# A Numerical Approximation of the Approximation of Stress Strength Reliability System in a View of Rayleigh and Half Normal Distribution

Osama Abdulaziz Alamri[1], Fahad M. Alharbi[2]

[1]Department of Statistics, Faculty of Science, University of Tabuk, Tabuk 71491, Saudi Arabia.

[2]Department of Mathematics, Faculty of Science, University of Tabuk, Tabuk 71491, Saudi Arabia.

**Article Info**
**Page Number: 2901-2905**
**Publication Issue:**
**Vol. 71 No. 4 (2022)**

**Article History**
**Article Received: 25** March 2022
**Revised: 30** April 2022
**Accepted: 15** June 2022
**Publication: 19** August 2022

**Abstract**
The significance topic recently stands the “stress-strength reliability” "SSR" expression that denotes to the measure probability P (X >Y) in the research papers. It reorganises a model by chance strength (X) in terms of a chance strength (Y), i.e.; a model fails in scenario the strength tends below the stress. This research aims to examine the "SSR" as the strength (X) obeys Rayleigh-half-normal distribution "RHND" whereas stress (Y1, Y2, Y3, and Y4) follows "RHND", exponential distribution, Rayleigh distribution, and half-normal distribution, correspondingly. This will contain defining the general designs of the reliabilities of a system. In addition, the maximum likelihood approximation method and method of moment (MOM) will be used to choose appropriate parameters values. Lastly, reliability has been achieved applying numerous values of stress and strength parameters.

## 1. Introduction

The “stress-strength models” “SSM” have been used wildly to describe a life component, such, in reliability concept, a random strength (X) varies in terms of a random stress (Y). such component fails when the strength tends underneath the stress. Note that, this component is scaled by R = P(Y < X). quantity takes a several of applications, particularly in the manufacturing of engineering.

The term stress- strength has firstly been presented by "Church and Harris" [1].Then, numerous distributions kinds are adopted for strength and stress. For example, numerous distributions are discussed in a view of the approximation issues of the “SSM” - see [2, 3, 4, 5]. Recently, Kotz et al. [6] summaries all approaches and consequences about the “SSR”. The approximation of reliability utilized a finite mixture of opposite Gaussian distributions are investigated by Akman et al. [7]. Similarly, AI-Hussaini [8] considered to investigate the approximation of R = P(Y < X) established on a finite mixture of lognormal components. For additional information, see [9–13].

## 2. *RHND* With a Finite Mixture

In Abd El-Monsef [14] have donated the distribution of Rayleigh-half-normal via RHN(θ). The following model is presented in a view of using a mixture of RHND with a parameter $1/\sqrt{2\theta}$ is applied to denote this case. It gives as

$$f(x,\theta) = Kf_R\left(\frac{x}{\sqrt{2\theta}}\right) + (1-K)f_{NH}\left(\frac{x}{\sqrt{2\theta}}\right) = K\left(2\theta x e^{-\theta x^2}\right) + (1-K)\left(2\sqrt{\frac{\theta}{\pi}}e^{-\theta x^2}\right), (1)$$

As $K = (1/(1+\sqrt{\pi\theta}))$.

Therefore, the *RHND* (pdf) is assumed through

$$f(x,\theta) = \frac{2\theta(x+1)e^{-\theta x^2}}{1+\sqrt{\pi\theta}}, \quad x,\theta > 0. \quad (2)$$

The corresponding (cdf) is assumed by

$$F(x,\theta) = 1 - e^{-\theta x^2} + \frac{\sqrt{\pi\theta}\,\mathrm{erf}\left(\sqrt{\theta}x\right)}{1+\sqrt{\pi\theta}}, \ x,\theta > 0, \ (3)$$

As $\mathrm{erf}(u)$ is the Gauss error function described as

$$\mathrm{erf}(u) = \frac{2}{\sqrt{\pi}}\int_0^u e^{-t^2}dt. \quad (4)$$

2.1 *The Survival, and Hazards Function*

The survival function might be defined as follows

$$S(x) = 1 - F(x) = e^{-\theta x^2} + \frac{\sqrt{\pi\theta}erfc\left(\sqrt{\theta}x\right)}{1+\sqrt{\pi\theta}}, \quad (5)$$

where $\mathrm{erf}(u)$ is the complementary error function, and it is defined as

$$erfc(u) = 1 - \mathrm{erf}(u) = \frac{2}{\sqrt{\pi}}\int_u^\infty e^{-t^2}dt. \quad (6)$$

The function of hazard rate of the RHND is known by

$$h(x) = \frac{f(x)}{S(x)} = \frac{2\theta(1+x)}{1+e^{-\theta x^2}\sqrt{\pi\theta}\,\mathrm{erf}\left(\sqrt{\theta}x\right)}. \ (7)$$

## 3. SSR Calculations

This section, the reliability was derived R = P(Y < X. So, let the strength (x) and the stress (y) be two statistically independent random variables. Thus, (*X*), and (*Y*) follows a joint probability density function f(x, $\theta$), and hence the component reliability yields

$$\mathrm{R} = \mathrm{P}(\mathrm{Y} < \mathrm{X}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dy\,dx \qquad (8)$$

In scenario that the random variables are independent, then $f(x,y) = f(x)\,g(y)$ so that

$$\mathrm{R} = \int_{-\infty}^{\infty}\int_{-\infty}^{x} f(x)g(y)dy\,dx\,, \qquad (9)$$

As f(x), g(y) are follow pdf's of *X* and *Y*.

In what follows, we assume that the Stress and the Strength meets "RHND". Since the strength X ~ RHN($\theta$) and $Y_1$ ~ RHN($\theta_1$), both are independent random variables with probability distribution function (pdf) f (x) and g($y_1$ ), correspondingly:

$$f(x) = \frac{2\theta(x+1)e^{-\theta x^2}}{1+\sqrt{\pi\theta}}, \quad x.\theta > 0, \; g(y_1) = \frac{2\theta(y_1+1)e^{-\theta y_1^2}}{1+\sqrt{\pi\theta}}, \quad y_1.\theta > 0 \qquad (10)$$

Now, Appling the first along with the second to (9), we reach, after some manipulation, to

$$R = \frac{1+\sqrt{\pi\theta_1}+2\sqrt{\theta\theta_1}\tan^{-1}\sqrt{\frac{\theta}{\theta_1}}+\left(\frac{\sqrt{\pi}(\theta-\theta_1)}{\sqrt{\theta+\theta_1}}\right)-\left(\frac{\theta_1}{\theta+\theta_1}\right)}{1+\sqrt{\pi\theta}+\sqrt{\pi\theta_1+\pi\sqrt{\theta\theta_1}}}. \qquad (11)$$

## 4. Numerical Evaluation and conclusion

With appropriated selections of the parameters values, the system readability R has evaluated. So, from Table 1, Figures 1 and 2, we clearly seen that the value of reliability decreases with increasing the parameter of strength values. In addition, the rate of reliability rises if the stress parameter rises.

In conclusion, this research investigated the "SSR" for "RHN" when the strength (*X*) obeys "RHN" distribution, while the stress (*Y*) takes "RHN" distribution. Based on the numerical results and figures, it can be emphasised that when the reliability value is decreased when stress parameter is increased, also, once the parameter of strength is increased, as well as reliability value increases.

**TABLE 1: Difference in *R* when the strength and stress has RHN distribution.**

| | $\theta_1$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\theta$ | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **1** |
| **0.1** | 0.208 | 0.359 | 0.472 | 0.558 | 0.624 | 0.677 | 0.718 | 0.752 | 0.780 | 0.803 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.2** | 0.149 | 0.267 | 0.364 | 0.442 | 0.507 | 0.561 | 0.606 | 0.645 | 0.678 | 0.706 |
| **0.3** | 0.121 | 0.222 | 0.307 | 0.379 | 0.440 | 0.493 | 0.538 | 0.577 | 0.611 | 0.641 |
| **0.4** | 0.105 | 0.194 | 0.271 | 0.337 | 0.395 | 0.445 | 0.490 | 0.529 | 0.563 | 0.594 |
| **0.5** | 0.093 | 0.174 | 0.245 | 0.307 | 0.362 | 0.410 | 0.453 | 0.491 | 0.526 | 0.556 |
| **0.6** | 0.085 | 0.159 | 0.225 | 0.284 | 0.336 | 0.382 | 0.424 | 0.461 | 0.495 | 0.526 |
| **0.7** | 0.078 | 0.148 | 0.210 | 0.265 | 0.315 | 0.360 | 0.400 | 0.437 | 0.470 | 0.500 |
| **0.8** | 0.073 | 0.138 | 0.197 | 0.250 | 0.297 | 0.341 | 0.380 | 0.416 | 0.448 | 0.478 |
| **0.9** | 0.068 | 0.130 | 0.186 | 0.237 | 0.283 | 0.325 | 0.363 | 0.398 | 0.430 | 0.459 |
| **1** | 0.065 | 0.124 | 0.177 | 0.226 | 0.270 | 0.311 | 0.348 | 0.382 | 0.413 | 0.443 |

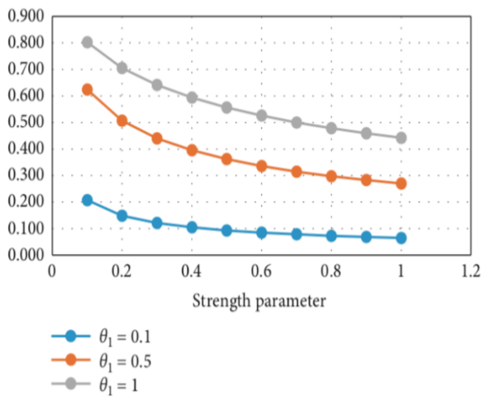


FIGURE 1: Variation in $R_1$ for constant stress.

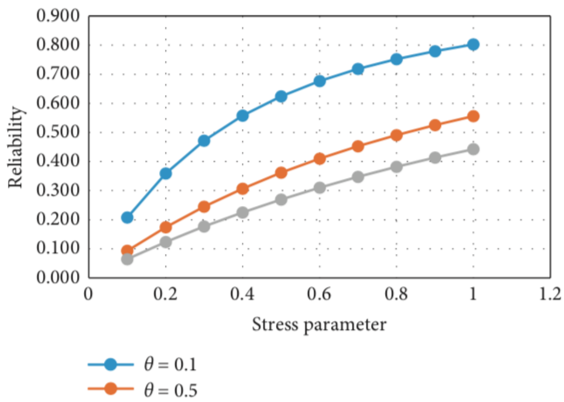


FIGURE 2: Variation in $R_1$ for constant strength.

## Reference

1. J. D. Church and B. Harris, "The estimation of reliability from stress-strength relationships," *Technometrics*, vol. 12, no. 1, pp. 49–54, 1970.
2. J. G. Surles and W. J. Padgett, "Inference for reliability and stress-strength for scald Burr *X* distribution," *Lifetime Data Analysis*, vol. 7, no. 2, pp. 187–200, 2001.
3. M. Z. Raqab and D. Kundu, "Comparison of different esti- mates of $P(Y < X)$ for a scaled Burr type *X* distribution," *Communications in Statistics - Simulation and Computation*, vol. 34, no. 2, pp. 465–483, 2005.
4. N. Mokhlis, "Reliability of a stress-strength model with Burr type III distributions," *Communications in Statistics—Theory and Methods*, vol. 34, no. 7, pp. 1643–1657, 2005.
5. B. Saraçog̃lu, İ. Kınacı, and D. Kundu, "On estimation of $(P(Y < X)$ for exponential distribution under progressive type-II censoring," *Journal of Statistical Computation and Simulation*, vol. 82, no. 5, pp. 729–744, 2011.
6. S. Kotz, Y. Lumelskii, and M. Pensky, *The Stress-Strength Model and its Generalizations: Theory and Applications*, World Scientific Publishing, Singapore, 2003.
7. O. Akman, P. Sansgiry, and M. C. Minnotte, "On the esti- mation of reliability based on mixture inverse Gaussian distributions," *Applied Statistical Science*, Vol. IV, Nova Science Publishers, New York, NY, USA, 1999.
8. E. K. AI-Hussaini, "Bayesian prediction under a mixture of two exponential components model based on type 1 cen- soring," *Journal of Applied Statistical Science*, vol. 8, pp. 173–185, 1999. [9]

9. M. M. M. El-Din, M. M. Amein, H. E. El-Attar, and E. H. Hafez, “Symmetric and asymmetric bayesian estimation for lindley distribution based on progressive first failure censored data,” *Mathematical Sciences Letters*, vol. 6, no. 3, pp. 255–260, 2017, þ.
10. J. Zhao, Z. Ahmad, E. Mahmoudi, E. H. Hafez, and M. M. Mohie El-Din, “A new class of heavy-tailed distribu- tions: modeling and simulating actuarial measures,” *Com- plexity*, vol. 2021, Article ID 5580228, , 2021.
11. H. M. Almongy, E. Almetwally, H. M. Aljohani, A. S. Alghamdi, and E. H. Hafez, “A new extended Rayleigh distribution with applications of COVID-19 data,” *Results in Physics*, vol. 23, no. 60, þ, Article ID 104012, 2021.
12. H. M. Almongy, F. Y. Alshenawy, E. M. Almetwally, and D. A. Abdo, “Applying transformer insulation using Weibull extended distribution based on progressive censoring scheme,” *Axioms*, vol. 10, no. 2, p. 100, 2021.
13. M. H Abu-Moussa, A. M. Abd-Elfattah, and E. H Hafez, “Estimation of stress-strength parameter for Rayleigh distri- bution based on progressive type-II censoring,” *Information Sciences Letters*, vol. 10, no. 1, p. 12, 2021.
14. M. M. E. Abd El-Monsef and M. M. Abd El-Raouf, “Rayleigh- half normal distribution for modeling tooth movement data,” *Journal of Biopharmaceutical Statistics*, vol. 30, no. 3, pp. 481–494, 2020