Limited Condition for Device that Perform Bipolar Fuzzy

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Abstract
Utilizing the idea of bipolar fuzzy sets, the ideas of limited condition for device that perform bipolar fuzzy (LCDBF), bipolar successor, bipolar subsystems, bipolar submachines, bipolar q-twins also, bipolar retrievable LCDBF are presented, and related properties are examined. Relations between bipolar q-twins and bipolar q-related LCDBF are given. A portrayal of a bipolar retrievable LCDBF is given.

Keywords- bipolar successor, bipolar exchange property, strongly bipolar connected, bipolar retrievable, bipolar *q*-related, bipolar *q*-twins.

1. Introduction

Bipolar fuzzy sets (BFS) are extension of fuzzy sets whose membership degree ranges from [−1, 1] [6]. It is worth mentioning that BFS and vague sets look to be comparable, however they are totally various sets. Despite the fact that the two sets handle with fragmented information, they won't adjust the vague or conflicting data which shows up in numerous areas. Distance measure is a significant tool which depicts difference among two items and deliberated as a double idea of similarity measure [3]. Bipolar fuzzy qualities or BFS in the system of penta-esteemed portrayal [1]. Malik et al. [4] presented the thoughts of limited condition device and subdevice of fuzzy, isolated and associated and talked about their fundamental properties. Kumbhojkar and Chaudhari [2] gave multiple approaches to building results of limited condition for device of fuzzy and their common relationship, through isomorphism and covers. Vasile Patrascu [5] introduced the concept of entropy and cardinality of bipolar fuzzy set and their similarity. In this paper, utilizing the idea of bipolar fuzzy sets, we present the ideas of limited condition for device that perform bipolar fuzzy (LCDBF), bipolar replacements, bipolar subsystems, bipolar submachines, bipolar q-twins, and bipolar retrievable LCDBF, and concentrate on correlated properties. We give relations between bipolar q-twins and a bipolar q-related LCDBF. We give a portrayal of a bipolar retrievable LCDBF.

2. Condition for device that perform bipolar fuzzy

Definition 2.1. Let $T=\left(\mu_T^P,\mu_T^N\right)$ be the BFS then Ψ denotes limited condition for device that perform bipolar fuzzy (LCDBF) denoted as $\Psi=\left(R,Z,T\right)$, where R and Z are finite set, the state set and the input symbol set, respectively.

Allow Z^* to indicate the arrangement of all expressions of components of Z of limited length. Let ς signify the vacant word in Z^* and $|Z|$ mean the length of Z for each $z \in Z^*$.

Definition 2.2. Let $\Psi = (R, Z, T)$ be LCDBF. Defined on $T^* = (\mu_{T^*}^P, \mu_{T^*}^N)$ by

$$\mu_{T^*}^P(r,\varsigma,s) = \begin{cases} 1 & if \quad r = s, \\ -1 & if \quad r \neq s, \end{cases} \qquad \mu_{T^*}^N(r,\varsigma,s) = \begin{cases} -1 & if \quad r = s, \\ 1 & if \quad r \neq s, \end{cases}$$

$$\mu_{T^*}^P(r,zd,s) = \bigvee_{e \in R} \left[\mu_{T^*}^P(r,z,e) \wedge \mu_{T^*}^P(e,z,s) \right]$$
$$\mu_{T^*}^N(r,zd,s) = \bigwedge_{e \in R} \left[\mu_{T^*}^N(r,z,e) \vee \mu_{T^*}^N(e,z,s) \right]$$

for all $r, s \in R, z \in Z^*$ and $d \in Z$.

Lemma 2.3. If $\Psi = (R, Z, T)$ be LCDBF. Then

$$\mu_{T^*}^P(r,zg,s) = \bigvee_{e \in R} \left[\mu_{T^*}^P(r,z,e) \wedge \mu_{T^*}^P(e,z,s) \right]$$
$$\mu_{T^*}^N(r,zg,s) = \bigwedge_{e \in R} \left[\mu_{T^*}^N(r,z,e) \vee \mu_{T^*}^N(e,z,s) \right]$$

for all $r, s \in R$ and $z, g \in Z^*$.

Proof. Through an induction on $|g| = n$, we prove the result. If n=0 then $g = \varsigma$ and $zg = z\varsigma = z$. Hence

$$\begin{aligned} \bigvee_{e \in R} \left[\mu_{T^*}^P(r,z,e) \wedge \mu_{T^*}^P(e,z,s) \right] &= \bigvee_{e \in R} \left[\mu_{T^*}^P(r,z,e) \wedge \mu_{T^*}^P(e,\varsigma,s) \right] \\ &= \mu_{T^*}^P(r,z,s) \\ &= \mu_{T^*}^P(r,zg,s) \end{aligned}$$

and

$$\begin{aligned} \bigwedge_{e \in R} \left[\mu_{T^*}^N(r,z,e) \vee \mu_{T^*}^N(e,z,s) \right] &= \bigwedge_{e \in R} \left[\mu_{T^*}^N(r,z,e) \vee \mu_{T^*}^N(e,\varsigma,s) \right] \\ &= \mu_{T^*}^N(r,z,s) \\ &= \mu_{T^*}^N(r,zg,s) \end{aligned}$$

Therefore, n = 0 holds for this result. Assume that the result is true for all $v \in Z^*$ with the $|v| = n-1, n > 0$. Let $g = vd$ where $v \in Z^*$ and $d \in Z$, and $|v| = n-1$. Then

$$\mu_{T^*}^{P}(r,zg,s)=\mu_{T^*}^{P}(r,zvd,s)$$
$$=\bigvee_{e\in R}\left[\mu_{T^*}^{P}(r,zv,e)\wedge\mu_{T^*}^{P}(e,d,s)\right]$$
$$=\bigvee_{e\in R}\left[\bigvee_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\wedge\mu_{T^*}^{P}(f,v,e)\right]\wedge\mu_{T}^{P}(e,d,s)\right]$$
$$=\bigvee_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\wedge\left(\bigvee_{e\in R}\mu_{T^*}^{P}(f,v,e)\wedge\mu_{T}^{P}(e,d,s)\right)\right]$$
$$=\bigvee_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\wedge\mu_{T^*}^{P}(f,vd,e)\right]$$
$$=\bigvee_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\wedge\mu_{T^*}^{P}(f,g,e)\right]$$

and

$$\mu_{T^*}^{N}(r,zg,s)=\mu_{T^*}^{N}(r,zvd,s)$$
$$=\bigwedge_{e\in R}\left[\mu_{T^*}^{N}(r,zv,e)\vee\mu_{T^*}^{N}(e,d,s)\right]$$
$$=\bigwedge_{e\in R}\left[\bigwedge_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\vee\mu_{T^*}^{P}(f,v,e)\right]\vee\mu_{T}^{P}(e,d,s)\right]$$
$$=\bigwedge_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\vee\left(\bigwedge_{e\in R}\mu_{T^*}^{P}(f,v,e)\vee\mu_{T}^{P}(e,d,s)\right)\right]$$
$$=\bigwedge_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\vee\mu_{T^*}^{P}(f,vd,e)\right]$$
$$=\bigwedge_{f\in R}\left[\mu_{T^*}^{P}(r,z,f)\vee\mu_{T^*}^{P}(f,g,e)\right]$$

Definition 2.4. Let $\Psi=(R,Z,T)$ be LCDBF. If there is $d\in Z$ such that $\mu_{T}^{P}(r,d,s)>-1$ and $\mu_{T}^{N}(r,d,s)<1$, then s is called an bipolar instant possible successor of r. s is referred to as an bipolar successor. If s is an bipolar possible successor of r then their exists $z\in Z^*$ such that $\mu_{T^*}^{P}(r,d,s)>-1$ and $\mu_{T^*}^{N}(r,d,s)<1$. The bipolar possible successor of r is denoted by B(r). A subset of R is denoted by K, the set of all bipolar possible successor of K is denoted by B(K), is characterized to be

the set $B(K)=\bigcup\{B(r)\mid r\in K\}$.

Proposition 2.5. Let $\Psi=(R,Z,T)$ be LCDBF. The following holds for any $r,s,a\in R$

(i) $r\in B(r)$.

(ii) If $s\in B(r)$ and $a\in B(s)$ then $a\in B(r)$.

Proof. (i) Since $\mu_{T^*}^{P}(r,\varsigma,r)=1>-1$ and $\mu_{T^*}^{N}(r,\varsigma,r)=-1<1$, We have $r\in B(r)$.

(ii) If $s \in B(r)$ and $a \in B(s)$ then their exist $z, g \in Z^*$ such that $\mu^P_{T^*}(r,z,s) > -1, \mu^N_{N^*}(r,z,s) < 1, \mu^P_{T^*}(s,g,a) > -1$ and $\mu^P_{N^*}(s,g,a) < 1$. We have

$$\mu^P_{T^*}(r,zg,a) = \bigvee_{f \in R}\left[\mu^P_{T^*}(r,z,f) \wedge \mu^P_{T^*}(f,g,a)\right]$$
$$\geq \mu^P_{T^*}(r,z,s) \wedge \mu^P_{T^*}(s,g,a) > -1$$

and

$$\mu^N_{T^*}(r,zg,a) = \bigwedge_{f \in R}\left[\mu^N_{T^*}(r,z,f) \vee \mu^N_{T^*}(f,g,a)\right]$$
$$\geq \mu^N_{T^*}(r,z,s) \vee \mu^N_{T^*}(s,g,a) < 1$$

Therefore $a \in B(r)$.

Proposition 2.6. Let $\Psi = (R, Z, T)$ be LCDBF. The following holds for the subset of R,

(i) If $T \subseteq C$, then $B(T) \subseteq B(C)$.

(ii) $B(B(T)) \subseteq B(T)$.

(iii) $B(T \cup C) = B(T) \cup B(C)$.

(iv) $B(T \cap C) \subseteq B(T) \cap B(C)$.

Proof. The straightforward proof are (i),(iii) and (iv).

(ii) Clearly $B(T) \subseteq B(B(T))$. $r \in B(s)$ follows from $r \in B(B(T))$ for some $s \in B(T)$. Since $s \in B(T)$, there exists $a \in T$ such that $s \in B(a)$. As a result of the preceding proposition $r \in B(a) \subseteq B(T)$ so that $B(B(T)) \subseteq B(T)$. (ii) holds true.

Definition 2.7. Let $\Psi = (R, Z, T)$ be LCDBF. Ψ is said to satisfy the bipolar exchange property altogether $s, r \in R$ and $K \subseteq R$, Whenever $s \in B(K \cup \{r\})$ and $s \notin B(K)$ then $r \in B(K \cup \{s\})$.

Theorem 2.8. Let $\Psi = (R, Z, T)$ be LCDBF. Then the following holds.

(i) Ψ satisfy bipolar exchange property.

(ii) $(\forall s, r \in R)(s \in B(r) \Leftrightarrow r \in B(s))$.

Proof. Suppose Ψ satisfy bipolar exchange property. If $s \in B(r) = B(\phi \cup \{r\})$,Where $s, r \in R$ .Try to be aware that $s \notin B(\phi)$ thus $r \in B(\phi \cup \{s\}) = B(s)$. Correspondingly, $r \in B(s)$then $s \in B(r)$. On the other hand (ii) is substantial. Let $s, r \in R$ and $K \subseteq R$. On the off chance that $s \in B(K \cup \{r\})$ and $s \notin B(K)$, then $s \in B(r)$. From (ii) it follows that $r \in B(s) \subseteq B(K \cup \{s\})$.

Therefore Ψ satisfy bipolar exchange property.

Definition 2.9. Let $\Psi=(R,Z,T)$ be LCDBF. Let $\overline{R}=\left(\mu_{\overline{R}}^{P},\mu_{\overline{R}}^{N}\right)$ be an BFS in R. Then $\left(R,\overline{R},Z,T\right)$ is called an bipolar subsystem of Ψ if for all $s,r\in R$ furthermore $d\in Z$,

$$\mu_{\overline{R}}^{P}(r)\geq \mu_{\overline{R}}^{P}(s)\wedge \mu_{T}^{P}(s,d,r),$$
$$\mu_{\overline{R}}^{N}(r)\leq \mu_{\overline{R}}^{N}(s)\vee \mu_{T}^{N}(s,d,r).$$

If $\left(R,\overline{R},Z,T\right)$ is an bipolar subsystem of Ψ, we essentially compose $\overline{R}$ for $\left(R,\overline{R},Z,T\right)$.

Theorem 2.10. Let $\Psi=(R,Z,T)$ be LCDBF. Let $\overline{R}=\left(\mu_{\overline{R}}^{P},\mu_{\overline{R}}^{N}\right)$ be an BFS in R. Then $\overline{R}$ is called an bipolar subsystem of Ψ if and only if

$$\mu_{\overline{R}}^{P}(r)\geq \mu_{\overline{R}}^{P}(s)\wedge \mu_{T^*}^{P}(s,z,r),$$
$$\mu_{\overline{R}}^{N}(r)\leq \mu_{\overline{R}}^{N}(s)\vee \mu_{T^*}^{N}(s,z,r).$$

for all $s,r\in R$ furthermore $z\in Z^*$.

Proof. Assume that $\overline{R}$ is an bipolar subsystem of ψ and $s,r\in R$ furthermore $z\in Z^*$. By induction method the proof on $|z|=n$. $z=\varsigma$ is necessary for $n=0$. Presently if $s=r$, then

$$\mu_{\overline{R}}^{P}(r)\wedge \mu_{T^*}^{P}(r,\varsigma,r)=\mu_{\overline{R}}^{P}(r)$$
$$\mu_{\overline{R}}^{N}(r)\vee \mu_{T^*}^{N}(r,\varsigma,r)=\mu_{\overline{R}}^{N}(r)$$

A sufficient condition for

$$\mu_{\overline{R}}^{P}(s)\wedge \mu_{T^*}^{P}(s,z,r)=-1\leq \mu_{\overline{R}}^{P}(r)$$
$$\mu_{\overline{R}}^{N}(s)\vee \mu_{T^*}^{N}(s,z,r)=1\geq \mu_{\overline{R}}^{N}(r)$$

is $r\neq s$.

Hence the outcome is valid for n = 0. Assume the outcome is valid for all $g\in Z^*$ with $|g|=n-1,n>0.$

|y| = n − 1, n >0. For the g above, let $z=gd$ where $d\in Z$ .Then, at that point,

$$\begin{aligned}\mu_{\overline{R}}^{P}(s)\wedge \mu_{T^*}^{P}(s,z,r)&=\mu_{\overline{R}}^{P}(s)\wedge \mu_{T^*}^{P}(s,gd,r)\\&=\mu_{\overline{R}}^{P}(s)\wedge\left(\bigvee_{a\in R}\left[\mu_{T^*}^{P}(s,g,a)\wedge \mu_{T}^{P}(a,d,r)\right]\right)\\&=\bigvee_{a\in R}\left[\mu_{\overline{R}}^{P}(s)\wedge \mu_{T^*}^{P}(s,g,a)\wedge \mu_{T}^{P}(a,d,r)\right]\\&\leq\bigvee_{a\in R}\left[\mu_{\overline{R}}^{P}(a)\wedge \mu_{T}^{P}(a,d,r)\right]\leq \mu_{\overline{R}}^{P}(r)\end{aligned}$$

and

$$\begin{aligned}\mu_{\bar{R}}^N(s)\wedge\mu_{T^*}^N(s,z,r) &= \mu_{\bar{R}}^N(s)\vee\mu_{T^*}^N(s,gd,r)\\ &= \mu_{\bar{R}}^N(s)\vee\left(\underset{a\in R}{\wedge}\left[\mu_{T^*}^N(s,g,a)\vee\mu_T^N(a,d,r)\right]\right)\\ &= \underset{a\in R}{\wedge}\left[\mu_{\bar{R}}^N(s)\vee\mu_{T^*}^N(s,g,a)\vee\mu_T^N(a,d,r)\right]\\ &\geq \underset{a\in R}{\wedge}\left[\mu_{\bar{R}}^N(a)\vee\mu_T^N(a,d,r)\right]\geq\mu_{\bar{R}}^N(r)\end{aligned}$$

The converse is trivial, concluding the proof.

Definition 2.11. Let $\Psi=(R,Z,T)$ be LCDBF. Let $K\subseteq R$. Let $U=(\mu_U^P,\mu_U^N)$ be an BFS in $K\times Z\times K$ and let $\varphi=(K,Z,U)$ be an LCDBF. Then φ is known as an bipolar submachine of Ψ if

(i) $T|_{K\times Z\times K}=U$, i.e., $\mu_T^P|_{K\times Z\times K}=\mu_U^P$ and $\mu_T^N|_{K\times Z\times K}=\mu_U^N$.

(ii) $B(K)\subseteq K$.

We expect to be that $\phi=(\phi,Z,U)$ is an bipolar submachine of Ψ. Clearly, K is an bipolar submachine of φ and φ is an bipolar submachine of Ψ, then, at that point, K is an bipolar submachine of Ψ.

Definition 2.12. If $s\in B(r)$ for each $s,r\in R$,an LCDBF $\Psi=(R,Z,T)$is said to be strongly bipolar connected. If $K\neq\phi$ and $K\neq R$, an bipolar submachine $\varphi=(K,Z,U)$ of an LCDBF Ψ is supposed to be appropriate.

Theorem 2.13. Let $\Psi=(R,Z,T)$ be LCDBF and $\varphi_b=(K_b,Z,U_b)$ be a bipolar submachine family of Ψ. Then at the point we have

(i) $\bigcap_{b\in\lambda}\varphi_b=\left(\bigcap_{b\in\lambda}K_b,Z,\bigcap_{b\in\lambda}U_b\right)$

(ii) $\bigcup_{b\in\lambda}\varphi_b=\left(\bigcup_{b\in\lambda}K_b,Z,E\right)$,where $E=(\mu_E^P,\mu_E^N)$ denoted as $\mu_E^P=\mu_Z^P|_{\bigcup_{b\in\lambda}K_b\times Z\times\bigcup_{b\in\lambda}K_b}$ and $\mu_E^N=\mu_Z^N|_{\bigcup_{b\in\lambda}K_b\times Z\times\bigcup_{b\in\lambda}K_b}$

both the condition are bipolar submachine of Ψ.

Proof. (i) Let $(r,z,s)\in\bigcap_{b\in\lambda}K_b\times Z\times\bigcap_{b\in\lambda}K_b$. At that point

$$\left(\underset{b\in\lambda}{\wedge}\mu_{U_b}^P\right)(r,z,s)=\underset{b\in\lambda}{\wedge}\mu_{U_b}^P(r,z,s)=\underset{b\in\lambda}{\wedge}\mu_T^P(r,z,s)=\mu_T^P(r,z,s)$$

And

$$\left(\bigvee_{b\in\lambda}\mu_{U_b}^N\right)(r,z,s)=\bigvee_{b\in\lambda}\mu_{U_b}^N(r,z,s)=\bigvee_{b\in\lambda}\mu_T^N(r,z,s)=\mu_T^N(r,z,s)$$

Therefore $T|_{\bigcap_{b\in\lambda}K_b\times Z\times\bigcap_{b\in\lambda}K_b}=\bigcap_{b\in\lambda}U_b$. Now

$$B\left(\bigcap_{b\in\lambda}K_b\right)\subseteq\bigcap_{b\in\lambda}B(K_b)\subseteq\bigcap_{b\in\lambda}K_b.$$

Hence $\bigcap_{b\in\lambda}\varphi_b$ is an bipolar submachine of ψ.

(ii)As $B\left(\bigcup_{b\in\lambda}K_b\right)\subseteq\bigcup_{b\in\lambda}B(K_b)\subseteq\bigcup_{b\in\lambda}K_b,\bigcup_{b\in\lambda}\varphi_b$ is an bipolar submachine of ψ.

Theorem 2.14. Let $\Psi=(R,Z,T)$ be LCDBF. If Ψ has no accurate bipolar submachines if and only if Ψ is strongly bipolar.

Proof. Assume $\Psi=(R,Z,T)$ has a strong bipolar connection. Allow $\varphi=(K,Z,U)$ to be an bipolar submachine of Ψ with $K\neq\phi$. After that, there is $r\in K$. Because Ψ is strongly bipolar, $s\in B(r)$ whenever $s\in R$. As a result $s\in B(r)\subseteq B(K)\subseteq K$ so that $K=R$. Therefore $\Psi=\varphi$, indicating that Ψ has no appropriate bipolar submachines. Assume Ψ, on the other hand, does not have any bipolar submachines. Let $s,r\in R$ and $\varphi=(B(r),Z,U)$ where $U=\left(\mu_U^P,\mu_U^N\right)$ is given by

$\mu_U^P=\mu_T^P|_{B(r)\times z\times B(r)}$ and $\mu_U^N=\mu_T^N|_{B(r)\times z\times B(r)}$.

Then φ is an bipolar submachine of Ψ and $B(r)\neq\phi$, thus $B(r)=R$. Subsequently $s\in B(r)$, and thusly Ψ is strong bipolar connected.

Theorem 2.15. Let $\Psi=(R,Z,T)$ be LCDBF and $\overline{R}$ stands for bipolar subsystem of Ψ. At that point

(i) $\varphi=\left(Supp\left(\overline{R}\right),Z,U\right)$ is bipolar submachine of , Ψ where $U=\left(\mu_U^P,\mu_U^N\right)$ is given by

$\mu_U^P=\mu_T^P|_{Supp(\overline{R})\times z\times Supp(\overline{R})}$ and $\mu_U^N=\mu_T^N|_{Supp(\overline{R})\times z\times Supp(\overline{R})}$.

(ii) $\varphi_k=\left(\overline{R}_k,Z,U_k\right)$ where $\overline{R}_k=\left\{z\in R\mid\mu_{\overline{R}}^P(z)\geq k\geq\mu_{\overline{R}}^N(z)\right\}$ where $U=\left(\mu_{U_k}^P,\mu_{U_k}^N\right)$ is given by

$\mu_{U_k}^P=\mu_T^P|_{\overline{R}_k\times z\times\overline{R}_k}$ and $\mu_{U_k}^N=\mu_T^N|_{\overline{R}_k\times z\times\overline{R}_k}$, $k\in[-1,1]$. If φ_k is bipolar submachine of Ψ for all $k\in[-1,1]$, then $\overline{R}$ is an bipolar subsystem of Ψ.

Proof. (i) Let $s\in B\left(Supp\left(\overline{R}\right)\right)$. Then, at that point, $s\in B(r)$ for some $r\in Supp\left(\overline{R}\right)$. Accordingly $\mu_{\overline{R}}^P(r)>-1$ and $\mu_{\overline{R}}^N(r)<1$. Then $s\in B(r)$, their exists $z\in Z^*$ such that

$\mu_{T^*}^P(r,z,s) > -1$ and $\mu_{T^*}^N(r,z,s) < 1$. Because $\overline{R}$ is an bipolar subsystem, theorem 2.11 states that

$\mu_{\overline{R}}^P(s) \geq \mu_{\overline{R}}^P(r) \wedge \mu_{T^*}^P(r,z,s) > -1, \mu_{\overline{R}}^N(s) \leq \mu_{\overline{R}}^N(r) \vee \mu_{T^*}^N(r,z,s) < 1$.

Hence $B(Supp(\overline{R})) \subseteq Supp(\overline{R})$, and therefore φ *is an bipolar submachine of* Ψ.

(ii) Let $r,s \in R$ and $z \in Z^*$. If $\mu_{\overline{R}}^P(r) = -1$ or $\mu_{T^*}^P(r,z,s) = -1$ then $\mu_{\overline{R}}^P(s) \geq -1 = \mu_{\overline{R}}^P(r) \wedge \mu_{T^*}^P(r,z,s)$.

If $\mu_{\overline{R}}^N(r) = 1$ or $\mu_{T^*}^N(r,z,s) = 1$ then $\mu_{\overline{R}}^N(s) \leq 1 = \mu_{\overline{R}}^N(r) \vee \mu_{T^*}^N(r,z,s)$. Suppose $\mu_{\overline{R}}^P(r) > -1$, $\mu_{T^*}^P(r,z,s) > -1$, $\mu_{\overline{R}}^N(r) < 1$ and $\mu_{T^*}^N(r,z,s) < 1$. Let $\mu_{\overline{R}}^P(r) \wedge \mu_{T^*}^P(r,z,s) = k = \mu_{\overline{R}}^N(r) \vee \mu_{T^*}^N(r,z,s)$.

Formerly $s \in \overline{R}_k$. Then φ_k *is an bipolar submachine of* Ψ, we have $B(\overline{R}_k) \subseteq \overline{R}_k$. Hence $s \in B(s) \subseteq B(\overline{R}_k) \subseteq \overline{R}_k$, and thus $\mu_{\overline{R}}^P(s) \geq k \geq \mu_{\overline{R}}^P(r) \wedge \mu_{T^*}^P(r,z,s)$ and $\mu_{\overline{R}}^N(s) \leq t \leq \mu_{\overline{R}}^N(r) \vee \mu_{T^*}^N(r,z,s)$. Therefore $\overline{R}$ is an bipolar subsystem of Ψ.

Definition 2.16. Let $\Psi = (R,Z,T)$ be LCDBF then Ψ is said to be bipolar retrievable if,

$$(\forall r \in R)(\forall g \in Z^*)\begin{pmatrix} (\exists k \in R)(\mu_{T^*}^P(r,g,k) > -1, \mu_{T^*}^N(r,g,k) < 1) \\ \Rightarrow (\exists z \in Z^*)(\mu_{T^*}^P(k,z,r) > -1, \mu_{T^*}^N(k,z,r) < 1) \end{pmatrix}$$

Definition 2.17. Let $\Psi = (R,Z,T)$ be LCDBF then $r,a,f \in R$. If $g \in Z^*$,a and f are said to be bipolar q-related then $\mu_{T^*}^P(r,g,a) > -1, \mu_{T^*}^P(r,g,f) > -1, \mu_{T^*}^N(r,g,a) < 1$ and $\mu_{T^*}^N(r,g,f) < 1$

If a and f are bipolar q-related and B(a) = B(f) then a and f are bipolar q-twins.

Lemma 2.18. Let $\Psi = (R,Z,T)$ be LCDBF. Then the following statements are equivalent.

(i) $(\forall r,a,s \in R)(\forall z,g \in Z^*)\begin{pmatrix} \mu_{T^*}^P(r,g,a) > -1, \mu_{T^*}^P(r,gz,s) > -1, \\ \mu_{T^*}^N(r,g,a) < 1, \mu_{T^*}^N(r,gz,s) < 1 \Rightarrow s \in B(a) \end{pmatrix}$.

(ii) $r,a,f \in R$, is a and f are bipolar q- related, then a and f are bipolar q- twins.

Proof. (i) implies (ii) Assume that (i) is valid. $r,a,f \in R$ be to such an extent that a and f are bipolar q- related. Then their exists $g \in Z^*$ such that $\mu_{T^*}^P(r,g,a) > -1, \mu_{T^*}^P(r,g,f) > -1, \mu_{T^*}^N(r,g,a) < 1$ and $\mu_{T^*}^N(r,g,f) < 1$. $s \in B(f)$ is sufficient for $\mu_{T^*}^P(f,z,s) > -1$ and $\mu_{T^*}^N(f,z,s) < 1$. Then

$$\mu_{T^*}^{P}(r,gz,s)=\bigvee_{k\in R}\left[\mu_{T^*}^{P}(r,g,k)\wedge\mu_{T^*}^{P}(k,z,s)\right]>-1 \text{ and}$$

$$\mu_{T^*}^{N}(r,gz,s)=\bigwedge_{k\in R}\left[\mu_{T^*}^{N}(r,g,k)\vee\mu_{T^*}^{N}(k,z,s)\right]<1.$$

As a result, $s\in B(a)$ by hypothesis. Likewise, $s\in B(f)$ whenever $s\in B(a)$. As a result, a and f are bipolar q-twins.

(ii) implies (i) Let $r,a,s\in R$ and $z,g\in Z^*$ be such that $\mu_{T^*}^{P}(r,g,a)>-1, \mu_{T^*}^{P}(r,gz,s)>-1, \mu_{T^*}^{N}(r,g,a)<1$ and $\mu_{T^*}^{N}(r,gz,s)<1$.

Then

$$\mu_{T^*}^{P}(r,gz,s)=\bigvee_{f\in R}\left[\mu_{T^*}^{P}(r,g,f)\wedge\mu_{T^*}^{P}(f,z,s)\right]>-1 \text{ and}$$

$$\mu_{T^*}^{N}(r,gz,s)=\bigwedge_{f\in R}\left[\mu_{T^*}^{N}(r,g,f)\vee\mu_{T^*}^{N}(f,z,s)\right]<1.$$

Thus a and f are bipolar q- related, then a and f are bipolar q- twins and B(a) = B(f).

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