

Solving Fuzzy Quadrating Programming Problem Using MATLAB

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Abstract

In this article, we have solved the Fuzzy Quadratic Programming Problem through MATLAB. Fuzzy numbers in the fuzzy quadratic programming are converted as crisp numbers and then solved the Quadratic Programming Problem using the quadprog function in MATLAB. Different ranking methods are applied for converting fuzzy number as crisp number. The procedures are explained in detail through some numerical examples.

Keywords: Fuzzy Quadratic Programming Problem, Fuzzy Numbers, Crisp Numbers.

1. INTRODUCTION

Fuzzy sets were introduced independently by professor Lotfi A. Zadeh and Dieter Klaua [de] in 1965 as an extension of the classical notion of set. Quadratic programming (QP) is an optimization problem

with a quadratic ideal function and direct equivalence or inequalities as constraints. QP is generally utilised in real- world problems to optimise portfolio selection, in retrogression to conduct the least square approach, in chemical shops to control scheduling, in successional quadratic programming, economics, and engineering design, among other operations. The QP is known as the NP-hard because it's the most interesting class of optimization. The styles and algorithms for solving the QP problem was proposed by Pardalos and Rosen (1), Horst and Tuy (2), and Bazaraa et al. (3). A necessary global optimality criterion was suggested for the nonconvex QP optimization problem with double constraints by Beck and Teboulle (4). Kochenberger et al. (5) delved the double quadratic programming issue with no constraints. For nonconvex quadratic optimization with double constraints, Xia (6) developed original optimality conditions for establishing new sufficient optimality conditions. Bonami et al. (7) proposed a computational fashion for working non-convex quadratic equations with box constraints grounded on effective direct programming. Abbasi (8) created a new model grounded on discriminational algebraic equations for working convex QP problems. For working multi-objective QP problems, Pramanik and Dey (9) suggested a precedence grounded fuzzy programming model. Despite expansive decision- timber experience, the decision maker isn't always suitable to describe the objects precisely. Bellman and Zadeh's (10) decision- making in a fuzzy terrain bettered and was a tremendous backing in operation decision difficulties. By assessing the upper and lower bounds of the objective value at the possibility position, the fuzzy QP is turned into a brace of two level fine programmes grounded on Zadeh's extension principle (11). The fuzzy set proposition and its operations were proposed by Zimmermann (12). Zimmermann (13) proposed fuzzy programming, which includes a variety of objective functions, fuzzy sets, and systems. QP was defined as a function of FN, with trapezoidal or triangular fuzzy figures defining the cost, constraints portions and right-hand side by Nasser (14). QP problem was converted as fuzzy figures in the portions and variables into a deterministic one and also produced an optimal result using fuzzy ranking and computation operations by Kheirfam (15). Allahviranloo and Moazam (16) presented a new notion called the alternate power of fuzzy figures, which provides a logical and approximate result to completely fuzzy quadratic equations. Specific type of fuzzy QPP concerning fuzziness in connection was solved by Taghi-Nezad and Taleshian (17) with new approach. Fuzzy For deriving quadratic minimization problems with either box or integer constraints, Gao and Ruan (18) proposed a canonical duality proposition. The duality gap between the double quadratic optimization problem and Sun et al. (19) developed a method it is

semi-definite programming relaxation. For working an inheritable QP problem with both equivalency and inequality constraints, Gill and Wong (20) presented an active set fashion. Interval valued variables quadratic programming was divided into two problems the best and worst optimum problems using two-position programming approach by Syaripuddin et.al. (21). To solve roughly minimizing convex quadratic function over the crossroad of affine and separable constraints Takapoui et.al. (22) suggested an algorithm. Quadratic programming problem with quadratic constraints was solved by Shi et al. (23) using effective algorithm. Working and assaying quadratic and nonlinear programming in fully fuzzy region, Gabr (24) presented a new methodology. QP problem with triangular fuzzy number in all of objective function portions and the fuzzy righthand side of the constraints was solved by Mirmohesni and Nasserii (25) using a new approach. To decide the fuzzy objective value for the problem introduced by Kuhn-Tucker conditions for working fuzzy QP problems, a new approach predicated by Maheswari (26).

In this present work, we are finding maximum or minimum value of the quadratic function with inequality or equality constraints. Quadratic programming problem is mostly used in day-to-day problems. Hence many of the researcher presented different approaches to solve fuzzy quadratic programming problem, here our approach for solving fully fuzzy quadratic programming problem can be converted into a crisp problem by using ranking method with the help of MATLAB.

Summary of this paper as follows: Section.2 provides the necessary definitions, some properties of fuzzy set theory. Section.3 describes the general form of quadratic programming problem and fuzzy quadratic programming problems defined. Section 4 calculation algorithm and examples provided, also MATLAB code for defuzzification using ranking method and conclusion given in Section 5.

2. FUZZY NUMBERS AND FUZZY ARITHMATIC (BASIC DEFINITIONS)

The basic definitions of fuzzy set theory are taken from [1]

Definition 2.1

A fuzzy set $\mathbf{A}$ in $\mathbb{R}$ is defined as a set of ordered pairs $\mathbf{A} = \{(x, \mu_{\mathbf{A}}(x)) | x \in \mathbb{R}\}$, where $\mu_{\mathbf{A}}(x)$ is the membership function for the fuzzy set and $\mu_{\mathbf{A}}(x): \mathbb{R} \rightarrow [0, 1]$.

Definition 2.2

Support of a fuzzy set A is defined as follows: $supp(A) = \{x \in \mathbb{R} | \mu_A(x) > 0\}$.

Definition 2.3

The set of all points x in $\mathbb{R}$ which $\mu_A(x) = 1$, is called **core** of the fuzzy A

Definition 2.4

There is at least one-point $x \in \mathbb{R}$ with $\mu_A(x) = 1$, is said to be normal of the fuzzy set A

Definition 2.5

The α -**cut** of a fuzzy set is a crisp set defined by: $A_\alpha = \{x \in \mathbb{R} | \mu_A(x) > \alpha\}$.

Definition 2.6

A fuzzy set A on $\mathbb{R}$ is **convex**, if any $x, y \in \mathbb{R}$ and $\lambda \in [0, 1]$, we have

$$\mu_A(\lambda x + (1 - \lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}.$$

Definition 2.7

A fuzzy set A on $\mathbb{R}$ is said to be **triangular fuzzy number**, if there exist real numbers (a, b, c) and $a, b, c \geq 0$, such that

$$A(x) = \mu_A(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x > c \end{cases}$$

Here is pictorial representation of triangular fuzzy number A Fig. 1.

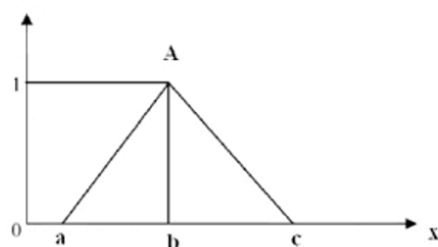


Fig. 1 Triangular Fuzzy Number.

Definition 2.8

Zero triangular fuzzy number can be defined as follows, $0 = (0, 0, 0)$

2.9 Fuzzy Arithmetic

For any two intervals, $[a, b]$ and $[c, d]$, the arithmetic operations are

$$\text{Addition} \quad [a, b] + [d, e] = [a + d, b + e]$$

$$\text{Subtraction} \quad [a, b] - [d, e] = [a - e, b - d]$$

$$\text{Multiplication} \quad [a, b] \cdot [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$$

$$\text{Power} \quad [a, b]^{[d, e]} = [\min(a^d, a^e, b^d, b^e), \min(a^d, a^e, b^d, b^e)]$$

$$\text{Division} \quad [a, b]/[d, e] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)], \text{ provided that } 0 \notin [d, e].$$

2.10 Fuzzy Ranking

The function $\mathcal{R}:F(R) \rightarrow R$ is a ranking function where the domain is fuzzy number and the image is real line. The order of $F(R)$ is defined as

- $a \geq b \iff R(a) \geq R(b)$
- $a > b \iff R(a) > R(b)$
- $a = b \iff R(a) = R(b)$

3. QUADRATIC PROGRAMMING AND FUZZY QUADRATIC PROGRAMMING PROBLEM

3.1 Quadratic Programming Problem

The general form of quadratic programming problem is taken from (Idhani, 1982)

$$\text{Min (Max)} Z = \sum_{j=1}^n c_j x_j + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n q_{ij} x_i x_j$$

$$\text{s. to } \begin{cases} \sum_{j=1}^n a_{ij} x_j \leq (\geq \text{ or } =) b_i, i \in N_m, \\ x_j \geq 0 \end{cases}$$

where

$c = (c_1, c_2, c_3, \dots, c_n)$ is a cost vector

$b^T = (b_1, b_2, b_3, \dots, b_n)$ is a right-hand side vector

$x = (x_1, x_2, \dots, x_n)^T$ is a variable which is in the form of vector

$A = [a_{ij}]_{m \times n}$, $i \in N_m$ and $j \in N_n$, is a constraint matrix and

$Q = [q_{ij}]_{m \times n}$ is a quadratic form matrix where $i \in N_n$ and $j \in N_n$.

From the above general form, then the matrix form of the function can be written as,

$$\begin{aligned} \text{Min(Max)} Z &= cx + \frac{1}{2} x^T Q x \\ \text{s. to } &\begin{cases} Ax \leq b \\ x \geq 0 \end{cases} \end{aligned}$$

where $Q = [q_{ij}]_{m \times n}$, $i \in N_m$ and $j \in N_n$ is a symmetric and positive semi-definite matrix. Many of the researchers have developed productive algorithms to solve quadratic programming problems with known parameters. Although quadratic programming functions are designed to find a way to do things in the future and parameter values to predict future conditions, it introduces a degree of uncertainty.

3.2 Fully Fuzzy Quadratic Programming

Consider the classic Quadratic Programming Problem (QPP). Fuzzy quadratic programming is a problem all of the parameters are fuzzy numbers. The problem's extensive form is as follows:

$$\begin{aligned} \text{Min } Z &= \sum_{j=1}^n C_j X_j + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n Q_{ij} X_i X_j \\ \text{s.to } &\begin{cases} \sum_{j=1}^n A_{ij} X_j \leq B_i, i \in N_m, \\ X \geq 0 \end{cases} \end{aligned}$$

Where A_{ij} , B_i , C_j , Q_{ij} and X_j are fuzzy numbers and fuzzy variables respectively for all $(i \in N_m, j \in N_n)$. Here sum and product are fuzzy arithmetic.

3.3 Introduction of Fuzzy Ranking

Jain was the first to introduce a fuzzy set grading process for multi-aspect decision-making in 1977. Since then, researchers have been interested in developing a general ranking measure capable of consistently discriminating the magnitude of fuzzy numbers. To convert the fuzzy quadratic programming problem to crisp quadratic programming, the fuzzy ranking approach is applied.

3.4 The Following Fuzzy Ranking are used this paper

- Let fuzzy number $A = (a_1, a_2, a_3)$ be a triangular fuzzy number, a new method of the ranking function was first proposed by Liou et al. [34] as follows

$$R(A) = \frac{(a_1 + 2a_2 + a_3)}{4}$$

- The Yager's ranking (12) as follows

$$R(A) = \int_0^1 0.5(d_\alpha^L + d_\alpha^U) d\alpha$$

where d_α^L = Lower α level cut, d_α^U = Upper α level cut

$$R(A) = \frac{(a_1 + a_2 + a_3)}{3}$$

- From [29] For every $A = (a_1, a_2, a_3) \in R$, The rating characteristic $R : F(R) \rightarrow R$ which maps every fuzzy wide variety into the actual line, in which an herbal order exists and it is

$$R(A) = \frac{(a_2 + 4a_1 + a_3)}{6}$$

- Maleki ranking method:

In [33] Khalifa et al. used a linear ranking function which is adopted by Maleki et al. [35]. Some $L-R$ fuzzy number $A = [a^L, a^U, \alpha, \beta]$ defined as follows:

$$R(A) = [a^L + a^U + \frac{(\beta - \alpha)}{2}]$$

- From Yager ranking $R(A) = \int_0^1 0.5(d_\alpha^L + d_\alpha^U) d\alpha$

which motivates BehrouzKheirfam et al. [17] proposed new ranking for the fuzzy number

$$A = [a^L, a^U, \alpha, \beta]$$

$$R(A) = [\frac{a^L + a^U}{2} + \frac{(\beta - \alpha)}{4}]$$

4. ALGORITHM, RESULTS AND DISCUSSION

4.1 Algorithm Fuzzy QPP

Step 1: Enter the fuzzy number and fuzzy matrices in the MATLAB.

Step 2: Convert the fuzzy number as crisp number using different ranking methods.

Step 3: Using cell array store the step 2 result in to separated array and matrices.

Step 4: Solve the QPP using the coefficient matrix of crisp numbers.

4.2 Result and Discussion

The five types of ranking are applied in the QPP are discussed and verified using MATLAB, here is some example problems are discussed

Example 1:

Consider the fuzzy quadratic programming problem with triangular fuzzy number [Liu et al. [38]]

$$\text{Min } Z = -(6,5,4)x_1 + (1,1.5,2)x_2 - [(2,3,4)x_1^2 + (3,2,1)x_1x_2 - (1,2,3)x_2^2]$$

$$\text{s. to. } \begin{cases} (1,1,1)x_1 + (0.5,1,1.5)x_2 \leq (1,2,3) \\ (1,2,3)x_1 + (-2,-1,-1/2)x_2 \leq (3,4,5) \\ \text{and } x_1, x_2 \geq 0 \end{cases}$$

Solution

Enter the size of row matrix H = 2

Enter the size of column matrix H = 2

Enter the row wise entries within [] for H matrix: [4,6,8]

Enter the row wise entries within [] for H matrix: [-3,-2,-1]

Enter the row wise entries within [] for H matrix: [-3,-2,-1]

Enter the row wise entries within [] for H matrix: [2,4,6]

$$H = \begin{bmatrix} 4 & 6 & 8 \\ -3 & -2 & -1 \end{bmatrix}$$

$$H = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$$

Enter the size of row matrix f = 2

Enter the size of column matrix $f = 1$

Enter the row wise entries within [] for f matrix: [-6,-5,-4]

Enter the row wise entries within [] for f matrix: [1,1.5,2]

$f =$ -6.0000 -5.0000 -4.0000

1.0000 1.5000 2.0000

$f =$ -5.0000

1.5000

Enter the size of row matrix $A = 2$

Enter the size of column matrix $A = 2$

Enter the row wise entries within [] for A matrix: [1,1,1]

Enter the row wise entries within [] for A matrix: [0.5,1,1.5]

Enter the row wise entries within [] for A matrix: [1,2,3]

Enter the row wise entries within [] for A matrix: [-2,-1,-.5]

$A =$ 1.0000 1.0000 1.0000, 0.5000 1.0000 1.5000

1.0000 2.0000 3.0000, -2.0000 -1.0000 -0.5000

$A =$ 1.0000 1.0000

2.0000 -1.1667

Enter the size of row matrix $b = 2$

Enter the size of column matrix $b = 1$

Enter the row wise entries within [] for b matrix: [1,2,3]

Enter the row wise entries within [] for b matrix: [3,4,5]

$b =$ 1 2 3

3 4 5

$$b = 2$$

$$4$$

Min function satisfies the given constraints

Optimization obtained.

The objective variables and the corresponding solutions are bellow,

$$\mathbf{x} = (0.8500, 0.0500)$$

$$\mathbf{fval} = -2.0875$$

Example 2:

Consider the fuzzy quadratic programming problem with triangular fuzzy number [Liu et al. [34]]

$$\text{Min } Z = (6,5,4)x_1 - (1,1.5,2)x_2 - (1,2,3)x_1^2 + (1,2,3)x_1x_2 - (0,1,2)x_2^2$$

$$s. to. \begin{cases} (1,1,1)x_1 + (0.5,1,1.5)x_2 \leq (1,2,3) \\ (1,2,3)x_1 + (-2,-1,-0.5)x_2 \leq (3,4,5) \\ x_1, x_2 \geq 0 \end{cases}$$

Solution

Enter the size of row matrix H = 2

Enter the size of column matrix H = 2

Enter the row wise entries within [] for H matrix: [1,2,3]

Enter the row wise entries within [] for H matrix: [-1,-2,-3]

Enter the row wise entries within [] for H matrix: [-1,-2,-3]

Enter the row wise entries within [] for H matrix: [0,1,2]

$$H = \begin{bmatrix} 1 & 2 & 3 & -1 & -2 & -3 \\ -1 & -2 & -3 & 0 & 1 & 2 \end{bmatrix}$$

$$H = \begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$$

Enter the size of row matrix $f = 2$

Enter the size of column matrix $f = 1$

Enter the row wise entries within [] for f matrix: [-6,-5,-4]

Enter the row wise entries within [] for f matrix: [1,1.5,2]

$$f = \begin{bmatrix} -6.0000 & -5.0000 & -4.0000 \\ 1.0000 & 1.5000 & 2.0000 \end{bmatrix}$$

$$f = \begin{bmatrix} -5.0000 \\ 1.5000 \end{bmatrix}$$

Enter the size of row matrix $A = 2$

Enter the size of column matrix $A = 2$

Enter the row wise entries within [] for A matrix: [1,1,1]

Enter the row wise entries within [] for A matrix: [0.5,1,1.5]

Enter the row wise entries within [] for A matrix: [1,2,3]

Enter the row wise entries within [] for A matrix: [-2,-1,-.5]

$$A = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000, & 0.5000 & 1.0000 & 1.5000 \\ 1.0000 & 2.0000 & 3.0000, & -2.0000 & -1.0000 & -0.5000 \end{bmatrix}$$

$$A = \begin{bmatrix} 1.0000 & 1.0000 \\ 2.0000 & -1.1667 \end{bmatrix}$$

Enter the size of row matrix $b = 2$

Enter the size of column matrix $b = 1$

Enter the row wise entries within [] for b matrix: [1,2,3]

Enter the row wise entries within [] for b matrix: [3,4,5]

$b = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}$

$b = 2$
4

The problem is non-convex.

Min function satisfies the given constraints

Optimization obtained

The values of the Min function and the corresponding values

$x = (1.4500, 0.5500)$

$fval = -3.5125$

Example 3:

This problem taken from Umamaheswari.P [29]

$$\text{Min } Z = (-5, -4, -3)x_1 + (-1, 0, 1)x_2 + (0, 1, 2)x_1^2 - (1, 2, 3)x_1x_2 + (1, 2, 3)x_2^2$$

$$s. to. \begin{cases} (1, 2, 3)x_1 + (0, 1, 2)x_2 \leq (5, 6, 7) \\ (1, 2, 3)x_1 + (-5, -4, -3)x_2 \leq (0, 0, 0) \\ x_1, x_2 \geq 0 \end{cases}$$

Solution

Enter the size of row matrix $H = 2$

Enter the size of column matrix $H = 2$

Enter the row wise entries within [] for H matrix: [0,2,4]

Enter the row wise entries within [] for H matrix: [-1,-2,-3]

Enter the row wise entries within [] for H matrix: [-1,-2,-3]

Enter the row wise entries within [] for H matrix: [2,4,6]

$H = \begin{bmatrix} 0 & 2 & 4 & -1 & -2 & -3 \\ -1 & -2 & -3 & 2 & 4 & 6 \end{bmatrix}$

$H = \begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$

Enter the size of row matrix $f = 2$

Enter the size of column matrix $f = 1$

Enter the row wise entries within [] for f matrix: [-5,-4,-3]

Enter the row wise entries within [] for f matrix: [-1,0,1]

$f = \begin{bmatrix} -5 & -4 & -3 \\ -1 & 0 & 1 \end{bmatrix}$

$f = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$

Enter the size of row matrix $A = 2$

Enter the size of column matrix $A = 2$

Enter the row wise entries within [] for A matrix: [1,2,3]

Enter the row wise entries within [] for A matrix: [0,1,2]

Enter the row wise entries within [] for A matrix: [0,1,2]

Enter the row wise entries within [] for A matrix: [-5,-4,-6]

$A = \begin{bmatrix} 1 & 2 & 3 & 0 & 1 & 2 \\ 0 & 1 & 2 & -5 & -4 & -6 \end{bmatrix}$

A = 2.0000 1.0000

1.0000 -4.5000

Enter the size of row matrix b = 2

Enter the size of column matrix b = 1

Enter the row wise entries within [] for b matrix: [5,6,7]

Enter the row wise entries within [] for b matrix: [-1,0,1]

b= 5 6 7

-1 0 1

b = 6

0

Min function satisfies the given constraints

Optimization obtained

The values of the Min function and the corresponding values

x = (2.4615, 1.0769)

fval = -6.7692

Example 4.

Consider the fuzzy quadratic programming problem with trapezoidal fuzzy number [Zhou et al. [33]]

$$\begin{aligned} \text{Min } Z &= \left(\frac{1}{2}, \frac{3}{2}, 1, 1\right) x_1^2 - (1, 1, 4, 2) x_1 x_2 + \left(\frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{1}{4}\right) x_2^2 \\ \text{s. to. } &\begin{cases} (2, 3, 2, 2) x_1 + (1, 3, 2, 3) x_2 \leq (5, 6, 2, 2) \\ (2, 3, 1, 3) x_1 + (1, 2, 3, 1) x_2 \leq (3, 5, 2, 4) \\ (1, -2, -1, -1) x_1 - (0, 2, 4, 2) x_2 \leq -(0, 1, 1, 1) \\ x_1, x_2 \geq 0 \end{cases} \end{aligned}$$

solution

Enter the size of row matrix $H = 2$

Enter the size of column matrix $H = 2$

Enter the row wise entries within [] for H matrix: [1/2,3/2,1,1]

Enter the row wise entries within [] for H matrix: [-1,-1,-4,-2]

Enter the row wise entries within [] for H matrix: [-1,-1,-4,-2]

Enter the row wise entries within [] for H matrix: [1/4,3/4,1/4,1/4]

$H = \begin{bmatrix} 0.5000 & 1.5000 & 1.0000 & 1.0000 & -1.0000 & -1.0000 & -4.0000 & -2.0000 \\ -1.0000 & -1.0000 & -4.0000 & -2.0000 & 0.2500 & 0.7500 & 0.2500 & 0.2500 \end{bmatrix}$

$H = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

Enter the size of row matrix $f = 2$

Enter the size of column matrix $f = 1$

Enter the row wise entries within [] for f matrix: [0,0,0,0]

Enter the row wise entries within [] for f matrix: [0,0,0,0]

$f = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Enter the size of row matrix $A = 3$

Enter the size of column matrix $A = 2$

Enter the row wise entries within [] for A matrix: [2,3,2,2]

Enter the row wise entries within [] for A matrix: [1,3,2,3]

Enter the row wise entries within [] for A matrix: [2,3,1,3]

Enter the row wise entries within [] for A matrix: [1,2,3,1]

Enter the row wise entries within [] for A matrix: [1,-2,-1,-1]

Enter the row wise entries within [] for A matrix: [-0,-2,-4,-2]

A = 2 3 2 2, 1 3 2 3
 2 3 1 3, 1 2 3 1
 1 -2 -1 -1, 0 -2 -4 -2

A = 5.0000 4.5000
 6.0000 2.0000
 -1.0000 -1.0000

Enter the size of row matrix b = 3

Enter the size of column matrix b = 1

Enter the row wise entries within [] for b matrix: [5,6,2,2]

Enter the row wise entries within [] for b matrix: [3,5,2,4]

Enter the row wise entries within [] for b matrix: [0,-1,-1,-1]

b = 5 6 2 2
 3 5 2 4
 0 -1 -1 -1

b = 11

9

-1

Min function satisfies the given constraints, Optimization obtained

The values of the Min function and the corresponding values are bellow,

x = (0.4000, 0.6000)

fval = 0.1000

Example 5.

Consider the fuzzy quadratic programming problem with trapezoidal fuzzy number [Zhou et al. [37]]

$$\text{Min } Z = (0,0,0,0)x_1 + (0,0,0,0)x_2 + (2,6,4,4)x_1^2 + (-2, -2, -8, -4)x_1x_2 + (1,3,2,2)x_2^2$$

$$s. to. \begin{cases} (2,3,2,2)x_1 + (1,3,2,3)x_2 \leq (5,6,2,2) \\ (2,3,1,3)x_1 + (1,2,3,1)x_2 \leq (3,5,2,4) \\ (1, -2, -1, -1)x_1 - (0,2,4,2)x_2 \leq -(0,1,1,1) \\ x_1, x_2 \geq 0 \end{cases}$$

solution

Enter the size of row matrix H = 2

Enter the size of column matrix H = 2

Enter the row wise entries within [] for H matrix: [2,6,4,4]

Enter the row wise entries within [] for H matrix: [-2,-2,-8,-4]

Enter the row wise entries within [] for H matrix: [-2,-2,-8,-4]

Enter the row wise entries within [] for H matrix: [1,3,2,2]

H = 2 6 4 4, -2 -2 -8 -4

 -2 -2 -8 -4, 1 3 2 2

H = 4 -1

 -1 2

Enter the size of row matrix f = 2

Enter the size of column matrix f = 1

Enter the row wise entries within [] for f matrix: [0,0,0,0]

Enter the row wise entries within [] for f matrix: [0,0,0,0]

f = 0 0 0 0

 0 0 0 0

f = 0

 0

Enter the size of row matrix A = 3

Enter the size of column matrix A = 2

Enter the row wise entries within [] for A matrix: [2,3,2,2]

Enter the row wise entries within [] for A matrix: [1,3,2,3]

Enter the row wise entries within [] for A matrix: [2,3,1,3]

Enter the row wise entries within [] for A matrix: [1,2,3,1]

Enter the row wise entries within [] for A matrix: [1,-2,-1,-1]

Enter the row wise entries within [] for A matrix: [0,-2,-4,-2]

```
A =  2   3   2   2   1   3   2   3
      2   3   1   3   1   2   3   1
      1  -2  -1  -1   0  -2  -4  -2
```

```
A = 2.5000  3.0000
      -0.5000  2.2500
      1.0000 -0.5000
```

Enter the size of row matrix b = 3

Enter the size of column matrix b = 1

Enter the row wise entries within [] for b matrix: [5,6,2,2]

Enter the row wise entries within [] for b matrix: [3,5,2,4]

Enter the row wise entries within [] for b matrix: [0,-1,-1,-1]

```
b = 5   6   2   2
      3   5   2   4
      0  -1  -1  -1
```

```
b = 5.5000
      4.5000
      -0.5000
```

Min function satisfies the given constraints

Optimization obtained

The values of the Min function and the corresponding values

x = (-0.3750, 0.2500)

fval = 0.4375

CONCLUSION

In this article, a new tool is developed to solve the fuzzy quadratic programming problem, by which both the objective function and the boundary conditions are viewed as a fuzzy number. Solve the fitted quadratic programming problem with the quadprog function in MATLAB. This approach is simple and flexible.

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