# The Accurate Distance - 2 Domination (Ad-2d) in Graphs

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**ABSTRACT**

An AD-2D collection in G is constructed as V/D has no D-2D set with |D|. The AD-2D number $\gamma_{a\leq 2}(G)$ is the least cardinality in all AD-2Dcollections. We received several bounds on AD-2D number. Precise values of $\gamma_{a\leq 2}(G)$ are acquire for few named graph, also their relationship with other parameters were extended. Named result: Nordhaus - Gaddum bounds were founded for this $\gamma_{a\leq 2}(G)$ parameter.

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**Keywords:** Domination number, AD set, ATD number, D-2D set, AD-2D set.

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## 1. INTRODUCTION

All diagrams concentrated here are without loops ,parallel edges, finite, there exists path across all vertices and without directin. We used the terminology of [4].

The following notions are utilized:

n : Number of vertices

m :Cardinality of edges

**Δ**(G) : Maximum degree of G

δ(G) : Minimum degree in vertices

$\lceil x \rceil$ : The smallest integer greater than or equivalent to x

$\lfloor x \rfloor$ : The greatest integer less than or same x

$\beta_0(G)$ : Maximum cardinality among the independent set

$\alpha_0(G)$ : Minimum cardinality of vertex cover

g(G) : length of the shortest cycle in G (girth)

c(G) :( circumference) distance of the longest cycle

A collection of nodes D is named as dominating set it hold the constrain, each points in V/D is adjacent to at least one node of D(G). The notion $\gamma$ (G) contains the cardinality of smallest minimal dominating collection of G.

Kulli,V.R., et al. Contributed accurate domination (AD) in G (2000) [6].

An AD set D holds the condition that, if V/D does not contains dominating collection with $|D|$.The AD number $\gamma_a(G)$ selected from least cardinality of minimal AD set [6]. The upper AD value $\Gamma_a(G)$ is highest cardinality of an AD set in G.

A D-2D sub-collection D contains the property, every points in V/D having at most length 2 at least 1 point in D. The smallest value of minimal D-2D set marked as $\gamma_{\leq 2}(G)$ [4].

Cockayne, Dawes, and Hedetniemi, initialized the concept total domination (TD) in graphs (1980) [4].

A TD set $D(G)$ holds the property such that $\langle D \rangle$ has no isolated points. Smallest collection in minimal TD set, denoted by $\gamma_t(G)$.

Accurate total domination (ATD), concept was contributed by Kattimani. M.B. et al. in 2012 [7].

An ATD collection with the property V/D has no TD set of $|D|$. An ATD number $\gamma_{at}(G)$ represent the least numerical value of minimal ATD set [7].

**Definition 1.1**

An AD-2D set D holds following properties,

(i). D is D-2D set

(ii). V/ D has no D-2D set of $|D|$

The AD-2D number $\gamma_{a\leq 2}(G)$ formed by selecting, numerically smallest value from minimal AD-2D in G. The upper AD-2D number written by $\Gamma_{a\leq 2}(G)$ is the highest value of an AD-2D collection.

**Example 1**

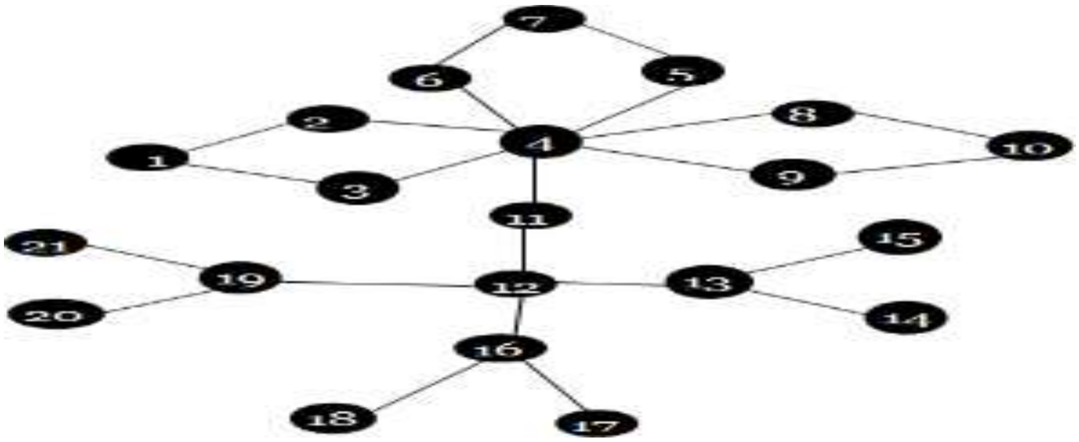



**Figure 1.**

Here AD-2D collections are {4, 12}, {1, 7, 10,12}, {1,7,10,11,14,17,20}, {4,13,16,19}. Thus, $\gamma_{a\leq 2}(G) = 2,\ \Gamma_{a\leq 2}(G) = 7$.

## 2. Few named graphs with their values of $\gamma_{a\leq 2}(G)$

### 2.1. Observation:

**1.** For $P_n$ ,

$$\gamma_{a\leq 2}(P_n) = \begin{cases} p, & \text{for } n = 5p, p = 1,2,3 \ldots \\ \lfloor \frac{n}{2} \rfloor + 1, & \text{for } n \neq 5p, p = 1,2,3 \ldots \end{cases}$$

**2.** In $C_n$,

$$\gamma_{a\leq 2}(C_n) = \lfloor \frac{n}{2} \rfloor + 1$$

**3.** For $W_n$,

$$\gamma_{a\leq 2}(W_n) = \lceil \frac{n}{2} \rceil + 1$$

**4.** In $F_n$,

$$\gamma_{a\leq 2}(F_n) = n + 1$$

**5.** For $K_n$ ,

$$\gamma_{a\leq 2}(K_n) = \lfloor \frac{n}{2} \rfloor + 1$$

**6.** In $K_{1,m}$ ,

$$\gamma_{a\leq 2}(K_{1,m}) = \lfloor \frac{1+m}{2} \rfloor + 1$$

**7.** For $K_{n,m}$ , for $m , n \geq 1$

$$\gamma_{a\leq 2}(K_{n,m}) = \lfloor\frac{n+m}{2}\rfloor + 1$$

**8.** In $B_n$ , $\gamma_{a\leq 2}(B_n) = n + 2$

**9.** For $H_n$ ,

$$\gamma_{a\leq 2}(H_n) = n + 1$$

**10.** For Grid graph $P_i$ X $P_j$ , for i, j ≥2

$$\gamma_{a\leq 2}(P_i XP_j) = \lfloor\frac{i X j}{2}\rfloor + 1$$

**11.** For n-Barbell graph,

$$\gamma_{a\leq 2}(n - \text{barbell}) = n + 1$$

**12.** For $L_n$, for n ≥1

$$\gamma_{a\leq 2}(L_n) = n + 1$$

**13.** For any Lollipop graph $I_{m,n}$, for m ≥1, n ≥1

$$\gamma_{a\leq 2}(I_{m,n}) = \lfloor\frac{m+n}{2}\rfloor + 1$$

**3. Relationship between $\gamma_{a\leq 2}(G)$ with other parameters**

**Proposition 3.1**

A Grid graph $P_2 XP_j$, then $\gamma_{a\leq 2}(P_2 XP_j) = \beta_0(G) + 1$.

**Proposition 3.2**

If G = $C_n$ or $K_n$, $\gamma_{a\leq 2}(G) = \gamma_a(G)$ .

**Proposition 3.3**

In Friendship graph $F_n$ , (i). $\gamma_{a\leq 2}(F_n) = \gamma_a(F_n) + n$, (ii) $\gamma_{a\leq 2}(F_n) = \alpha_0$.

**Proposition 3.4**

For Helm graph $H_n$, $\gamma_{a\leq 2}(H_n) = \gamma_a(H_n) + 1$.

**Proposition 3.5**

For Book graph $B_n$, (i). $\gamma_{a\leq 2}(B_n) = \gamma_a(B_n) + n$, (ii) $\gamma_{a\leq 2}(B_n) = \alpha_0$.

**Proposition 3.6**

In Wheel graph $W_n$, $\gamma_{a\leq 2}(W_n) + \beta_0 = n$,

**Proposition 3.7**

An AD – 2D number of $H_n$ is same to AD– 2D value of $F_n$ ,

**Proof**

From observation2.1, we concluded $\gamma_{a\leq 2}(H_n) = \gamma_{a\leq 2}(F_n)$.

**Proposition 3.8**

For $W_n$, (or) $H_n$, $\gamma_{a\leq 2}(W_n) = \alpha_0 = \gamma_{a\leq 2}(H_n)$

**Proposition 3.9**

In connected graph, for $n \geq 3$, then $\gamma_{a\leq 2}(G) + \gamma(G) \leq n$,

**Proposition 3.10**

If $G = H_n$ , then $\gamma_{a\leq 2}(G) + \gamma_a(G) = n$.

**Proposition 3.11**

For fan graph $F_{n,m}$,, then $\gamma_{a\leq 2}(F_{n,m}) \leq \lceil\frac{n+m}{2}\rceil + 1$

**Proof**

Based on the construction of $F_{n,m}$ , maximum length between each node is one. So AD-2D set contains $\lceil\frac{n+m}{2}\rceil + 1$ vertices. Hence, $\gamma_{a\leq 2}(F_{n,m}) \leq \lceil\frac{n+m}{2}\rceil + 1$.

**Proposition 3.12**

If $G = C_n$ , (or ) $K_n$, $\gamma_{a\leq 2}(G) = \gamma_t(G) = \gamma_{at}(G)$

**Proposition 3.13**

For $H_n$, $\gamma_{a\leq 2}(H_n) = \gamma_{at}(H_n) + 1$.

**4. Upper and Lower bounds of $\gamma_{a\leq 2}(G)$**

**Theorem 4.1**

In G, $\gamma_{\leq 2}(G) \leq \gamma_{a\leq 2}(G)$

**Proof**

Every AD- 2D set in G is D– 2D set for same G.

We conclude, $\gamma_{\leq 2}(G) \leq \gamma_{a\leq 2}(G)$.

**Theorem 4.2**

For G, $\gamma(G) \leq \gamma_{a\leq 2}(G)$

**Proof**

Each AD- 2D collection of graph satisfies domination condition.

Hence, $\gamma(G) \leq \gamma_{a\leq 2}(G)$.

**Theorem 4.3**

Except a path with points 5n, for n ≥ 1, then $\gamma_a(G) \leq \gamma_{a\leq 2}(G)$.

**Proof**

From the observation 2.1 (1), all AD- 2D selection of G is an AD set in G.

We have, $\gamma_a(G) \leq \gamma_{a\leq 2}(G)$.

**Theorem 4.4**

If G contains an isolated point, then minimal D – 2D collection of G is an AD- 2D set.

**Proof**

Every isolated nodes are in D-2D collection D in G. Then V-/D has no D-2D set with |D|. Hence the desired result follows.

**Corollary 4.5**

If $\gamma_{\leq 2}(G) = \gamma_{a\leq 2}(G)$, where G has isolates.

**Theorem 4.6**

In G, an AD-2D set has $\lfloor\frac{n}{2}\rfloor + 1$ .

**Proof**

Suppose D contained in Vis a D- 2D set with $\lfloor\frac{n}{2}\rfloor + 1$ vertices. So D contains more than half of points inV. Hence D treated as AD- 2D set.

**Theorem 4.7**

In non disconnected graph, $\gamma_{a\leq 2}(G) \leq n - \gamma_{\leq 2}(G) + 1$, this bound is sharp.

**Proof**

Consider a minimum D–2D collection, then take $v \in D, (V/D) \cup \{v\}$ forms an AD-2D collection in graph.

Hence, $\gamma_{a\leq 2}(G) \leq |(V/D) \cup \{v\}| = n - \gamma_{\leq 2}(G) + 1$.

**Corollary 4.8**

In G, $\gamma_{a\leq 2}(P_4) = n - \gamma(P_4) + 1$ if G = $P_4$.

**Theorem 4.9**

If $\gamma_{a\leq 2}(G) \leq \gamma_{a\leq 2}(H)$ where H is spanning connected sub-graph.

**Theorem 4.10**

The maximum limit and minimum limit of $\gamma_{a\leq 2}(G)$,

$$\frac{n}{\Delta+1} \leq \gamma_{a\leq 2}(G) \leq \frac{n\Delta}{\Delta+1} + 1,$$

**Proof**

By the reference [4], we have $\frac{n}{\Delta+1} \leq \gamma(G)$ also from reference [6] $\gamma(G) \leq \gamma_a(G)$

By Theorem 4.3 we obtained, $\gamma_a(G) \leq \gamma_{a\leq 2}(G)$. Using this $\frac{n}{\Delta+1} \leq \gamma_{a\leq 2}(G)$.

From Theorem 4.7 , $\gamma_{a\leq 2}(G) \leq n - \gamma_{\leq 2}(G) + 1$

$$\leq n - \frac{n}{\Delta+1} + 1$$

$$\leq \frac{n\Delta}{\Delta+1} + 1.$$

Hence, $\frac{n}{\Delta+1} \leq \gamma_{a\leq 2}(G) \leq \frac{n\Delta}{\Delta+1} + 1$.

**Theorem 4.11**

If G, not a complete bipartite graph, then $\gamma_{a\leq 2}(G) \leq \alpha_0 + 1$. Furthermore, equality obtained if G = $H_n$ or $P_4$.

**Theorem 4.12**

For $T_n$ with m cut nodes, $\gamma_{a\leq 2}(T_n) \leq m + 1$.

**Proof**

Choose D, collection of all cut points with |D|= m. Then any pendent point $v \in T_n$, $D \cup \{v\}$ makes an AD-2D set in $T_n$. Here, each dominating collections form D-2D, hence that collection defines an AD-2D. $\therefore \gamma_{a\leq 2}(G) \leq m + 1$.

**Corollary 4.13**

In tree, $\gamma_{a\leq 2}(T_n) \leq n - k + 1$ , where pendent node |k|

**Theorem 4.14**

For G, not $K_{n,m}$, then $\gamma_{a\leq 2}(G) \leq n - \beta_0 + 1$. Furthermore, same values obtains when G = $H_n$.

**Corollary 4.15**

In $K_{n,m}$ , $\gamma_{a\leq 2}(G) \leq \beta_0(G)$.

**Theorem 4.16**

If $2 \leq \gamma_{a\leq 2}(G) \leq n + 2$,when G without isolates.

**Proof**

From observation 2.1, we reach the bounds.

**Nordhas - Gaddum Type results**

**Theorem 4.17**

If G and $\bar{G}$ has no isolated nodes,

(i). $4 \leq \gamma_{a\leq 2}(G) + \gamma_{a\leq 2}(\bar{G}) \leq 2(n + 2)$

(ii). $4 \leq \gamma_{a\leq 2}(G).\gamma_{a\leq 2}(\bar{G}) \leq (n + 2)^2$

**Theorem 4.18**

If $\gamma_{a\leq 2}(G) + \chi(G) \leq 2n + 2$, where $\chi(G)$,chromatic cardinality in G.

**Proof**

We know $\chi(G) \leq n$, and by Theorem 4.12. $\gamma_{a\leq 2}(G) \leq n + 2$.

$$\therefore \gamma_{a\leq 2}(G) + \chi(G) \leq 2n + 2.$$

**Theorem 4.19**

In cubic connected graph ,$\gamma_{a\leq 2}(G) = \lfloor \frac{n}{2} \rfloor + 1$.

**Theorem 4.20**

If $\gamma_{a\leq 2}(G) \leq \gamma_t(G) + 1$, where G contains pendent points. Furthermore, if $G = H_n$.we arrive equality.

**Proof**

Consider v, end point and D total dominating collection of graph. Then we found u which is at most length two to v.

**Case 1.** Suppose $v \in D$. Then $u \in D$, Hence D becomes an AD-2D collection.

We have, $\gamma_{a\leq 2}(G) = |D| \leq \gamma_t(G)$.

**Case 2.** Suppose $v \notin D$. And $u \in D$. Then $D \cup \{v\}$ changed as AD-2D set. Thus $\gamma_{a\leq 2}(G) \leq |D \cup \{v\}| \leq \gamma_t(G) + 1$.

In Helm graph $H_n$, $\gamma_{a\leq 2}(H_n) = \gamma_t(H_n) + 1$.

**5. AD-2D number for few special graph families**

**Theorem 5.1**

For path, $\gamma_{a\leq 2}(C(P_n)) = \lceil\frac{n}{2}\rceil$.

**Proof**

Method of constructing central graph follows these steps:

(i).join all non ad joint points

(ii).add new points in each edge of original graph

Apply the above steps in $P_n$, we received 2n-1 vertices. Choose $\lceil\frac{n}{2}\rceil$ nodes its forms AD-2D set. Hence, $\gamma_{a\leq 2}(C(P_n)) = \lceil\frac{n}{2}\rceil$

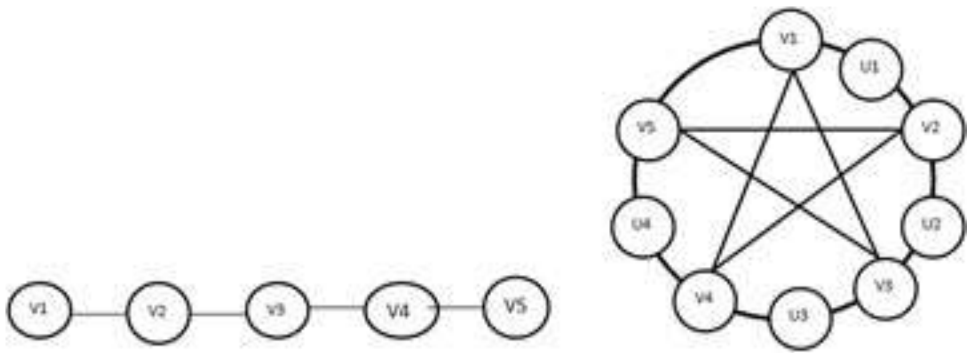



**Figure 2: The graph $C_5$, $C(C_5)$ and $\gamma_{a\leq 2}(C(P_5)) = 5$.**

**Theorem 5.2**

In $C_n$, $\gamma_{a\leq 2}(C(C_n)) = n + 1$.

**Proof**

Assume that $V(C_n) = \{v_1, v_2, v_3, \ldots v_n\}$, $E(C_n) = \{e_1, e_2, e_3, \ldots e_n\}$ with $e_n = v_n v_1$ and $e_i = v_i v_{i+1}$, $(1 \leq i \leq n - 1)$.

In $C(C_n)$ contains the nodes $\{v_i\} \cup \{u_i\}$ where $u_i$ is points produced by sub-dividing all lines.

Select all $\{v_i\}$ together one of $\{u_i\}$ its becomes AD-2D set .

$$\therefore \ \gamma_{a\leq 2}(C(C_n)) = n + 1.$$

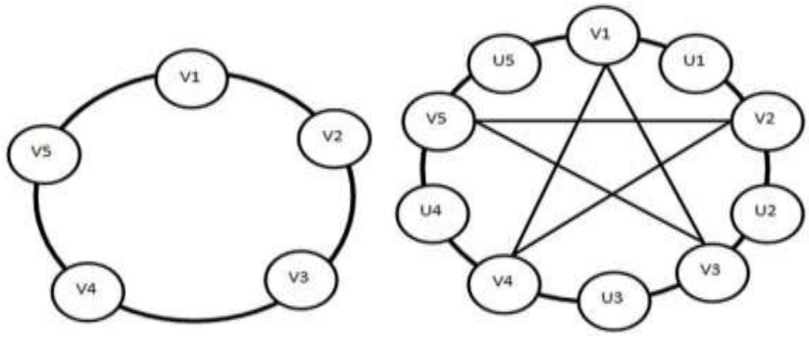



**Figure 3: The graph $C_5$, $C(C_5)$ and $\gamma_{a\leq2}(C(C_5)) = 6$.**

**Theorem 5.3.**

For any star $K_{1,n}$, $\gamma_{a\leq2}[C(K_{1,n})] = n + 1$.

**Proof**

Consider $V(K_{1,n}) = \{v, v_1, v_2, v_3, \ldots v_n\}$ where deg $v = n$. Centralization of star, we mark the nodes of sub-division by $u_1, u_2, u_3, \ldots, u_n$. Naming the lines, $e_i = v_iu_i$ and $e'_i = vu_i$. In $C(K_{1,n})$, the sub-graph made by the nodes $\{v_1, v_2, v_3, \ldots v_n\}$ is $K_n$.

Consider the nodes in $K_n$ with $\{v\}$, totally n+1 points makes an AD-2D collection. Finally,we have $\gamma_{a\leq2}[C(K_{1,n})] = n + 1$.

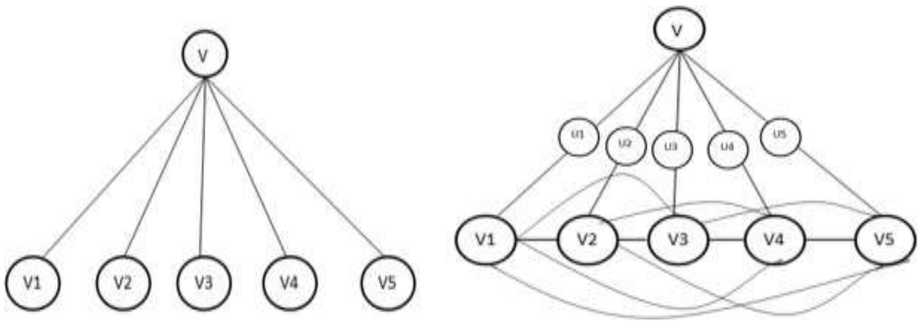



**Figure 4: The graph $K_{1,5}$ , $C(K_{1,5})$and $\gamma_{a\leq2}(C(K_{1,5})) = 6$.**

**Theorem 5.4**

For $P_n$ , $\gamma_{a\leq2}(M(P_n)) = \lceil\frac{n}{2}\rceil$.

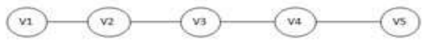
**Proof**

Based on Middle graph construction apply the steps:

(i). In original graph include new nodes to every lines

(ii). Form a complete graph using new points

In $M(P_n)$, we found 2n-1 points. Now, take $\lceil \frac{n}{2} \rceil$ vertices, this forms AD-2D collection. Hence, $\gamma_{a\leq 2}(M(P_n)) = \lceil \frac{n}{2} \rceil$



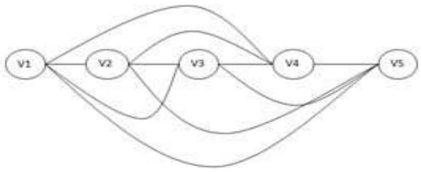




**Figure 5: The graph $P_5$ and $M(P_5)$ and , $\gamma_{a\leq 2}(M(P_5)) = 3$**

**Theorem 5.5**

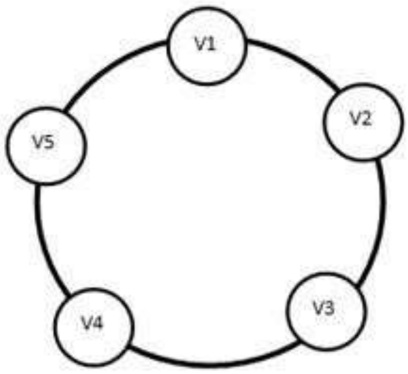
For $C_n$ , $\gamma_{a\leq 2}(M(C_n)) = n + 1$.

**Proof**

Consider, $\{v_1, v_2, v_3, \ldots v_n\}$ and $\{e_1, e_2, e_3, \ldots e_n\}$ are points, lines in $C_n$. In $M(C_n)$ , $V(C_n) \cup E(C_n)$ be node set .Each $e_i$ adjoint with $e_{i+1}$ for $i = 1,2,3 \ldots, n - 1$, $e_n$ is adjoint with $v_1$.

In $M(C_n)$, we select $\{v_i\}$ and one node in $\{u_i\}$ ,here $u_i$ is vertices produced by lines sub-division . This collection forms AD-2D set.

$$\therefore \gamma_{a\leq 2}(M(C_n)) = n + 1.$$



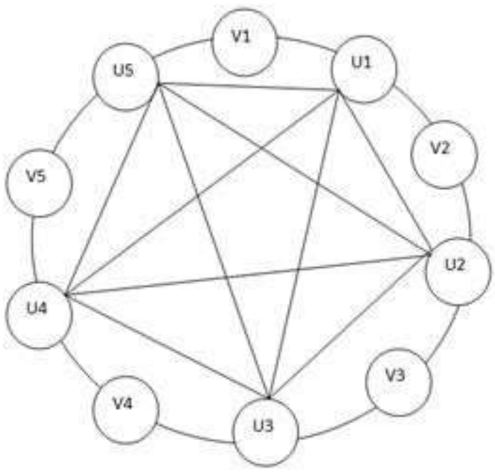




**Figure 6: The graph of $C_n$, $M(C_n)$ and $\gamma_{a\leq 2}(M(C_5)) = 6$.**

**Theorem 5.6**

For any star graph $K_{1,n}$, $\gamma_{a\leq 2}[M(K_{1,n})] = n + 1$.

**Proof**

Let $V(K_{1,n}) = \{v, v_1, v_2, v_3, \dots v_n\}$ and $E(K_{1,n}) = \{e_1, e_2, e_3, \dots e_n\}$. In $V[M(K_{1,n})] = \{v\} \cup \{e_i\} \cup \{v_i\}$ in which the vertices $e_1, e_2, e_3, \dots e_n, v$ induces a clique of order $n + 1$.

In $M(K_{1,n})$, Consider , $\{\{v\} \cup \{v_i\}\}$ produces a AD-2D collection.

Hence we received, $\gamma_{a\leq 2}[M(K_{1,n})] = n + 1$.

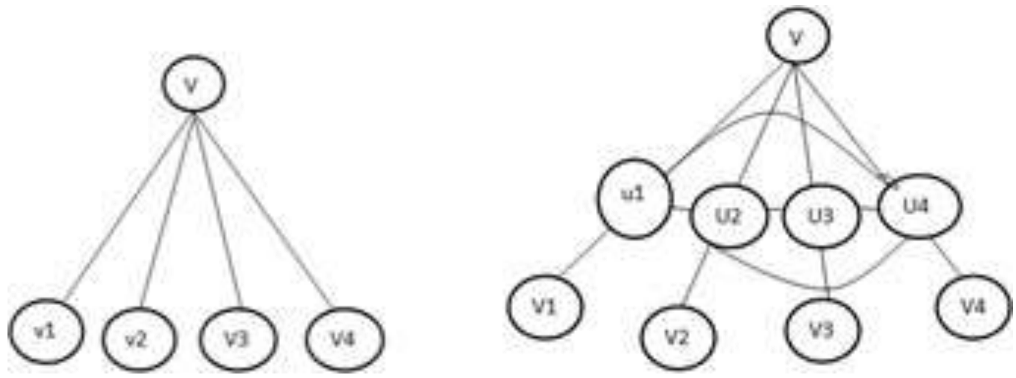



**Figure 7: The graph of $K_{1,4}$ , $M(K_{1,4})$ and $\gamma_{a\leq 2}[M(K_{1,4})] = 5$.**

**Theorem 5.7**

For any closed helm graph $CH_n$, $\gamma_{a\leq 2}[CH_n] = 1$.

**Proof**

Let the set D has the apex vertex of the closed helm graph. The remaining vertices in <V-D> are all within the distance two from the apex vertex in D. Then D frms a distance - 2 dominating set of the closed helm graph. Thus $\gamma_{\leq 2}[CH_n] = 1$.

But in <V-D> we cannot able to find a distance -2 dominating set with the cardinality 1. Hence $\gamma_{a\leq 2}[CH_n] = 1$.

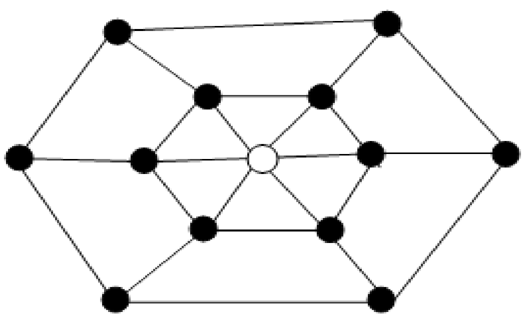


**Figure 8: Closed helm with $\gamma_{a\leq 2}\ (CH_6) = 1$.**

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