Mathematical Statistician and Engineering Applications
ISSN: 2094-0343
2326-9865

Momentary and Immovable State Conduct of Heterotypical Queues with Three Types of Ingress Sources Administered by A Third Order Stochastic Matrix

Gajendra Kumar Saraswat [1, a)] and Vijay Kumar [2, b)]
[1]Professor, Department of Mathematics, Modern Institute of Technology & Research Centre, Alwar-301028, Rajasthan, India
[2]Assistant Professor, Department of Applied Sciences and Humanities, Govt. Engineering College, Lakhisarai, Bihar, India
[a)]telllakshya01@gmail.com,[b)]vijay.math82@gmail.com,

Article Info
Page Number: 3055 - 3072
Publication Issue:
Vol 71 No. 4 (2022)

Abstract
The present research problem analyses Markovian heterotypical queuing structure where into the advent process remains in three states i.e., operative, semi-operative and inoperative states with advent rates λ_o, λ_s or zero respectively. The employ times in each state are μ. When the ingress origin operates in operative state, it tends to switches to the alternative state i.e., semi-operative or inoperative, with Poisson intensity η_{os} and η_{oi} respectively; where o, s and i denote operative, semi-operative and inoperative states serially. The state of the ingress operating with advent rate λ_o, λ_s or zero is indicated by P, L and Q respectively. The Poisson rates through if the ingress origin switches by state P to L or P to Q and by L to Q or L to P and by state Q to P or Q to L are denoted by η_{os}, η_{oi} and η_{si}, η_{so} and η_{io}, η_{is} respectively. The stochastic procedures contiguous, namely, inter-advent period of units and employ period of clients are not dependent of inter alia.
In the segment I, the immovable state conduct of the queuing channel in bounded place is examined by the support of probability generating function, and many special concerns are analyzed in detail. In the segment II, we analyses the momentary state conduct of the system by using Matrix Method Technique (MMT).

KEYWORDS: Poisson distribution, Exponential distribution, Probability generating function, Stochastic Process, Matrix method technique (MTT).

Article History
***Article Received**: 25 March 2022*
***Revised**: 30 April 2022*
***Accepted**: 15 June 2022*
***Publication**: 19 August 2022*

1. PREAMBLE

A general condition that often happens in daily life is waiting in long lines at bus stations, petrol stations, ticket offices, clinics, restaurants, traffic lights, bank counters, etc. There are also queues in the workshops, where machines are waiting to be repaired; trucks waiting to be loaded, planes waiting to take off and land, etc.
Typically, a waiting line is built up in a production/operating channel when a client (human being or natural entity) requests a pending employ because the quantity of customers more than the number of employ facility or employ facility is not working proficiently / takes longer than specified period to attend one customer.

Queuing theory may be applicable to many conditions where it is not doable to purely vaticinate a client's advent rate (or period) and employ rate (or period) of employ facility/facilities. In exclusive, it can be used to find the level of employ (both the employ rate and the quantity of employ facilities) that balances the under mentioned two opposing charges.

(i) Employ delivery charges

(ii) Charges of incurred due to late delivering employ

Primarily is related to the execution facilities and their manipulations, and the secondarily is the charge of client waiting for employ.

Clearly, a growth in current employ facilities will decrease client wait times. On the contrary, a drop in employ levels will be the result of long queues. In this approach, an increase in employ levels increases the operating charges of employ facilities, however decreases the charge of clients waiting for employ. Figure 1 is an example of each charge category as a function of employ level. Mixed employ charges and client waiting charges form a U-shaped because of their trade-off relationship. The overall charge is minimized at the low point of the overall charge curve.

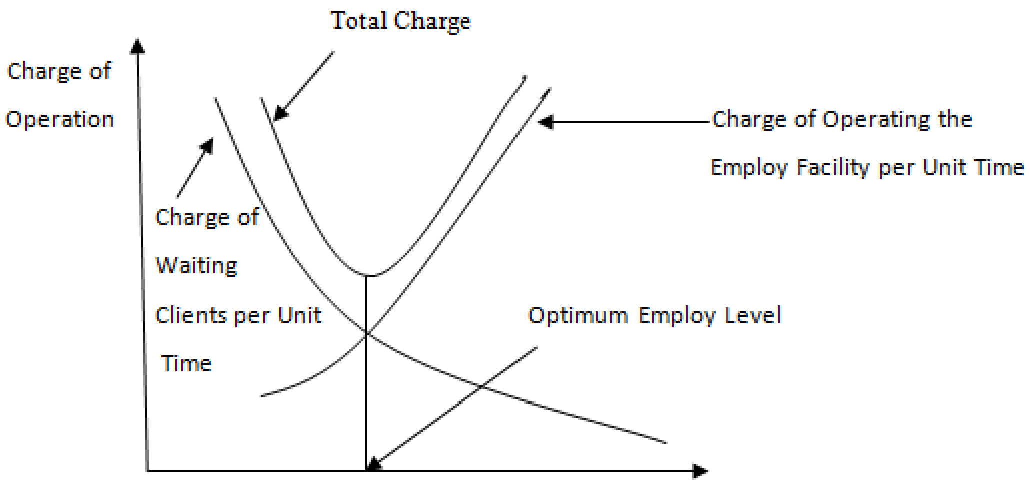

Fig. 1

Since herculean will assess a client's charge of waiting for employ, it is often estimated based on a shortfall in selling or credit when the client is human being and has no compassion for the employ channel. However, if the client is a device awaiting refurbishment, the waiting charge is estimated by the turned off production charge.

Keeping those views, the queuing structures with heterotypical are greater relevant as evaluate to their homogeneous counterparts, due to the fact in real-existence conditions the employ affairs at various rates. Multifarious service networks are scheduling techniques that permit clients to obtain various quality of employ. Extreme activities in the production structure have diversified employ mechanisms. That's why; queuing structures with heterogeneity have received crucial interest in the research community.

Morse [1] has placed the idea of a heterogeneous service. Saaty [2] enhances Morse's statement and deduces steady state probabilities and averages in the channel. Sharma and Dass [3] examine the ingoing usage time of the multichannel probabilistic queuing channel and get the statement of its density function in the limited shape. Kumar and Sharma [5] receive the momentary result of the heterogeneous two-server probabilistic queuing system with the conception of leaving clients. Sarawat, G. [9] examined an exponential queue of a single counter with heterogeneity in the arrival. Krishanamoorthy [6] assumes a Poisson line by two multifarious servers with improved queuing disciplines. Multifarious findings in the field of waiting opinions were measured by Yechiali and Naor [10] and Neuts [12]. In the waiting sample explored in [10], the advent framework at a employ centre is Poisson, the service time dispensation is contracted as negative exponential and the parameters fall under on the atmosphere. In the waiting version purported by Murari and Agarwal [13], the advent procedure cracks below by two advent rapidities namely, λ and 0. The shape by 0 advent rates is repaired after which added to level I having λ as mean advent rate. The quest of Saraswat G.K. and Agarwal R.K. [16] welcomed stochasti queuing version by heterotaxia in advent procedure dominated with the aid of using second order stochastic matrix. Saraswat G. K. and Agarwal R. K. [21] centered probabilistic waiting pattern with heterotaxia and clogging in service of their studies. In a studies version with the aid of using I. M. Premachandra and Liliana Gongalez [15], the employ procedure includes three tiers featured in series with the aid of using two servers. In his recent research work, Saraswat G. K. and Kumar, Vijay [33] examined Markovian heteromorphic limited space queue with a random count input sources operating at a time.

In this problem, a more realistic supposition is that in which the transition are not cyclic, i.e. a state of the ingress is achieved from any one of the three states, in which the ingress origin remains for a random time. Quantities of fields recommend themselves wherever this channel is of advantageous utilization. For instance, take into account a device who might be in one in all the three states, active condition (operative state), sick condition (semi-operative state), or passive condition (inoperative state). Suppose a device in active condition will stay this manner for a random period and can then move either the sick condition or the passive condition. A device within the sick condition will stay such way for at random period and will certainly move at any one of the active condition or the passive condition. A passive device are repaired and once repaired it should either move active condition or to the sick condition. If the device is producing stipulated things certainly it can be known like an ingress origin. The conduct of ingress origin is strictly identical as device. Thus, the ingress origin stays in above mentioned three states and it's ruled by third order random (stochastic) matrix. Keeping this in view, the problem possesses following characteristics.

NARRATION OF THE PROBLEM

A flow of Poisson-type units reach at a singular employ terminal. The advent arrangement is heterotypical, i.e., there exist three advent intensities at which the ingress origin is efficient of functioning. The ingress origin oscillates between three doable states, (i) operative state, (ii) semi-operative state, (iii) inoperative state. The time intermission until which the ingress functions in any one state is an exponentially distributed random mutable. It is supposed that any perception of time interval associated with uniform advent rate is independent of earlier history. Furthermore, the employ period is considered to be exponentially distributed with parameter μ for all the three states of the ingress. The firmness of the ingress origin at any state is administered by a random procedure: if the ingress origin functions in operative state, mean advent rate of units is λ_o, it tends to jump to the alternative state, semi-operative or inoperative, with Poisson intensity η_{os} or η_{oi} respectively: where o, s and i denote operative, semi-operative and inoperative states respectively. The mean advent rates of units, when the ingress functions in semi-operative or inoperative state, are λ_s or zero serially. The state of the ingress functioning with advent rate λ_o, λ_s or zero is indicated by P, L and Q respectively. The Poisson rates at which the ingress origin switches by state P to L or P to Q and by L to Q or L to P and by state Q to P or Q to L are denoted by η_{os}, η_{oi} and η_{si}, η_{so} and η_{io}, η_{is} serially. The transition from t to $t+\Delta$ be administered by the following stochastic matrix:

$$\begin{array}{l} \text{State} \\ \text{Operative} \\ \text{Semi-operative} \\ \text{Inoperative} \end{array} \begin{array}{c} : \text{Operative} \quad \text{Semi-operative} \quad \text{Inoperative} \\ \begin{bmatrix} 1-\eta_{os}\Delta-\eta_{oi}\Delta & \eta_{os}\Delta & \eta_{oi}\Delta \\ \eta_{so}\Delta & 1-\eta_{so}\Delta-\eta_{si}\Delta & \eta_{si}\Delta \\ \eta_{io}\Delta & \eta_{is}\Delta & 1-\eta_{io}\Delta-\eta_{is}\Delta \end{bmatrix} \end{array}$$

The transition of three ingress states is also depicting by the following figure 2:

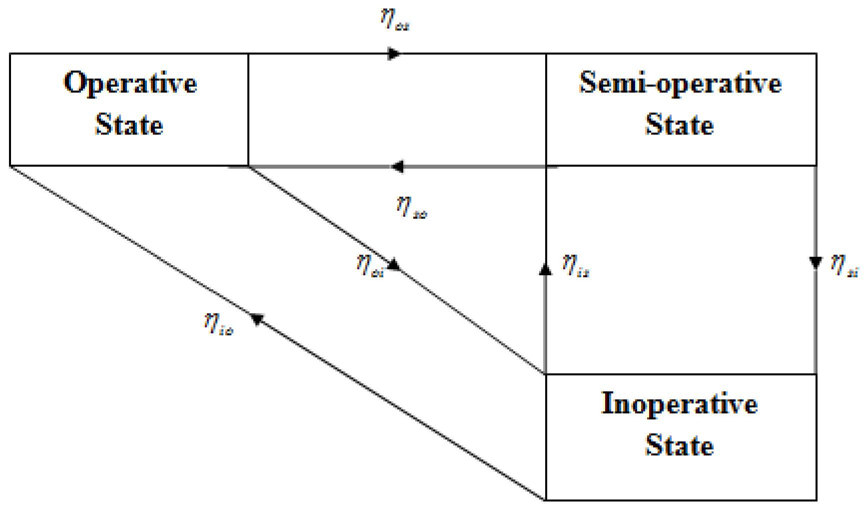

Fig. 2

Further, it is supposed that if at any instant the units in channel are N, then the adventing unit will be supposed to lose for the channel. The stochastic procedures contiguous, namely, inter-advent period of units and employ period of clients are not dependent of inter alia.

The mathematical structure of the waiting channel is submitted. The momentary and immovable state solutions of the subject are explained. In redress segment I, the immovable state conduct of the queuing channel in bounded place is studied by the support of probability generating function, and various special concerns are explicated in detail. In redress segment II, we explicate the momentary state conduct of the channel using Matrix Method Technique (MTT).

2. REDRESS OF THE PROBLEM

Let us settle

$P_n(t) \equiv$ The likelihood at period t, the channel is in the operative state P and n units are in the line, inclusive of one in employ.

$L_n(t) \equiv$ The likelihood at period t, the channel is in the semi-operative state L and n units are in the line, inclusive of one in employ.

$Q_n(t) \equiv$ The likelihood at period t, the channel is in the inoperative state Q and n units are in the line, inclusive of one in employ

$R_n(t) \equiv$ The likelihood, n units are in the line at time t inclusive of one in employ.

Clearly

$$R_n(t) = P_n(t) + L_n(t) + Q_n(t)$$

Allow the time t be calculated by the instantaneous as queue duration is zero, and the channel is in the state I. Ingoing concerns are,

$$P_n(0) = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases}$$

$$L_n(0) = 0, \quad n \geq 0$$

$$Q_n(0) = 0, \quad n \geq 0$$

3.1 SEGMENT 'I' (IMMOVABLE STATE CONDUCT)

The balance equalities in immovable state regulating the channel are as:

For Operative State

$$(f - \mu)P_0 = \mu P_1 + \eta_{io} Q_0 + \eta_{so} L_0$$

(1.1)

$$fP_n = \mu P_{n+1} + \lambda_o P_{n-1} + \eta_{io} Q_n + \eta_{so} L_n, \quad 1 \leq n < N \tag{1.2}$$

$$(f - \lambda_o)P_N = \lambda_o P_{N-1} + \eta_{io} Q_N + \eta_{so} L_N,$$

(1.3)

Where,

$$f = (\lambda_o + \mu + \eta_{os} + \eta_{oi})$$

For Semi-operative State

$$(g - \mu)L_0 = \mu L_1 + \eta_{os} P_0 + \eta_{is} Q_0$$

(1.4)

$$gL_n = \mu L_{n+1} + \lambda_s L_{n-1} + \eta_{os} P_n + \eta_{is} Q_n, \quad 1 \leq n < N$$

(1.5)

$$(g - \lambda_s)L_N = \lambda_s L_{N-1} + \eta_{os} P_N + \eta_{is} Q_N,$$

(1.6)

Where,

$$g = (\lambda_s + \mu + \eta_{si} + \eta_{so})$$

For Inoperative State

$$(h-\mu)Q_0 = \mu Q_1 + \eta_{si} L_0 + \eta_{oi} P_0$$
(1.7)

$$hQ_n = \mu Q_{n+1} + \eta_{si} L_n + \eta_{oi} P_n, \quad 1 \le n < N$$
(1.8)

$$hQ_N = \eta_{si} L_N + \eta_{oi} P_N,$$
(1.9)

Where,

$$h = (\mu + \eta_{io} + \eta_{is})$$

Settle the probability generating functions of P_n, L_n and Q_n by

$$P(z) = \sum_{n=0}^{N} z^n P_n \tag{1.10}$$

$$L(z) = \sum_{n=0}^{N} z^n L_n \tag{1.11}$$

$$Q(z) = \sum_{n=0}^{N} z^n Q_n \tag{1.12}$$

Increasing (1.1) to (1.9) by apposite degrees of z and applying (1.10) to (1.12), we obtain

$$K_o(z)P(z) = \mu(z-1)P_0 + z\eta_{so}L(z) + z\eta_{io}Q(z) - z^{N+2}\lambda_o P_N \tag{1.13}$$

$$K_s(z)L(z) = \mu(z-1)L_0 + z\eta_{os}P(z) + z\eta_{is}Q(z) - z^{N+2}\lambda_s L_N \tag{1.14}$$

$$K_i(z)Q(z) = \mu(z-1)Q_0 + z\eta_{oi}P(z) + z\eta_{si}Q(z) \tag{1.15}$$

Where,

$$K_o(z) = [z\{\lambda_o(1-z) + \mu + \eta_{os} + \eta_{oi}\} - \mu] \tag{1.16}$$

$$K_s(z) = [z\{\lambda_s(1-z) + \mu + \eta_{si} + \eta_{so}\} - \mu] \tag{1.17}$$

$$K_i(z) = [z(\mu + \eta_{io} + \eta_{is}) - \mu] \tag{1.18}$$

Solving equations (1.13) and (1.18), we get

$$P(z) = \frac{\begin{array}{l}\mu(z-1)[P_0\{K_s(z)K_i(z) - z^2\eta_{si}\eta_{is}\} + L_0\{z\eta_{so}K_i(z) + z^2\eta_{si}\eta_{io}\} \\ +Q_0\{z\eta_{io}K_s(z) + z^2\eta_{so}\eta_{is}\}] - z^{N+2}[\lambda_o P_N\{K_s(z)K_i(z) - z^2\eta_{si}\eta_{is}\} \\ +\lambda_s L_N\{z\eta_{io}K_i(z) + z^2\eta_{si}\eta_{io}\}]\end{array}}{\begin{array}{l}K_o(z)K_s(z)K_i(z) - z^2[K_o(z)\eta_{si}\eta_{is} + K_s(z)\eta_{io}\eta_{oi} + K_i(z)\eta_{os}\eta_{so}] \\ -z^3(\eta_{os}\eta_{si}\eta_{io} + \eta_{so}\eta_{oi}\eta_{is})\end{array}} \tag{1.19}$$

$$L(z) = \frac{\begin{array}{l}\mu(z-1)[P_0\{z\eta_{os}K_i(z) + z^2\eta_{oi}\eta_{is}\} + L_0\{K_o(z)K_i(z) - z^2\eta_{io}\eta_{oi}\} \\ +Q_0\{z\eta_{is}K_o(z) + z^2\eta_{io}\eta_{os}\}] - z^{N+2}[\lambda_o P_N\{z\eta_{os}K_i(z) + z^2\eta_{oi}\eta_{is}\} \\ +\lambda_s L_N\{K_o(z)K_i(z) - z^2\eta_{oi}\eta_{io}\}]\end{array}}{\begin{array}{l}K_o(z)K_s(z)K_i(z) - z^2[K_o(z)\eta_{si}\eta_{is} + K_s(z)\eta_{io}\eta_{oi} + K_i(z)\eta_{os}\eta_{so}] \\ -z^3(\eta_{os}\eta_{si}\eta_{io} + \eta_{so}\eta_{oi}\eta_{is})\end{array}} \tag{1.20}$$

$$Q(z)=\frac{\begin{array}{l}\mu(z-1)[P_0\{z\eta_{oi}K_s(z)+z^2\eta_{os}\eta_{si}\}+L_0\{z\eta_{si}K_o(z)+z^2\eta_{oi}\eta_{so}\}\\+Q_0\{K_o(z)K_s(z)-z^2\eta_{os}\eta_{so}\}]-z^{N+2}[z\lambda_oP_N\{z\eta_{os}\eta_{si}+\eta_{oi}K_s(z)\}\\+z\lambda_sL_N\{z\eta_{oi}\eta_{so}+\eta_{si}K_o(z)\}]\end{array}}{\begin{array}{l}K_o(z)K_s(z)K_i(z)-z^2[K_o(z)\eta_{si}\eta_{is}+K_s(z)\eta_{io}\eta_{oi}+K_i(z)\eta_{os}\eta_{so}]\\-z^3(\eta_{os}\eta_{si}\eta_{io}+\eta_{so}\eta_{oi}\eta_{is})\end{array}} \quad (1.21)$$

$$R(z)=P(z)+L(z)+Q(z)$$

So,

$$R(z)=\frac{\begin{array}{l}\mu(z-1)[\sum_{o,s,i}P_0^o\{K_s(z)K_i(z)-z^2\eta_{is}\eta_{si}\}+\sum_{o,s,i}P_0^s\{z\eta_{so}K_i(z)+z^2\eta_{si}\eta_{io}\}\\+\sum_{o,s,i}P_0^i\{z\eta_{io}K_s(z)+z^2\eta_{so}\eta_{is}\}]-z^{N+2}[\sum_{o,s,i}\lambda_oP_N^o\{K_s(z)K_i(z)-z^2\eta_{si}\eta_{is}\}\\+\sum_{o,s,i}\lambda_sP_N^s\{z\eta_{so}K_i(z)+z^2\eta_{si}\eta_{io}\}+\sum_{o,s,i}\lambda_iP_N^i\{z\eta_{io}K_s(z)+z^2\eta_{so}\eta_{is}\}]\end{array}}{\begin{array}{l}K_o(z)K_s(z)K_i(z)-z^2[K_o(z)\eta_{si}\eta_{is}+K_s(z)\eta_{io}\eta_{oi}+K_i(z)\eta_{os}\eta_{so}]\\-z^3(\eta_{os}\eta_{si}\eta_{io}+\eta_{so}\eta_{oi}\eta_{is})\end{array}}$$

Where $\sum$ runs cyclically over o,s,i, and $P_N^i=0, P_N^o=P_N, P_N^s=L_N, P_0^o=P_0, P_0^s=L_0, P_0^i=Q_0$. The term P_N^i is added here to write the results in cyclic order, and it may be clearly noted that P_N^i does not represent Q_N and that its value is zero here.

$R(z)$ is a omnibus. The five zeros of its divisors need dissipate its fraction, who is of (N+5) extent, putting growth to set of five equalities in five unknowns, namely, P_0, L_0, Q_0, P_N and L_N. Settling these five equalities, the five unknowns can be obtained.

Thus, $R(z)$ must be determinate tee totally.

SPECIAL CONCERNS:

The under mentioned special concerns are exists:

(i) As by means of inoperative state, the channel makes not move to semi-operative state, the commensurate solution may be got by putting η_{is} equal to zero i.e., $\eta_{is}=0$

$$P(z)=\frac{\begin{array}{l}\mu(z-1)[P_0\{K_s(z)K_i(z)\}+L_0\{z\eta_{so}K_i(z)+z^2\eta_{si}\eta_{io}\}+Q_0\{z\eta_{io}K_s(z)\}]\\-z^{N+2}[\lambda_oP_NK_s(z)K_i(z)+\lambda_sL_N\{z\eta_{io}K_i(z)+z^2\eta_{si}\eta_{io}\}]\end{array}}{K_o(z)K_s(z)K_i(z)-z^2[K_s(z)\eta_{io}\eta_{oi}+K_i(z)\eta_{os}\eta_{so}]-z^3(\eta_{os}\eta_{si}\eta_{io})}$$

Where,

$$K_i(z)=[z\{\mu+\eta_{io}\}-\mu]$$

$$L(z)=\frac{\begin{array}{l}\mu(z-1)[P_0\{z\eta_{os}K_i(z)\}+L_0\{K_o(z)K_i(z)-z^2\eta_{io}\eta_{oi}\}+Q_0\{z^2\eta_{io}\eta_{os}\}]\\-z^{N+2}[\lambda_oP_N\{z\eta_{os}K_i(z)\}+\lambda_sL_N\{K_o(z)K_i(z)-z^2\eta_{oi}\eta_{io}\}]\end{array}}{K_o(z)K_s(z)K_i(z)-z^2[K_s(z)\eta_{io}\eta_{oi}+K_i(z)\eta_{os}\eta_{so}]-z^3(\eta_{os}\eta_{si}\eta_{io})}$$

$$Q(z)=\frac{\begin{array}{l}\mu(z-1)[P_0\{z\eta_{oi}K_s(z)+z^2\eta_{os}\eta_{si}\}+L_0\{z\eta_{si}K_o(z)K_i(z)+z^2\eta_{oi}\eta_{so}\}\\+Q_0\{K_o(z)K_s(z)-z^2\eta_{os}\eta_{so}\}]-z^{N+2}[z\lambda_oP_N\{z\eta_{os}\eta_{si}+\eta_{oi}K_s(z)\}\\+z\lambda_sL_N\{z\eta_{oi}\eta_{so}+\eta_{si}K_o(z)\}]\end{array}}{K_o(z)K_s(z)K_i(z)-z^2[K_s(z)\eta_{io}\eta_{oi}+K_i(z)\eta_{os}\eta_{so}]-z^3(\eta_{os}\eta_{si}\eta_{io})}$$

(ii) As transitions are possible only from operative to semi-operative, semi-operative to inoperative state, and from inoperative to operative state, then, corresponding solution can be obtained by making η_{so},η_{is} and η_{oi} equal to zero.

Therefore,

$$P(z)=\frac{\begin{array}{l}\mu(z-1)[P_0K_s(z)K_i(z)+L_0z^2\eta_{si}\eta_{io}+Q_0z\eta_{io}K_s(z)]\\-z^{N+2}[\lambda_oP_NK_s(z)K_i(z)+\lambda_sL_Nz^2\eta_{si}\eta_{io}\}]\end{array}}{K_o(z)K_s(z)K_i(z)-z^3(\eta_{os}\eta_{si}\eta_{io})}$$

$$L(z)=\frac{\begin{array}{l}\mu(z-1)[P_0z\eta_{os}K_i(z)+L_0K_o(z)K_i(z)+Q_0z^2\eta_{io}\eta_{os}\}]\\-z^{N+2}[\lambda_oP_Nz\eta_{os}K_i(z)+\lambda_sL_N\{K_o(z)K_i(z)]\end{array}}{K_o(z)K_s(z)K_i(z)-z^3(\eta_{os}\eta_{si}\eta_{io})}$$

$$Q(z)=\frac{\begin{array}{l}\mu(z-1)[P_0z^2\eta_{os}\eta_{si}+L_0z\eta_{si}K_o(z)+Q_0K_o(z)K_s(z)]\\-z^{N+2}[\lambda_oP_Nz^2\eta_{os}\eta_{si}+z\lambda_sL_N\eta_{si}K_o(z)]\end{array}}{K_o(z)K_s(z)K_i(z)-z^3(\eta_{os}\eta_{si}\eta_{io})}$$

Where,

$$K_o(z)=[z\{\lambda_o(1-z)+\mu+\eta_{os}\}-\mu]$$
$$K_s(z)=[z\{\lambda_s(1-z)+\mu+\eta_{si}\}-\mu]$$
$$K_i(z)=[z\{\mu+\eta_{io}\}-\mu]$$

(iii) As the channel makes not have semi-operative state, though, it moves by operative state to inoperative state and by inoperative to operative state; the compatible solution may be possessed by letting

$$\eta_{os}\to\eta_{oi},\eta_{is}\to\eta_{io},\eta_{so}\to\infty \text{ and } \eta_{si}\to\infty \text{ and } L_N\to 0, n\le 0\le N.$$

This may be possessed by special concern (ii)

$$P(z)=\frac{\mu(z-1)[P_0K_i(z)+Q_0z\eta_{io}]-\lambda_oz^{N+2}P_NK_i(z)}{K_o(z)K_i(z)-z^2\eta_{oi}\eta_{io})}$$

$$Q(z)=\frac{\mu(z-1)[P_0z\eta_{oi}+Q_0K_o(z)]-\lambda_oz^{N+3}\eta_{oi}P_N}{K_o(z)K_i(z)-z^2\eta_{oi}\eta_{io})}$$

Where,

$$K_o(z)=[z\{\lambda_o(1-z)+\mu+\eta_{os}\}-\mu]$$
$$K_i(z)=[z\{\mu+\eta_{io}\}-\mu]$$

3.2 SEGMENT 'II' (MOMENTARY STATE CONDUCT)

KOLMOGOROV'S onward equalities depicting the problem lead to the under mentioned:

Discriminative equalities for operative state

$$P_0^{'}(t) = (f+\mu)P_0(t) + \mu P_1(t) + \eta_{io}Q_0(t) + \eta_{so}L_0(t)$$

(1.22)

$$P_1^{'}(t) = \lambda_o P_0(t) + fP_1(t) + \mu P_2(t) + \eta_{io}Q_1(t) + \eta_{so}L_1(t)$$

$$P_n^{'}(t) = \lambda_o P_{n-1}(t) + fP_n(t) + \mu P_{n+1}(t) + \eta_{io}Q_n(t) + \eta_{so}L_n(t \qquad 1 \le n < N$$

(1.23)

$$P_N^{'}(t) = \lambda_o P_{N-1}(t) + (f+\lambda_o)P_N(t) + \eta_{io}Q_N(t) + \eta_{so}L_N(t),$$

(1.24)

Where.

$$f = -(\lambda_o + \mu + \eta_{os} + \eta_{oi})$$

Discriminative equalities for semi-operative state

$$L_0^{'}(t) = (g+\mu)L_0(t) + \mu L_1(t) + \eta_{os}P_0(t) + \eta_{is}Q_0(t)$$

(1.25)

$$L_1^{'}(t) = \lambda_s L_0(t) + gL_1(t) + \mu L_2(t) + \eta_{os}P_1(t) + \eta_{is}Q_1(t)$$

$$L_n^{'}(t) = \lambda_s L_{n-1}(t) + gL_n(t) + \mu L_{n+1}(t) + \eta_{os}P_n(t) + \eta_{is}Q_n(t), \quad 1 \le n < N$$

(1.26)

$$L_N^{'}(t) = \lambda_s L_{N-1}(t) + (g+\lambda_s)L_N(t) + \eta_{os}P_N(t) + \eta_{is}Q_N(t),$$

(1.27)

Where,

$$g = -(\lambda_s + \mu + \eta_{si} + \eta_{so})$$

Discriminative equalities for inoperative state

$$Q_0^{'}(t) = (h+\mu)Q_0(t) + \mu Q_1(t) + \eta_{si}L_0(t) + \eta_{oi}P_0(t)$$

(1.28)

$$Q_1^{'}(t) = hQ_1(t) + \mu Q_2(t) + \eta_{si}L_1(t) + \eta_{oi}P_1(t)$$

$$Q_n^{'}(t) = hQ_n(t) + \mu Q_{n+1}(t) + \eta_{si}L_n(t) + \eta_{oi}P_n(t), \quad 1 \le n < N$$

(1.29)

$$Q_N^{'}(t) = hQ_N(t) + \eta_{si}L_N(t) + \eta_{oi}P_N(t),$$

(1.30)

Where,

$$h = -(\mu + \eta_{io} + \eta_{is})$$

Employing matrix notation, equations (1.22 to 1.24), (1.25 to 1.27) and (1.28 to 1.30) give

$$\vec{P}^{'}(t) = A_1\vec{P}(t) + \eta_{io}I_{N+1}\vec{Q}(t) + \eta_{so}I_{N+1}\vec{L}(t) \qquad (1.31)$$

$$\vec{L}(t) = B_1\vec{L}(t) + \eta_{os}I_{N+1}\vec{P}(t) + \eta_{is}I_{N+1}\vec{Q}(t) \qquad (1.32)$$

$$\vec{Q}^{'}(t) = C_1\vec{Q}(t) + \eta_{si}I_{N+1}\vec{L}(t) + \eta_{oi}I_{N+1}\vec{P}(t) \qquad (1.33)$$

Where,

$$\vec{P}(t)=\begin{bmatrix} P_0(t) \\ P_1(t) \\ .... \\ .... \\ P_{N-1}(t) \\ P_N(t) \end{bmatrix}, \qquad \vec{L}(t)=\begin{bmatrix} L_0(t) \\ L_1(t) \\ .... \\ .... \\ L_{N-1}(t) \\ L_N(t) \end{bmatrix}, \qquad \vec{Q}(t)=\begin{bmatrix} Q_0(t) \\ Q_1(t) \\ .... \\ .... \\ Q_{N-1}(t) \\ Q_N(t) \end{bmatrix}$$

I_{N+1} = Unit matrix of order (N+1)

$\vec{P}'(t) = D\vec{P}(t)$ etc., $D \equiv \dfrac{d}{dt}$

$$A_1=\begin{bmatrix} f+\mu & \mu & C & 0 & .. & .. & .. & 0 & 0 & 0 & 0 \\ \lambda_o & f & \mu & 0 & .. & .. & .. & 0 & 0 & 0 & 0 \\ 0 & \lambda_o & f & \mu & .. & .. & .. & 0 & 0 & 0 & 0 \\ .. \\ .. \\ .. \\ .. \\ 0 & 0 & 0 & 0 & .. & .. & .. & 0 & \lambda_o & f & \mu \\ 0 & 0 & 0 & 0 & .. & .. & .. & 0 & 0 & \lambda_o & (f+\lambda_o) \end{bmatrix}$$

$\equiv$ Tri-diagonal matrix of order (N+1)

$$B_1=\begin{bmatrix} g+\mu & \mu & 0 & 0 & .. & .. & .. & 0 & 0 & 0 & 0 \\ \lambda_s & g & \mu & 0 & .. & .. & .. & 0 & 0 & 0 & 0 \\ 0 & \lambda_s & g & \mu & .. & .. & .. & 0 & 0 & 0 & 0 \\ .. \\ .. \\ .. \\ .. \\ 0 & 0 & 0 & 0 & .. & .. & .. & 0 & \lambda_s & g & \mu \\ 0 & 0 & 0 & 0 & .. & .. & .. & 0 & 0 & \lambda_s & (g+\lambda_s) \end{bmatrix}$$

$\equiv$ Tri-diagonal matrix of order (N+1)

$$C_1 = \begin{bmatrix} h+\mu & \mu & 0 & 0 & .. & .. & .. & 0 & 0 & 0 & 0 \\ 0 & h & \mu & 0 & .. & .. & .. & 0 & 0 & 0 & 0 \\ 0 & 0 & h & \mu & .. & .. & .. & 0 & 0 & 0 & 0 \\ .. \\ .. \\ .. \\ .. \\ 0 & 0 & 0 & 0 & .. & .. & .. & 0 & 0 & h & \mu \\ 0 & 0 & 0 & 0 & .. & .. & .. & 0 & 0 & 0 & h \end{bmatrix}$$

$\equiv$ Tri-diagonal matrix of order (N+1) with leading diagonal and super diagonal.

Extricating equalities (1.31), (1.32) and (1.34), we obtain a six – order differential equations. Applying matrix notations in order, it would be describe in a form that involves first order discriminative multipliers.

$$D^6\vec{P}(t) - aD^5\vec{P}(t) - bD^4\vec{P}(t) - cD^3\vec{P}(t) + dD^2\vec{P}(t) - eD\vec{P}(t) + f\vec{P}(t) = 0$$

(1.34)

Where,

$$a = A^* + B^* + E^*$$

$$b = (B^* + A^*C^* + D^*) - (A^* + C^*)E^* + F^*$$

$$c = \{(A^*D^* + B^*C^*) + (B^* + A^*C^* + D^*)E^* + (A^* + C^*)F^*\} + \eta_{io}\eta_{os}\eta_{si}I_{N+1}$$

$$d = [\{(A^*D^* + B^*C^*)E^* + (B^* + A^*C^* + D^*)F^* + B^*C^*\} + \{(A_1 + B_1 + C_1) \\ \eta_{os}\eta_{si}\eta_{io}I_{N+1} + \eta_{io}\eta_{os}\eta_{so}\eta_{oi}I_{N+1} + \eta_{so}\eta_{is}\eta_{os}\eta_{si}I_{N+1} + \eta_{io}\eta_{si}\eta_{is}\eta_{oi}I_{N+1}\}]$$

$$e = [\{(A^*D^* + B^*C^*)F^* + B^*E^*D^*\} + \{(A_1B_1 + A_1C_1 + B_1C_1)\eta_{io}\eta_{os}\eta_{si}I_{N+1} \\ + (B_1 + C_1)\eta_{io}\eta_{os}\eta_{so}\eta_{oi}I_{N+1} + (A_1 + C_1)\eta_{so}\eta_{is}\eta_{os}\eta_{si}I_{N+1} + (A_1 + B_1)\eta_{io}\eta_{si}\eta_{is}\eta_{oi}I_{N+1} \\ (\eta_{so}^2\eta_{os}\eta_{is}\eta_{oi} + \eta_{oi}^2\eta_{io}\eta_{so}\eta_{is} + \eta_{is}^2\eta_{si}\eta_{so}\eta_{oi})\}]$$

$$f = B^*D^*F^* + A_1B_1C_1\eta_{io}\eta_{os}\eta_{si}I_{N+1} + B_1C_1\eta_{io}\eta_{os}\eta_{so}\eta_{oi}I_{N+1} + A_1C_1\eta_{so}\eta_{is}\eta_{os}\eta_{si}I_{N+1} \\ + A_1B_1\eta_{io}\eta_{si}\eta_{is}\eta_{oi}I_{N+1} + C_1\eta_{so}^2\eta_{os}\eta_{is}\eta_{oi} + B_1\eta_{oi}^2\eta_{io}\eta_{so}\eta_{is} + A_1\eta_{is}^2\eta_{si}\eta_{so}\eta_{oi} + \eta_{so}^2\eta_{oi}^2\eta_{is}^2$$

and $A^* = A_1 + B_1,\ B^* = A_1B_1 - \eta_{so}\eta_{os}I_{N+1},\ C^* = A_1 + C_1,\ D^* = A_1C_1 - \eta_{io}\eta_{oi}I_{N+1},\ E^* = B_1 + C_1$,

$$F^* = B_1C_1 - \eta_{is}\eta_{si}I_{N+1}$$

In sequence to solve (1.34), we go with **PIPES AND HARVILL** (1970)

Let $\vec{S}_1(t) = \vec{P}'(t)$ (1.35)

$$\vec{S}_2(t) = \frac{d}{dt}\vec{S}_1(t) = \vec{P}''(t) \quad (1.36)$$

$$\vec{S}_3(t) = \frac{d}{dt}\vec{S}_2(t) = \vec{P}'''(t) \quad (1.37)$$

$$\vec{S}_4'(t) = \frac{d}{dt}\vec{S}_3(t) = \vec{P}''''(t) \quad (1.38)$$

$$\vec{S}_5^{'}(t)=\frac{d}{dt}\vec{S}_4(t)=\vec{P}'''''(t) \tag{1.39}$$

$$\vec{S}_6^{'}(t)=\frac{d}{dt}\vec{S}_5(t)=\vec{P}''''''(t) \tag{1.40}$$

Therefore (1.34) assumes in the form

$$\vec{S}_5^{'}(t)-a\vec{S}_5(t)-b\vec{S}_4(t)-c\vec{S}_3(t)+d\vec{S}_2(t)-e\vec{S}_1(t)+f\vec{P}(t) \tag{1.41}$$

From (1.35) to (1.41),we have

$$\vec{v}^{'}(t)=A\vec{v}(t) \tag{1.42}$$

Where,

$$\vec{v}^{'}(t)=\begin{bmatrix}\vec{P}(t)\\ \vec{S}_1(t)\\ \vec{S}_2(t)\\ \vec{S}_3(t)\\ \vec{S}_4(t)\\ \vec{S}_5(t)\end{bmatrix},$$ a (6N+46 column vector whose first (N+1) elements are $\vec{P}(t)$,

second (N+1)elements are of $\vec{S}_1(t)$ and so on and whose last (N+1) elements are of $\vec{S}_5(t)$

$$A=\begin{bmatrix}0 & I_{N+1} & 0 & 0 & 0 & 0\\ 0 & 0 & I_{N+1} & 0 & 0 & 0\\ 0 & 0 & 0 & I_{N+1} & 0 & 0\\ 0 & 0 & 0 & 0 & I_{N+1} & 0\\ 0 & 0 & 0 & 0 & 0 & I_{N+1}\\ -f & e & -d & c & b & a\end{bmatrix},$$ a square matrix of (6N+6)$^{\text{th}}$ order

Where,

$0\equiv$ null matrix of order (N+1)

$I_{N+1}\equiv$ Unit matrix of order (N+1)

The set of ingoing conditions involved with (1.42) are

$$\vec{P}(0)=\begin{bmatrix}P_0(0)\\ P_1(0)\\ P_2(0)\\ ....\\ ....\\ P_{N-1}(0)\\ P_N(0)\end{bmatrix}=\begin{bmatrix}1\\0\\0\\..\\..\\0\\0\end{bmatrix},\qquad \vec{L}(0)=\begin{bmatrix}L_0(0)\\ L_1(0)\\ L_2(0)\\ ....\\ ....\\ L_{N-1}(0)\\ L_N(0)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\..\\..\\0\\0\end{bmatrix},\qquad \vec{Q}(0)=\begin{bmatrix}Q_0(0)\\ Q_1(0)\\ Q_2(0)\\ ....\\ ....\\ Q_{N-1}(0)\\ Q_N(0)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\..\\..\\0\\0\end{bmatrix},$$

$$P'(0)=S_1(0)=\begin{bmatrix} P_0'(0)\\ P_1'(0)\\ P_2'(0)\\ ....\\ ....\\ P_{N-1}'(0)\\ P_N'(0)\end{bmatrix}=\begin{bmatrix} -(\lambda_o+\eta_{os}+\eta_{oi})\\ \lambda_o\\ 0\\ ....\\ ....\\ 0\\ 0\end{bmatrix},$$

$$P''(0)=S_2(0)=\begin{bmatrix} P_0''(0)\\ P_1''(0)\\ P_2''(0)\\ ....\\ ....\\ P_{N-1}''(0)\\ P_N''(0)\end{bmatrix}=\begin{bmatrix} -(\lambda_o+\eta_{os}+\eta_{oi})^2+\mu\lambda_o\\ \lambda_o(2\lambda_0+\mu+2\eta_{os}+2\eta_{oi})\\ \lambda_o{}^2\\ ....\\ ....\\ 0\\ 0\end{bmatrix}$$

$$P'''(0)=S_3(0)=\begin{bmatrix} P_0'''(0)\\ P_1'''(0)\\ P_2'''(0)\\ P_3'''(0)\\ P_4'''(0)\\ ....\\ ....\\ P_{N-1}'''(0)\\ P_N'''(0)\end{bmatrix}=\begin{bmatrix} (\lambda_o+\eta_{os}+\eta_{oi})^3-2\mu\lambda_o(\lambda_0+\eta_{os}+\eta_{oi})-\mu\lambda_o(\lambda_o+\mu+\eta_{os}+\eta_{oi})\\ -f\lambda_o(2\lambda_o+\mu+2\eta_{os}+2\eta_{oi})-\lambda_o(\lambda_o+\eta_{os}+\eta_{oi})^2+2\mu\lambda_o{}^2\\ -\lambda_o{}^2(3\lambda_o+\mu+2\eta_{os}+2\eta_{oi})+f\lambda_o{}^2\\ \lambda_a{}^3\\ 0\\ ....\\ ....\\ 0\\ 0\end{bmatrix}$$

$$P''''(0)=S_4(0)=\begin{bmatrix} P_0''''(0)\\ P_1''''(0)\\ P_2''''(0)\\ P_3''''(0)\\ P_4''''(0)\\ P_5''''(0)\\ ....\\ ....\\ P_{N-1}''''(0)\\ P_N''''(0)\end{bmatrix}=\begin{bmatrix} (f+\mu)P_0'''(0)+\mu P_1'''(0)\\ fP_1'''(0)+\mu P_2'''(0)+\lambda_o P_0'''(0)\\ \lambda_o P_1'''(0)+fP_2'''(0)+\mu P_3'''(0)\\ \lambda_o P_2'''(0)+fP_3'''(0)\\ \lambda_o{}^4\\ 0\\ ....\\ ....\\ 0\\ 0\end{bmatrix},$$

$$P'''''(0) = S_5(0) = \begin{bmatrix} P_0^{'''''}(0) \\ P_1^{'''''}(0) \\ P_2^{'''''}(0) \\ P_3^{'''''}(0) \\ P_4^{'''''}(0) \\ P_5^{'''''}(0) \\ P_6^{'''''}(0) \\ \ldots.. \\ \ldots.. \\ P_{N-1}^{'''''}(0) \\ P_N^{'''''}(0) \end{bmatrix} = \begin{bmatrix} (f+\mu)P_0^{''''}(0) + \mu P_1^{''''}(0) \\ fP_1^{''''}(0) + \mu P_2^{''''}(0) + \lambda_o P_0^{''''}(0) \\ \lambda_o P_1^{''''}(0) + fP_2^{''''}(0) + \mu P_3^{''''}(0) \\ \lambda_o P_2^{''''}(0) + fP_3^{''''}(0) + \mu P_4^{''''}(0) \\ \lambda_o P_3^{''''}(0) + fP_4^{''''}(0) \\ \lambda_o{}^5 \\ 0 \\ \ldots.. \\ \ldots.. \\ 0 \\ 0 \end{bmatrix},$$

$$P''''''(0) = S_5^{'}(0) = \begin{bmatrix} P_0^{''''''}(0) \\ P_1^{''''''}(0) \\ P_2^{''''''}(0) \\ P_3^{''''''}(0) \\ P_4^{''''''}(0) \\ P_5^{''''''}(0) \\ P_6^{''''''}(0) \\ P_7^{''''''}(0) \\ \ldots.. \\ \ldots.. \\ P_{N-1}^{''''''}(0) \\ P_N^{''''''}(0) \end{bmatrix} = \begin{bmatrix} (f+\mu)P_0^{'''''}(0) + \mu P_1^{'''''}(0) \\ fP_1^{'''''}(0) + \mu P_2^{'''''}(0) + \lambda_o P_0^{'''''}(0) \\ \lambda_o P_1^{'''''}(0) + fP_2^{'''''}(0) + \mu P_3^{'''''}(0) \\ \lambda_o P_2^{'''''}(0) + fP_3^{'''''}(0) + \mu P_4^{'''''}(0) \\ \lambda_o P_3^{'''''}(0) + fP_4^{'''''}(0) + \mu P_5^{'''''}(0) \\ \lambda_o P_4^{'''''}(0) + fP_5^{'''''}(0) \\ \lambda_o{}^6 \\ 0 \\ \ldots.. \\ \ldots.. \\ 0 \\ 0 \end{bmatrix},$$

Adding the above six sets, we obtain

$$\vec{v}(0) = \begin{bmatrix} \vec{P}(0) \\ \vec{S}_1(0) \\ \vec{S}_2(0) \\ \vec{S}_3(0) \\ \vec{S}_4(0) \\ \vec{S}_5(0) \end{bmatrix}, \quad \text{a (6N+6) column vector}$$

In order to the solution of (1.42), we assume the under mentioned two concerns:-

CONCERN I :

As the Multiplier matrix A has different characteristic roots. Contemplate the Inear modification by $\vec{v}(t)$ to a modern dependent vector $\vec{z}$ by way of the modal matrix of A,

$$w = (\vec{w}_1, \vec{w}_2, \vec{w}_3, \ldots\ldots\ldots, \vec{w}_M, \ldots\ldots\ldots, \vec{w}_{6N+6})$$

Where $\vec{w}_M$, is the M^{th} characteristic vector of A.?

Therefore $\vec{v}(t) = w\vec{z}$

(1.43)

Substituting (1.43) into (1.42) and pre-multiplying by w^{-1}

$$w^{-1}\vec{v}\,'(t) = w^{-1}Aw\vec{z}$$

Or $\vec{z}\,' = w^{-1}Aw\vec{z}$ (1.44)

So far as w is a modal matrix of A, the term $w^{-1}Aw$ is a diagonal matrix with the characteristic roots of A on the diagonal

$$\vec{z}\,' = \begin{bmatrix} \searrow & & \\ & x_i & \\ & & \nwarrow \end{bmatrix}\vec{z}$$

(1.45)

Where x_i indicates the characteristic roots of A on the diagonal.

The outcome is that the modified channel is disjoint, though (1.45) can be denoted by the singular scalar discriminative equality.

$\vec{z}_M^{'} = x_M \vec{z}_M$, M=1, 2, …………, 6N+6

(1.46)

Its common solution is

$$z_M = C_M e^{tx_M}$$

(1.47)

Where C_M is an arbitrary constant? Hence, common solution of (1.45) may be written as.

$$\vec{z} = \begin{bmatrix} C_1 e^{tx_1} \\ C_2 e^{tx_2} \\ C_3 e^{tx_3} \\ .... \\ .... \\ C_{6N+6} e^{tx_{6N+6}} \end{bmatrix}$$

(1.48)

From (1.43), the common solution of (1.42) in preconditions of characteristic vector elaboration

$$\vec{v}(t) = (\vec{w}_1, \vec{w}_2, \vec{w}_3, ..., \vec{w}_{6N+6}) \begin{bmatrix} C_1 e^{tx_1} \\ C_2 e^{tx_2} \\ C_3 e^{tx_3} \\ .... \\ .... \\ C_{6N+6} e^{tx_{6N+6}} \end{bmatrix}$$

(1.49)

Or

$$\vec{v}(t) = \sum_{M=1}^{6N+6} \vec{w}_M C_M e^{tx_M}$$

This equality proves that the common settlement can be described as a inear addition of the characteristic vectors. By using the ingoing concern t=0, the (6N+6) constants C_M can be estimated by (1.49)

$$\vec{v}(0) = \sum_{M=1}^{6N+6} \vec{w}_M C_M \tag{1.50}$$

$$= (\vec{w}_1, \vec{w}_2, \vec{w}_3, ..., \vec{w}_{6N+6}) \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ .... \\ .... \\ C_{6N+6} \end{bmatrix} = Cw$$

So $C = w^{-1}\vec{v}(0)$

The characteristic roots x_i (i = 1, 2, 3, ………,6N+6) of the matrix A may be obtained by its characteristic equality:

$$\left| A - xU \right| = 0$$

(1.51)

If the numeral values of the parameters $\lambda_0, \lambda_s, \mu, \eta_{os}, \eta_{si}, \eta_{oi}, \eta_{so}, \eta_{io}, \eta_{is}$ and N are given, the factual solution of the equation (1.51) can be known.

CONCERN II:

As, the multiplier matrix A has few multiplicative characteristic roots. While multiplicative characteristic roots get, an improved viewpoint is applied because from general manners we may not obtain all the characteristic vectors allied with the various roots. As this condition gets, the optional manner is rooted on the manner of unexpected multipliers.

In order to comfort, assume so the first characteristic roots of a (6N+6)th order channel has a plurality of 4; so it is $x_1 = x_2 = x_3 = x_4 = x$ (say) and $x_5, x_6, ..., x_{6N+6}$ are different.

To solve (1.42) considers a solution in the form

$$\vec{v}(t) = (\vec{w}_1 + t\vec{w}_2 + t^2\vec{w}_3 + t^3\vec{w}_4)e^{xt} + \sum_{M=5}^{6N+6} C_M e^{tx_M} \vec{w}_M \tag{1.52}$$

Putting this considered solution into (1.31), which gives

$$x(\vec{w}_1 + t\vec{w}_2 + t^2\vec{w}_3 + t^3\vec{w}_4)e^{xt} + (\vec{w}_2 + 2t\vec{w}_3 + 3t^2\vec{w}_4)e^{xt} + \sum_{M=5}^{6N+6} C_M e^{tx_M} \vec{w}_M$$

$$= A(\vec{w}_1 + t\vec{w}_2 + t^2\vec{w}_3 + t^3\vec{w}_4)e^{xt} + \sum_{M=5}^{6N+6} C_M e^{tx_M} A\vec{w}_M$$

(1.53)

By applying the explanation of characteristic roots and characteristic vectors, the additions on all direction of the similarity can be rejected. The rest description gets

$$x(\vec{w}_1 + t\vec{w}_2 + t^2\vec{w}_3 + t^3\vec{w}_4) + (\vec{w}_2 + 2t\vec{w}_3 + 3t^2\vec{w}_4)$$

$$= A(\vec{w}_1 + t\vec{w}_2 + t^2\vec{w}_3 + t^3\vec{w}_4)$$

(1.54)

Equating like powers of t

$$x\vec{w}_1 + \vec{w}_2 = A\vec{w}_1$$

$$x\vec{w}_2 + 2\vec{w}_3 = A\vec{w}_2$$

$$x\vec{w}_3 + 3\vec{w}_4 = A\vec{w}_3$$

$$x\vec{w}_4 = A\vec{w}_4$$

This may be obtained serially for the four unknown characteristic vectors? On time the unknown characteristic vectors have been estimated, the common settlement obtains

$$\vec{v}(t) = (C_1\vec{w}_1 + C_2t\vec{w}_2 + C_3t^2\vec{w}_3 + C_4t^3\vec{w}_4)e^{xt} + \sum_{M=5}^{6N+6} C_M e^{tx_M}\vec{w}_M \qquad (1.55)$$

For which the (6N+6) self-willed constants can be estimated in the common manner by application of the ingoing situations. The settlement for $\vec{v}(t)$ will allow the settlement of $\vec{P}(t)$ and $\vec{S}_1(t) = \vec{P}^{'}(t)$. Applying the measures of $\vec{P}(t)$ and $\vec{P}^{'}(t)$ in (1.31), we get $\vec{Q}(t)$ in terms of $\vec{L}(t)$. Therefore, from (1.32) and (1.33), we get $\vec{L}(t)$ and $\vec{Q}(t)$.

Thus $P_n(t)$, $L_n(t)$ and $Q_n(t)$ are completely known. Hence $R_n(t)$ can be determined.

REFERENCES

1. Morse, P. M. Queues, Inventories and Management, Wiley, New York, 1958.
2. Saaty, T. L. Elements of Queuing Theory with Applications. Mc-Graw Hill, New York, 1961.
3. Sharma, O. P. and Dass, J. (1989). Initial busy period analysis for a multichannel Markovian
4. Dharmaraja, S. (2000). Transient solution of a two-processor heterogeneous system. Math. Comput. Model., 32:1117-1123.
5. Kumar R. and Sharma, S. (2019). Transient Solution of a Two-Heterogeneous Servers' Queuing System with Retention of Reneging Customers. Bull. Malays. Math. Sci. Soc., 42:223-240.
6. Krishanamoorthy, B (1963). On Poisson queues with heterogeneous servers. Operations Research, 11:321-330.
7. Prabhu, N, U. : Queues and Inventories, New York, Wiley 1965
8. Singh, V.P. (1970) Two-server Markovian queues with balking: heterogeneous vs homogeneous servers. Operations Research. 18:145-159.
9. Saraswat G. K. (2012); "*Explicit results in a single counter exponential queues with heterogeneity in arrival process*", in International J. of .Math. Sci. & Engg. Appls. , Vol 6 (2), pp. 335- 347.
10. Yechili, U. and Naor, P. (1971): Queuing problems with heterogeneous arrivals and service, Operations Research, 19, 722-734.
11. Kleinrock, L. : Queueing Systems (Vol. I and II) Wiley New York. 1975
12. Neuts, M.F. (1978) : The M / M / 1 queue with randomly varying arrival and service rates, Opsearch 15 (4), 139-157.
13. Murari, K. and R. K. Agarwal (1981): 'Explicit results in heterogeneous queues with general distribution,' Cahiers du Centre d' Etudes de Recherche Operationnelle, Bruxelles, Volume 23.
14. Sharma O. P. and Dass, J. (1990). Limited space double channel Markovian queue with heterogeneous servers, Trabajos De Investigacion Dperativa, 5:73-78.
15. Premchandra, I. M. and Liliana Gongalez (1994) : System with multipurpose counters, Computers Ops. Res. Vol. 21 No. 9. PP 969-977.

16. Saraswat G. K. and Agarwal R. K. (2013); "Markovian Queuing Model with Heterogeneity in Arrival Process Ruled by Second Order Stochastic Matrix", in Int J.Engg. Sci. & Mgmt., Vol. III, pp.70-79.
17. Madan, K. C. (2001) : On a single server queue with two stage heterogeneous service and deterministic server vacations, International Journal of System Science, 32, 113-125.
18. Green, L. V., Kolesar, P. J. and W. Whitt (2007): Coping with time-varying demand when setting staffing requirement for a service system, Production Oper. Management16(1)13-39.
19. Krishna Kumar, B., Madheswari Pavai, S. and Venkatakrishnan (2007) Transient solution of an M/M/2 queue with heterogeneous servers ubject to catastrophes, Information and Management Sciences Vol.18(1) 63-80.
20. Saraswat G. K. and Agarwal R. K. (2012), Limited waiting space heterogeneous queuing system with input depending on queue length, Global Journal of Pure and Applied Mathematics, Vol. 8, pp. 349-358.
21. Saraswat G. K. and Agarwal R. K. (2014), Markovian Queuing System with Heterogeneity and Blocking in Service, Mathematics Applied in Science and Technology (MAST), Vol. 6, 15-23.
22. Wang Jiang-Ping, Huang Yong (2008): The monitoring of the network traffic based on queuing theory, ISORA 'O8, ORSC & APORC, pp. 60-65.
23. Rakesh Kumar and Sumeet Kumar Sharma (2014), Two-Heterogeneous Server Markovian Queueing Model with Discouraged Arrivals, Reneging and Retention of Reneged Customer, International Journal of Operations Research,Vol. 11, No-2, 064-068.
24. Saraswat G. K. (2018), Single Counter Markovian Queuing Model with Multiple Inputs, IJMTT, Vol. 60, 20 – 34.
25. Bansal P. and Saraswat G. K. (2018) "An M/M/1 Queuing System With Three Types Inputs Sources" in (IJMAA), Vol. 6(4), 175-186.
26. Saraswat G. K. and Agarwal R. K. (2012); "Limited waiting space heterogeneous queuing system with input depending on queue length", in Global Journal of Pure and Applied Mathematics, Vol. 8, pp. 349-358.
27. Sudhesh, R, Savitha P., Dharmaraja, S. (2017). Transient analysis of a two-heterogeneous servers queue with system disaster, server repair and customers impatience. TOP, 25:179– 205.
28. Saraswat G. K. and Bansal P. (2020) "Markovian Multifarious queue with service rate depending on duration of time in one state", in Studies in Indian Place Names (SIPN) Vol. 40 (3), 1164-1177.
29. Saraswat G.K. and Kumar Vijay (2021) "Confined Space Queue with a Random Number of Input Sources Acting at a Time" in Malaya Journal of Matematik, Vol. S. No. 1, pp. 346- 348.
30. Kumar, B. K., Madheshwari, S. P. and Venkatakrishanan, K. S. (2007). Transient solution of an M/M/2 queue with heterogeneous servers subject to catastrophes. Int. J. of Inf. Manag. Sci., 18:63-80.
31. Ammar, S.I. (2014). Transient behavior of a two-processor heterogeneous system with catastrophes, server failures and repairs. Appl. Math. Model., 38:2224-2234.
32. Saraswat, G.K. and Kumar, Vijay (2022) "Markovian Heterogeneous Queue with Service Rate Depending On Duration of Time in Two States" in Stochastic Modeling & Applications Journal, Vol. 26 No.3 (January – June, Special Issue 2022 part – 2), 910-924,
33. Saraswat, G.K. and Kumar, Vijay (2022) "Markovian Heteromorphic Limited Space Queue With a Random Count Input Sources Operating At A Time" in International Journal of Mechanical Engineering, Vol. 7 No. 4 April 2022, 97-105.