & $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ be the unbiased estimators of population means μ_X and μ_Y respectively

i.e. $E(\bar{x}) = \mu_X$ and $E(\bar{y}) = \mu_Y$. But in the presence of Measurement errors. $s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 and$

$s_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are not unbiased estimators of the population variances $\sigma_X^2\ and\ \sigma_Y^2$. The expected value of s_y^2 in the presence of measurement errors is given by

$E(s_y^2) = \sigma_Y^2 + \sigma_u^2.$

Let error variances $\sigma_u^2 and\ \sigma_v^2$ are known to prior, then in the presence of measurement errors unbiased estimators of population variances are.

$\hat{\sigma}_Y^2 = s_y^2 - \sigma_u^2 > 0$

$\hat{\sigma}_X^2 = s_x^2 - \sigma_v^2 > 0$

Further let,

$C_Y = \frac{\sigma_Y}{\mu_Y}$

$C_X = \frac{\sigma_X}{\mu_X}$

$\gamma_{2Y} = \beta_{2Y} - 3, \gamma_{2X} = \beta_{2X} - 3, \gamma_{2u} = \beta_{2u} - 3$

$\gamma_{2v} = \beta_{2v} - 3, \beta_{2Y} = \frac{\mu_4(Y)}{\mu_2^2(Y)}$

$\beta_{2X} = \frac{\mu_4(X)}{\mu_2^2(X)}, \beta_{2u} = \frac{\mu_4(u)}{\mu_2^2(u)}$

$\beta_{2v} = \frac{\mu_4(v)}{\mu_2^2(v)}, \gamma_{1(X)} = \sqrt{\beta_1(X)},\ \beta_1(X) = \frac{\mu_3^2}{\mu_2^3}$

$\mu_{qrst} = E[(X - \mu_X)^q (Y - \mu_Y)^r v^s u^t]$

$\mu_{2000} = \sigma_X^2$

$\mu_{0200} = \sigma_Y^2$

$\mu_{0020} = \sigma_v^2$

$\mu_{0002} = \sigma_u^2$.

To estimate the population mean, an estimator in presence of measurement errors is proposed as

$$\hat{\bar{y}}_{ME} = \bar{y} + b(\mu_X - \bar{x}) + k_1\left(\frac{\hat{\sigma}_X^2}{C_X^2} - \bar{x}^2\right) + k_2\left(\frac{\hat{\sigma}_Y^2}{C_Y^2} - \bar{y}^2\right) \quad (1.1)$$

2. Bias and Mean Squared Error

Here we consider the approximations as

$\bar{y} = \mu_Y(1 + e_0)$

$\bar{x} = \mu_X(1 + e_1)$

$\hat{\sigma}_Y^2 = \sigma_Y^2(1 + e_2)$

$\hat{\sigma}_X^2 = \sigma_X^2(1 + e_3)$

$\hat{\sigma}_{XY} = \sigma_{XY}(1 + e_4)$

so that $E(e_0) = E(e_1) = E(e_2) = E(e_3) = E(e_4) = 0$

From Singh and Karpe (2009), we have

$E(e_0^2) = \frac{C_Y^2}{n\theta_Y} and\ E(e_1^2) = \frac{C_X^2}{n\theta_X}$, where $\theta_Y = \frac{\sigma_Y^2}{\sigma_Y^2+\sigma_u^2}$ and $\theta_X = \frac{\sigma_X^2}{\sigma_X^2+\sigma_v^2}$

$$E(e_1e_3) = \frac{\mu_{3000}}{n\sigma_X^2\mu_X}$$

$$E(e_3^2) = \frac{A_X}{n}, where\ A_X = \gamma_{2X} + \gamma_{2v}\frac{\sigma_v^4}{\sigma_X^4} + 2\left(1+\frac{\sigma_v^2}{\sigma_X^2}\right)^2$$

$$E(e_0e_2) = \frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}$$

$$E(e_0e_3) = \frac{\mu_{2100}}{n\sigma_X^2\mu_Y}$$

$$E(e_1e_2) = \frac{\mu_{1200}}{n\sigma_Y^2\mu_X}$$

$$E(e_0e_1) = \frac{\sigma_{XY}}{n\mu_X\mu_Y} = \frac{\rho C_X C_Y}{n}$$

$$E\left(e_2^2\right) = \frac{A_Y}{n}, where\ A_Y = \gamma_{2Y} + \gamma_{2u}\frac{\sigma_u^4}{\sigma_Y^4} + 2\left(1+\frac{\sigma_u^2}{\sigma_Y^2}\right)^2$$

$$E(e_2e_3) = \frac{\delta-1}{n}, where\ \delta = \frac{\mu_{2200}}{\sigma_X^2\sigma_Y^2}$$

$$E(e_1e_4) = \frac{\mu_{2100}}{n\sigma_{XY}\mu_X}$$

Expressing (1.1) in terms of e_i's, we have

$$\bar{y}_{ME} = \mu_Y + e_0\mu_Y + \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X^2}(-\mu_X e_1) + k_1\left\{\frac{\sigma_X^2(1+e_3)}{C_X^2} - \mu_X^2(1+e_1)^2\right\}$$
$$+k_2\left\{\frac{\sigma_Y^2(1+e_2)}{C_Y^2} - \mu_Y^2(1+e_0)^2\right\}$$

$$\bar{y}_{ME} = \mu_Y + e_0\mu_Y - \frac{\sigma_{XY}(1+e_4)}{\sigma_X^2(1+e_4)}(\mu_X e_1) + k_1\{\mu_X^2(1+e_3) - \mu_X^2(1+e_1)^2\}$$
$$+k_2\{\mu_Y^2(1+e_2) - \mu_Y^2(1+e_0)^2\}$$

$$= \mu_Y + e_0\mu_Y - \frac{\sigma_{XY}}{\sigma_X^2}(1+e_4)(1-e_3+e_3^2)\mu_X e_1 + k_1\mu_X^2\{1+e_3-1-e_1^2-2e_1\}$$
$$+k_2\mu_Y^2\{1+e_2-1-e_0^2-2e_0\}$$
$$= \mu_Y + e_0\mu_Y - \frac{\sigma_{XY}}{\sigma_X^2}\mu_X e_1(1-e_3+e_3^2+e_4-e_3e_4) + k_1\mu_X^2\{e_3-2e_1-e_1^2\}$$
$$+k_2\mu_Y^2\{e_2-2e_0-e_0^2\}$$
$$= \mu_Y + e_0\mu_Y - \frac{\sigma_{XY}}{\sigma_X^2}\mu_X(e_1-e_1e_3+e_1e_4) + k_1\mu_X^2\{e_3-2e_1-e_1^2\} + k_2\mu_Y^2\{e_2-2e_0-e_0^2\}$$

or

$$\hat{\bar{y}}_{ME} - \mu_Y = \mu_Y e_0 - \frac{\sigma_{XY}}{\sigma_X^2}\mu_x(e_1-e_1e_3+e_1e_4) + k_1\mu_X^2\left(e_3-2e_1-e_1^2\right) + k_2\mu_Y^2\left(e_2-2e_0-e_0^2\right) \qquad (2.1)$$

Taking expectation in both the sides of (2.1), Bias to the first degree of approximation is given by

$$Bias(\hat{\bar{y}}_{ME}) = E(\hat{\bar{y}}_{ME} - \mu_Y) = \frac{\sigma_{XY}}{\sigma_X^2}\mu_X\left(\frac{\mu_{3000}}{n\sigma_X^2\mu_X} - \frac{\mu_{2100}}{n\sigma_{XY}\mu_X}\right) - k_1\mu_X^2\frac{C_X^2}{n\theta_X} - k_2\mu_Y^2\frac{C_Y^2}{n\theta_y}$$

or $$Bias(\hat{\bar{y}}_{ME}) = \frac{1}{n}\left\{\frac{\sigma_{XY}\mu_{3000}}{\sigma_X^4} - \frac{\mu_{2100}}{\sigma_X^2} - \frac{k_1\mu_X^2C_X^2}{\theta_X} - \frac{k_2\mu_Y^2C_Y^2}{\theta_Y}\right\} \tag{2.2}$$

Now to get the m.s.e of the proposed estimator, first on squaring (2.1) both the sides, we have

$$\left(\hat{\bar{y}}_{ME} - \mu_Y\right)^2 = \Big[\mu_Y e_0 - \frac{\sigma_{XY}}{\sigma_X^2}\mu_X e_1 + k_1\mu_X^2(e_3 - 2e_1) + k_2\mu_Y^2(e_2 - 2e_0) + \frac{\sigma_{XY}}{\sigma_X^2}\mu_X(e_1e_3 - e_1e_4)$$
$$- k_1\mu_X^2e_1^2 - k_2\mu_Y^2e_0^2\Big]^2 \tag{2.3}$$

Solving (2.3) and approximating to the first degree, we have

$$\left(\hat{\bar{y}}_{ME} - \mu_Y\right)^2 = \mu_Y^2e_0^2 + \frac{\sigma_{XY}^2\mu_X^2}{\sigma_X^4}e_1^2 - \frac{2\mu_Y\sigma_{XY}\mu_X}{\sigma_X^2}e_0e_1 + k_1^2\mu_X^4\left(e_3^2 + 4e_1 - 4e_1e_3\right)$$
$$+ k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right) + 2k_1\left\{\mu_X^2\mu_Y(e_0e_3 - 2e_0e_1) - \frac{\sigma_{XY}\mu_X^3}{\sigma_X^2}\left(e_1e_3 - 2e_1^2\right)\right\}$$
$$+ 2k_2\left\{\mu_Y^3\left(e_0e_2 - 2e_0^2\right) - \frac{\sigma_{XY}\mu_X\mu_Y^2}{\sigma_X^2}(e_1e_2 - 2e_0e_1)\right\}$$
$$+ 2k_1k_2\mu_X^2\mu_Y^2(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1) \tag{2.4}$$

Now taking expectation on both sides of (2.4) and using values of the expectations given earlier, the MSE to the first degree of approximation is given by

$$MSE\left(\hat{\bar{y}}_{ME}\right) = E\left(\hat{\bar{y}}_{ME} - \mu_Y\right)^2$$
$$= \left(\mu_Y^2\frac{C_Y^2}{n\theta_Y} + \frac{\sigma_{XY}^2\mu_X^2}{\sigma_X^4}\frac{C_X^2}{n\theta_X} - 2\frac{\mu_Y\sigma_{XY}\mu_X}{\sigma_X^2}\frac{\rho C_XC_Y}{n}\right)$$
$$+ k_1^2\mu_X^4\left\{\frac{A_X}{n} + 4\frac{C_X^2}{n\theta_X} - 4\frac{\mu_{3000}}{n\sigma_X^2\mu_X}\right\} + k_2^2\mu_Y^4\left\{\frac{A_Y}{n} + 4\frac{C_Y^2}{n\theta_Y} - 4\frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}\right\}$$
$$+ 2k_1\left\{\mu_X^2\mu_Y\left(\frac{\mu_{2100}}{n\sigma_X^2\mu_Y} - 2\rho\frac{C_XC_Y}{n}\right) - \frac{\sigma_{XY}\mu_X^3}{\sigma_X^2}\left(\frac{\mu_{3000}}{n\sigma_X^2\mu_X} - 2\frac{C_X^2}{n\theta_X}\right)\right\}$$
$$+ 2k_2\left\{\mu_Y^3\left(\frac{\mu_{0300}}{n\sigma_Y^2\mu_Y} - 2\frac{C_Y^2}{n\theta_Y}\right) - \frac{\sigma_{XY}\mu_X\mu_Y^2}{\sigma_X^2}\left(\frac{\mu_{1200}}{n\sigma_Y^2\mu_X} - 2\frac{\rho C_XC_Y}{n}\right)\right\}$$
$$+ 2k_1k_2\mu_X^2\mu_Y^2\left(\frac{\delta - 1}{n} - 2\frac{\mu_{2100}}{n\sigma_X^2\mu_Y} - 2\frac{\mu_{1200}}{n\sigma_Y^2\mu_X} + 4\rho\frac{C_XC_Y}{n}\right)$$

$$MSE\left(\hat{\bar{y}}_{ME}\right) = \left(\mu_Y^2\frac{C_Y^2}{n\theta_Y} + \frac{\sigma_{XY}^2}{\sigma_X^2 n\theta_X} - 2\rho^2\frac{\sigma_Y^2}{n}\right) + k_1^2\mu_X^4\left\{\frac{A_X}{n} + 4\frac{C_X^2}{n\theta_X} - 4\frac{\mu_{3000}}{n\sigma_X^2\mu_X}\right\}$$

$$+k_2^2\mu_Y^4\left\{\frac{A_Y}{n}+4\frac{C_Y^2}{n\theta_Y}-4\frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}\right\}$$

$$+2k_1\left\{\mu_X^2\mu_Y\left(\frac{\mu_{2100}}{n\sigma_X^2\mu_Y}-2\rho\frac{C_XC_Y}{n}\right)-\frac{\sigma_{XY}\mu_X^3}{\sigma_X^2}\left(\frac{\mu_{3000}}{n\sigma_X^2\mu_X}-2\frac{C_X^2}{n\theta_X}\right)\right\}$$

$$+2k_2\left\{\mu_Y^3\left(\frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}-2\frac{C_Y^2}{n\theta_Y}\right)-\frac{\sigma_{XY}\mu_X\mu_Y^2}{\sigma_X^2}\left(\frac{\mu_{1200}}{n\sigma_Y^2\mu_X}-2\frac{\rho C_XC_Y}{n}\right)\right\}$$

$$+2k_1k_2\mu_X^2\mu_Y^2\left(\frac{\delta-1}{n}-2\frac{\mu_{2100}}{n\sigma_X^2\mu_Y}-2\frac{\mu_{1200}}{n\sigma_Y^2\mu_X}+4\rho\frac{C_XC_Y}{n}\right) \tag{2.5}$$

$$MSE\left(\hat{\bar{y}}_{ME}\right)=\left(\mu_Y^2\frac{C_Y^2}{n\theta_Y}+\frac{\sigma_{XY}^2}{\sigma_X^2n\theta_X}-2\rho^2\frac{\sigma_Y^2}{n}\right)+k_1^2\delta_{11}+k_2^2\delta_{22}+2k_1\delta_{10}+2k_2\delta_{02}+2k_1k_2\delta_{12} \tag{2.6}$$

where $\delta_{11}=\mu_X^4\left(\frac{A_X}{n}+4\frac{C_X^2}{n\theta_X}-4\frac{\mu_{3000}}{n\sigma_X^2\mu_x}\right)$

$$\delta_{22}=\mu_Y^4\left(\frac{A_Y}{n}+4\frac{C_Y^2}{n\theta_Y}-4\frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}\right)$$

$$\delta_{10}=\left\{\mu_X^2\mu_Y\left(\frac{\mu_{2100}}{n\sigma_X^2\mu_Y}-2\rho\frac{C_XC_Y}{n}\right)-\frac{\sigma_{XY}\mu_X^3}{\sigma_X^2}\left(\frac{\mu_{3000}}{n\sigma_X^2\mu_X}-\frac{2C_X^2}{n\theta_X}\right)\right\}$$

$$\delta_{02}=\left\{\mu_Y^3\left(\frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}-2\frac{C_Y^2}{n\theta_Y}\right)-\frac{\sigma_{XY}\mu_X\mu_Y^2}{\sigma_X^2}\left(\frac{\mu_{1200}}{n\sigma_Y^2\mu_X}-2\rho\frac{C_XC_Y}{n}\right)\right\}$$

$$\delta_{12}=\left\{\mu_x^2\mu_Y^2\left(\frac{\delta-1}{n}-2\frac{\mu_{2100}}{n\sigma_X^2\mu_Y}-2\frac{\mu_{1200}}{n\sigma_Y^2\mu_X}+4\rho\frac{C_XC_Y}{n}\right)\right\}$$

For optimizing (2.5) w.r.t k_1& k_2, we have the two normal equations as

$$\delta_{11}k_1+\delta_{12}k_2+\delta_{10}=0 \tag{2.7}$$

$$\delta_{12}k_1+\delta_{22}k_2+\delta_{02}=0 \tag{2.8}$$

On solving these two normal equations for k_1 & k_2, the optimum values of k_1& $k_2$are given by

$$k_1=\frac{\delta_{22}\delta_{10}-\delta_{02}\delta_{12}}{\delta_{12}^2-\delta_{11}\delta_{22}} \tag{2.9}$$

$$k_2=\frac{\delta_{11}\delta_{02}-\delta_{12}\delta_{10}}{\delta_{12}^2-\delta_{11}\delta_{22}} \tag{2.10}$$

For these optimum values of k_1 and $k_2$the minimum mean squared error of $\hat{\bar{y}}_{ME}$is given by

$$MSE\left(\hat{\bar{y}}_{ME}\right)_{\min}=\left(\mu_Y^2\frac{C_Y^2}{n\theta_Y}+\frac{\sigma_{XY}^2}{\sigma_X^2n\theta_X}-2\rho^2\frac{\sigma_Y^2}{n}\right)-\frac{\left(\delta_{11}\delta_{02}^2+\delta_{22}\delta_{10}^2-2\delta_{02}\delta_{10}\delta_{12}\right)}{\left(\delta_{11}\delta_{22}-\delta_{12}^2\right)} \tag{2.11}$$

3. Theoretical Comparision

For comparing efficiency over the usual mean per unit estimator,Let's use the usual mean per unit estimator y in the presence of measurement error to comparing efficiency to it.

$$\bar{y}_m = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{3.1}$$

$$\bar{y}_m = \mu_Y(1+e_0)$$

Expressing (1.1) in terms of e_i's, $\bar{y}_m$ becomes

$$\bar{y}_m = \mu_Y(1+e_0)$$

$$\bar{y}_m - \mu_Y = \mu_Y e_0$$

Therefore $Bias(\bar{y}_m) = 0$ (3.2)

$$MSE(\bar{y}_m) = \frac{\sigma_Y^2}{n}\left(1+\frac{\sigma_X^2}{\sigma_Y^2}\right) \tag{3.3}$$

Now, the proposed estimator $\hat{\hat{y}}_{ME}$ will be more efficient than the usual mean per unit estimator in presence of measurement error if

$$MSE(\bar{y}_m) - MSE(\hat{\hat{y}}_{ME}) > 0$$

or $\frac{1}{2\sigma_Y^2}\left(\frac{\mu_Y^2\sigma_X^2}{\theta_Y} + \frac{\sigma_{XY}^2}{\sigma_X^2\theta_Y} - \sigma_Y^2 - \sigma_X^2 - h\right) < \rho^2$

or $h > \frac{1}{2\sigma_Y^2}\left(\frac{\mu_Y^2\sigma_X^2}{\theta_Y} + \frac{\sigma_{XY}^2}{\sigma_X^2\theta_Y} - \sigma_Y^2 - \sigma_X^2\right) - \rho^2$, where $h = \frac{\left(\delta_{11}\delta_{02}^2 + \delta_{22}\delta_{10}^2 - 2\delta_{02}\delta_{10}\delta_{12}\right)}{\left(\delta_{11}\delta_{22} - \delta_{12}^2\right)}$. (3.4)

Hence the proposed estimator $\hat{\hat{y}}_{ME}$ will be more efficient than the usual mean per unit estimator in presence of measurement error if the condition (3.4) is satisfied by the data set.

4. Empirical Study

In this section, using a known population data set, we compare the performance of the suggested estimator that was used in this paper. The following is a description of the population set.

Data statistics: The data used for empirical study has been taken from Guajarati and Sangeetha (2012) pg. 509.

Y= True Consumption Expenditure

X= True Income

y_i= Measured consumption expenditure

x_i = Measure Income, with the help of hypothetical data we get the following values

$n = 10$,

$\bar{X} = 170$,

$\bar{Y} = 127$,

$\sigma_X^2 = 3300$,

$\sigma_Y^2 = 1278$,

$\sigma_u^2 = 32.4001$,

$\sigma_v^2 = 32.3998$

$C_Y = 0.2815$,

$C_X = 0.3379$,

$\rho_{XY} = 0.9641$,

$\beta_{2Y} = 1.9026$,

$\beta_{2X} = 1.7758$,

$\beta_{2u} = 1.7186,$
$\beta_{2v} = 1.8409$
The calculated MSE's of the estimators with measurement errors are given by,

$MSE(\bar{y}_m) = 131.033$
$MSE(\bar{y}_m) = 12.969.$
Showing the enhanced efficiency of the proposed estimator.

5. Conclusion

We are interested in exploring how the suggested estimate performs when there are measurement errors in the data. We used mean squared error as a criterion for testing the performance of estimators in order to evaluate their performance. When the proposed estimator is compared to the mean per unit estimator, it is found that the proposed estimator is more efficient in terms of MSE.

The relative efficiency (PRE) of the proposed estimator over the mean per unit estimator under measurement error is calculated using the above MSEs.is 1010, showing the enhanced efficiency of the proposed estimator.

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