# On $\mathcal{N}g\omega\alpha$-Continuous Functions in Nano Topological Spaces

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***Abstract***

The purpose of this paper is to define and study a new class of functions called $\mathcal{N}g\omega\alpha$-continuous functions and $\mathcal{N}g\omega\alpha$-irresolute functions in nano topological spaces.Also we study the relationship between $\mathcal{N}g\omega\alpha$-continuous functions and other existing nano continuous functions. Further obtain some of the basic properties and also we have given an appropriate examples to understand the abstract concepts clearly.

**Keywords :** Nano continuous, Nano $g$-continuous, Nano $\alpha$-continuous, Nano $\hat{g}$-continuous, Nano $\hat{g}\alpha$-continuous and $\mathcal{N}g\omega\alpha$-continuous.

## 1. Introduction:

One of the main concepts of topology is continuous functions. In 1963, Levine [11] introduced and concepts of semi open sets and semi continuity in topological spaces. The generalizations of generalized continuity were introduced and studied by Balachandranet. al.[1], Mashhour et. al.[14], Gnanambal [9], Devi et. al.[7], sundaram and shaik John[17] and Benchalli et.al.[2] namely $g$-continuity, $\alpha$-continuity and Pre-continuity, $gpr$-continuity, $g\alpha$-continuity and $\alpha g$-continuity, $\omega$-continuity, $\omega\alpha$-continuity and $g\omega\alpha$-ccontinuity in topological spaces.

The noyion of nano topology was introduced by Lellisthivagar [12], which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it.He has also defined a nano continuous, nano open mappings, nano closed mappings and their representations in terms of nano closure ansnano interior.Bhuvaneshwariet. al [3,4,5,6] introduced the concept of Nano $\alpha g$-continuous and Nano $g\alpha$-continuous, Nano $rg$-continuous and Nano $gr$-

continuous, Nano $gp$-continuous and Nano $pg$-continuous in nano topological spaces.Recently, Parvathy et.al [15], Lalitha et.al. [10] andRajendran et.al. [16],introduced the concepts of Nano $gpr$-continuous, Nano $\hat{g}$-continuous and Nano$\hat{g}\alpha$-continuous functions in nano topological spaces.

The structure of this manuscript is as follows. In section 2, we recall some existing definitions which are more important to prove our main results. In section 3, we induct and study the notion of $\mathcal{N}g\omega\alpha$-continuous functions in nano topological spaces. In section 4, we introduce and examine some theorems which satisfy the condition of $\mathcal{N}g\omega\alpha$-irresolute functions.

**2. Preliminaries:**

In this section, we recall some basic definitions and results in nano topological spaces are given, which are more important to prove our main results.

**Definitions 2.1.**[12]Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation spaces. Let $X \subseteq U$.

(i) The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and its denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x$.

(ii) The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and its denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of $X$ with respect to $R$ is the set of all objects, which can be for certain classified neither as $X$ nor as not $X$ with respect to $R$ and its denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

**Preposition 2.2 [**12**]**If $(U, R)$ is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and$L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv)$U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi)$L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$and$U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii)$U_R(X^c) = [L_R(X)]^c$and$L_R(X^c) = [U_R(X)]^c$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**Definition 2.3.**[12]Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by Property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) $U$and$\emptyset \in \tau_R(X)$

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is $\tau_R(X)$

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on $U$ called the nano topology on $U$ with respect to $X$. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$.

**Remark 2.4.**[12]If $\tau_R(X)$ is the nano topology on $U$ with respect to $X$, then the set $B = \{U, \emptyset, L_R(X),\ B_R(X)\}$ is the basis for $\tau_R(X)$.

**Definition 2.5.** [12]If $(U, \tau_R(X))$ is a nano topological space with respect to $X$ and if $A \subseteq U$, then

- The nano interior of A is defined as the union of all nano open subsets of $A$ and it is denoted by $Nint(A)$.That is, $Nint(A)$ is the largest nano open subset of $A$.
- The nano closure of A is defined as the intersection of all nano closed subsets of $A$ and it is denoted by $Ncl(A)$.That is, $Ncl(A)$ is the smallest nano closed set containing $A$.

**Definition 2.6.**Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then $A$ is said to be

(i) nano semi open [12], if $A \subseteq \mathcal{N}cl(\mathcal{N}int(A))$

(ii) nanopre open [12], if $A \subseteq \mathcal{N}int(\mathcal{N}cl(A))$

(iii) nano α-open [12], if $A \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(A)))$

(iv) nano regular open [12], if $A = \mathcal{N}int(\mathcal{N}cl(A))$

**Definition 2.6.**A subset $A$ of a nano topological space $(U, \tau_R(X))$ is called

(i) $\mathcal{N}g$-closed [3], if $\mathcal{N}cl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano open in $U$.

(ii) $\mathcal{N}\hat{g}$-closed [10], if $\mathcal{N}cl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano semi open in $U$.

(iii) $\mathcal{N}\hat{g}\alpha$-closed [16], if $\mathcal{N}\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is nano $\mathcal{N}\hat{g}$-open in $U$.

(iv) $\mathcal{N}\alpha g$-closed [4], if $\mathcal{N}\alpha cl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano open in $U$.

(v) $\mathcal{N}g\alpha$-closed [4], if $\mathcal{N}\alpha cl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano $\alpha$-open in $U$.

(vi) $\mathcal{N}gr$-closed [5], if $\mathcal{N}rcl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano open in $U$.

(vii) $\mathcal{N}rg$-closed [5], if $\mathcal{N}rcl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano regular open in $U$.

(viii) $\mathcal{N}gp$-closed [6], if $\mathcal{N}pcl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano open in $U$.

(ix) $\mathcal{N}gpr$-closed [15], if $\mathcal{N}pcl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano regular open in $U$.

(x) $\mathcal{N}g\omega\alpha$-closed [8], if $\mathcal{N}\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $\mathcal{N}\omega\alpha$-open set in $U$.

**Definition 2.7.**A function$f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$ is said to be

(i) Nano continuous [13], if the inverse image of every nanoopen set in$V$ is open set in $U$.

(ii) $\mathcal{N}g$-continuous [13], if the inverse image of every nano closed set in$V$ is $\mathcal{N}g$-closed set in $U$.

(iii) $\mathcal{N}\hat{g}$-continuous [10], ifthe inverse image of every nano closed set in $V$ is $\mathcal{N}\hat{g}$-closed set in $U$.

(iv) $\mathcal{N}\hat{g}\alpha$-continuous [16], if the inverse image of every nano closed set in $V$ is $\mathcal{N}\hat{g}\alpha$-closed set in $U$.

(v) $\mathcal{N}\alpha g$-continuous [4], if $f^{-1}(S)$ is $\mathcal{N}\alpha g$-closed set in $U$, for every Nano closed set $S$in $V$.

(vi) $\mathcal{N}g\alpha$-continuous [4], if $f^{-1}(S)$ is $\mathcal{N}g\alpha$-closed set in $U$, for every Nano closed set $S$in $V$.

(vii) $\mathcal{N}gr$-continuous [5], if the inverse image of every nano closed set in $V$ is $\mathcal{N}gr$-closed set in $U$.

(viii) $\mathcal{N}rg$-continuous [5], if the inverse image of every nano closed set in $V$ is $\mathcal{N}rg$-closed set in $U$.

(ix) $\mathcal{N}gp$-continuous [6], if the inverse image of every nano closed set in $V$ is $\mathcal{N}gp$-closed set in $U$.

(x) $\mathcal{N}gpr$-continuous [15], if the inverse image of every nano closed set in $V$ is $\mathcal{N}gpr$-closed set in $U$.

## 3. $\mathcal{N}g\omega\alpha$-continuous functions

In this section, we define and study the notions of $\mathcal{N}g\omega\alpha$-continuous functionsin nano topological spaces. Also we discuss some of its properties and study the relationship between other existing nano continuous functionsin nanotopological spaces.

**Definition 3.1.**A function$f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$ is said to be$\mathcal{N}g\omega\alpha$-continuous, if the inverse image of every nano closed set in $V$ is $\mathcal{N}g\omega\alpha$-closedset in $U$.

**Theorem 3.2.**A function $f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-continuous if and only if the inverse image of every nano closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed set in $U$.

**Proof.**Let $f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$ be $\mathcal{N}g\omega\alpha$-continuous function and $F$ be nano closed set in $V$. That is $V - F$ is nano open set in $V$. Since $f$ is$\mathcal{N}g\omega\alpha$-continuous,$f^{-1}(V - F)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. That is, $f^{-1}(V) - f^{-1}(F) = U - f^{-1}(F)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. Hence $f^{-1}(F)$ is $\mathcal{N}g\omega\alpha$-closed set in $U$.Thus the inverse of every nano closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed set in $U$.

Conversely, let the inverse image of every nano closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed set in $U$. Let $H$ be a nano closed set in $V$. Then $V - H$ is nano open set in $V$ and $f^{-1}(V - H)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. That is, $f^{-1}(V) - f^{-1}(F) = U - f^{-1}(H)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. Hence $f^{-1}(H)$ is $\mathcal{N}g\omega\alpha$-closed set in $U$. This imples that, $f$ is $\mathcal{N}g\omega\alpha$-continuous.

**Example 3.3.** Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$and $X = \{a, b\} \subseteq U$. Then$\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $\mathcal{N}g\omega\alpha$-closed set $= \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\},$ $\{ b, c, d\}\}$ .Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{w\}, \{y, z\}\}$ and $Y = \{x, z\} \subseteq V$ . Then $\tau_{R'}(Y) = \{V, \emptyset, \{x\}, \{y, z\}, \{x, y, z\}\}$. Define $f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$ as $f(a) = x$ , $f(b) = y$ , $f(c) = w$ and $f(d) = z$. Then $f^{-1}(\{w\}) = \{c\}$, $f^{-1}(\{x, w\}) = \{a, c\}$ and $f^{-1}(\{y, z, w\}) = \{b, c, d\}$. That is, the inverse image of every nano closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed setin $U$. Therefore $f$ is $\mathcal{N}g\omega\alpha$-continuous.

**Theorem 3.4.**Let $f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$ be a function. If $f$ is nano continuous function, then $f$ is $\mathcal{N}g\omega\alpha$-continuous function.

**Proof.** Let $F$ be any nano closed set in $V$. Since $f$ is nano continuous, $f^{-1}(F)$ is nano closed set in $U$. Since every nano closed set is $\mathcal{N}g\omega\alpha$-closed set. Hence $f\ is\ \mathcal{N}g\omega\alpha$-continuous function.

**Theorem 3.5.** A function $f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$, then every nano$\alpha$-continuous function is $\mathcal{N}g\omega\alpha$-continuous function.

**Proof.** Let $F$ be any nano closed set in $V$. Since $f$ is nano $\alpha$-continuous function, $f^{-1}(F)$ is nano $\alpha$-closed set in $U$. Since every nano $\alpha$-closed set is $\mathcal{N}g\omega\alpha$-closed set. This implies that, $f^{-1}(F)$ is$\mathcal{N}g\omega\alpha$-closed setin $U$. Hence $f\ is\ \mathcal{N}g\omega\alpha$-continuous function.

**Theorem 3.6.** A function $f: \left(U, \tau_R(X)\right) \rightarrow (V, \tau_{R'}(Y))$, then every nano regular-continuous function is $\mathcal{N}g\omega\alpha$-continuous function.

**Proof.** Let $F$ be any nano closed set in $V$. Since $f$ is nano regular-continuous function, $f^{-1}(F)$ is nano regular-closed set in $U$. Since every nano regular-closed set is $\mathcal{N}g\omega\alpha$-closed set. This implies that, $f^{-1}(F)$ is $\mathcal{N}g\omega\alpha$-closed set in $U$. Hence $f\ is\ \mathcal{N}g\omega\alpha$-continuous function.

**Remark 3.7.** The converse of the above theorem 3.4, 3.5 and 3.6 need not be true as seen from the following example.

**Example 3.8.** Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\} \subseteq U$ . Then $\tau_R(X) = \{U, \emptyset, \{a\}\}$ and Nano α-closed set$= \{U, \emptyset, \{b\}, \{c\}, \{b, c\}\}$, Nano regular closed set$= \{U, \emptyset\}$ and $\mathcal{N}g\omega\alpha$ -closed set $= \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . Let $V = \{x, y, z\}$ with $V/R = \{\{x\}, \{y, z\}\}$and $Y = \{y, z\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{y, z\}\}$. Define $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$as $f(a) = x$, $f(b) = y$ and$f(c) = z$. Then $f^{-1}(\{x\}) = \{a\}$ is $\mathcal{N}g\omega\alpha$-closed set in $U$ but not in nano closedset, nano α-closed set and nano regular closed set.Therefore $f$ is $\mathcal{N}g\omega\alpha$-continuous but not nano continuous, nano$\alpha$-continuous and nano regular continuous.

**Theorem 3.9.** Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a function. If $f$ is nano $g$-continuous(nano $g\alpha$-continuous, nano $g$r-continuous) function, then $f$ is $\mathcal{N}g\omega\alpha$-continuous function.

**Proof.** Let $F$ be any nano closed set in $V$. Since $f$ is nano $g$-continuous(nano $g\alpha$-continuous, nano $g$r-continuous) function, $f^{-1}(F)$ is nano $g$-closed(nano $g\alpha$-closed, nano $gr$-closed) set in $U$. Since every nano $g$-closed(nano $g\alpha$-closed, nano $gr$-closed) set is $\mathcal{N}g\omega\alpha$-closed set in $U$. Therefore $f^{-1}(F)$ is $\mathcal{N}g\omega\alpha$-closed set in $U$. Hence $f$ is $\mathcal{N}g\omega\alpha$-continuous function but not in nano $g$-continuous(nano $g\alpha$-continuous, nano $g$r-continuous) function.

**Example 3.10.**From Example 3.8, $f^{-1}(\{x\}) = \{a\}$ is $\mathcal{N}g\omega\alpha$-closed set in $U$ but not in nano $g$-closed(nano $g\alpha$-closed, nano $gr$-closed) set. Therefore $f$ is $\mathcal{N}g\omega\alpha$-continuous function but not in nano $g$-continuous(nano $g\alpha$-continuous, nano $g$r-continuous) function.

**Theorem 3.11.** A function $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$, then the following hold.

(i) Every $\mathcal{N}g\omega\alpha$-continuous function is nano $\alpha g$-continuous function.

(ii) Every $\mathcal{N}g\omega\alpha$-continuous function is nano $gp$-continuous function.

(iii) Every $\mathcal{N}g\omega\alpha$-continuous function is nano $rg$-continuous function.

(iv) Every$\mathcal{N}g\omega\alpha$-continuous function is nano $gpr$-continuous function.

**Proof.**(i) Let $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$ be $\mathcal{N}g\omega\alpha$-continuous function. Let $F$ be any nano closed set in $V$. Since $f$ is $\mathcal{N}g\omega\alpha$-continuous, $f^{-1}(F)$ is $\mathcal{N}g\omega\alpha$-closed set in $U$. Since every $\mathcal{N}g\omega\alpha$-closed set is nano $\alpha g$-closed set. Therefore $f^{-1}(F)$ is nano $\alpha g$-closed set in $U$. Hence $f$ is nano $\alpha g$-continuous function.

Proof of (ii) to (iv) is as follows from the Proof (i).

**Remark 3.12.** The converse of the above theorem need not be true as seen from the following examples.

**Example 3.13.** Let $U=\{a,b,c,d\}$ with $U/R=\left\{\{a\},\{b\},\{c,d\}\right\}$and $X=\{a\}\subseteq U$. Then$\tau_R(X)=\{U,\emptyset,\{a\}\}$. Herenano$\alpha g$-closed set = $\{U,\emptyset,\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},$ $\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ , nano $gp$ -closed set = $\{U,\emptyset,\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and $\mathcal{N}g\omega\alpha$-closed set = $\{U,\emptyset,\{b\},\{c\},\{d\},$ $\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}$. Let $V=\{x,y,z,w\}$ with $V/R=\left\{\{x\},\{z\},\{y,w\}\right\}$and $Y=\{x,z\}\subseteq V$. Then $\tau_{R'}(Y)=\{V,\emptyset,\{x,z\}\}$. Define $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$as $f(a)=w$, $f(b)=y$, $f(c)=z$ and $f(d)=x$. Then $f^{-1}(\{y,w\})=\{a,b\}$ is nano $\alpha g$-closed set and nano$gp$-closed set but not in $\mathcal{N}g\omega\alpha$-closed set. Therefore $f$ is nano $\alpha g$-continuous and nano $gp$-continuous but $f$ is not $\mathcal{N}g\omega\alpha$-continuous.

**Example 3.14.** Let $U=\{a,b,c,d\}$ with $U/R=\left\{\{a\},\{d\},\{b,c\}\right\}$and $X=\{a,c\}\subseteq U$. Then$\tau_R(X)=\{U,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. Herenano$rg$-closed set = $\{U,\emptyset,\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},$ $\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ ,nano $gpr$ -closed set = $\{U,\emptyset,\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$and$\mathcal{N}g\omega\alpha$-closed set = $\{U,\emptyset,\{d\},\{a,d\},\{b,d\},$ $\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let $V=\{x,y,z,w\}$ with $V/R=\left\{\{x\},\{y\},\{z,w\}\right\}$and $Y=\{x\}\subseteq V$. Then $\tau_{R'}(Y)=\{V,\emptyset,\{x\}\}$. Define $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$as $f(a)=w$, $f(b)=z$, $f(c)=y$ and $f(d)=x$. Then $f^{-1}(\{y,z,w\})=\{a,b,c\}$ isnano$rg$-closed set and nano$gpr$-closed setbut not in $\mathcal{N}g\omega\alpha$-closed set. Therefore $f$ isnano $rg$-continuous and nano $gpr$-continuous but $f$ is not$\mathcal{N}g\omega\alpha$-continuous.

**Theorem 3.15.**A function : $\left(U,\tau_R(X)\right) \to (V,\tau_{R'}(Y))$, then the following conditions are equivalent.

(i) $f$ is $\mathcal{N}g\omega\alpha$-continuous.

(ii) for every subset $A$ of $U$, $f(\mathcal{N}g\omega\alpha - cl(A)) \subseteq \mathcal{N}cl(f(A))$.

(iii) for every subset $B$ of $V$, $\mathcal{N}g\omega\alpha - cl(f^{-1}(B)) \subseteq f^{-1}(\mathcal{N}cl(B))$.

(iv) for every subset $B$ of $V$, $f^{-1}(\mathcal{N}int(B)) \subseteq \mathcal{N}g\omega\alpha - int(f^{-1}(B))$.

**Proof.**

**(i) ⇒ (ii).** Suppose (i) holds and let $A$ be any subset of $U$. Since $A \subseteq f^{-1}(f(A)) \subset f^{-1}(\mathcal{N}cl(f(A)))$. Now $\mathcal{N}cl(f(A))$ is closed in $V$ and $f$ is $\mathcal{N}g\omega\alpha$-continuous, implies $f^{-1}(\mathcal{N}cl(f(A)))$is $\mathcal{N}g\omega\alpha$-closed set in $U$containing$A$.

Consequently,$\mathcal{N}g\omega\alpha - cl(A) \subseteq f^{-1}(\mathcal{N}cl(f(A)))$.

Therefore $f\left(\mathcal{N}g\omega\alpha - cl(A)\right) \subseteq f(f^{-1}(\mathcal{N}cl\left(f(A)\right))) \subseteq \mathcal{N}cl(f(A))$.

Hence $f(\mathcal{N}g\omega\alpha - cl(A)) \subseteq \mathcal{N}cl(f(A))$.

**(ii) ⇒ (iii).**Suppose (ii) holds and let $B$ be any subset of$V$, then $f^{-1}(B)$ is subset in $U$.

By (ii), $f\left(\mathcal{N}g\omega\alpha - cl(f^{-1}(B))\right) \subseteq \mathcal{N}cl\left(f(f^{-1}(B))\right) \subseteq \mathcal{N}cl(B)$.

Therefore$\mathcal{N}g\omega\alpha - cl(f^{-1}(B)) \subseteq f^{-1}(\mathcal{N}cl(B))$.

**(iii) ⇒ (iv).**Suppose (iii) holds. Let $B \subseteq V$, then $V - B \subseteq V$.

By (iii), $\left(\mathcal{N}g\omega\alpha - cl(f^{-1}(V-B))\right) \subseteq f^{-1}(\mathcal{N}cl(V-B))$.

This implies that $U - \left(\mathcal{N}g\omega\alpha - int\left(f^{-1}(B)\right)\right) \subseteq U - (f^{-1}(\mathcal{N}int(B)))$.

Therefore, $f^{-1}(\mathcal{N}int(B)) \subseteq \mathcal{N}g\omega\alpha - int(f^{-1}(B))$.

**(iv) ⇒ (i).**Suppose (iv) holds. If $F$ is any closed set in $V$, then $V - F$ is an open set in $V$. Therefore $f^{-1}(V-F) = f^{-1}\left(\mathcal{N}int(V-F)\right) \subseteq \mathcal{N}g\omega\alpha - int\left(f^{-1}(V-F)\right) = U - (\mathcal{N}g\omega\alpha - cl\left(f^{-1}(F)\right))$.

This implies that, $\mathcal{N}g\omega\alpha - cl\left(f^{-1}(F)\right) \subseteq f^{-1}(F)$.

But $f^{-1}(F) \subseteq \mathcal{N}g\omega\alpha - cl\left(f^{-1}(F)\right)$ is always true.

Therefore, $f^{-1}(F) = \mathcal{N}g\omega\alpha - cl\left(f^{-1}(F)\right)$.

This implies, $f^{-1}(F)$ is $\mathcal{N}g\omega\alpha$-closed set. Therefore $f$ is $\mathcal{N}g\omega\alpha$-continuous function.

**Theorem 3.16.** Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$ -continuous and $g: (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is continuous, then $g \circ f: (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Proof.** Let $g$ be a nano continuous function and $B$ be any nano open set in $W$, then $g^{-1}(B)$ is nanoopen in $V$. Since $f$ is $\mathcal{N}g\omega\alpha$-continuous, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is $\mathcal{N}g\omega\alpha$-open in $U$. Hence $g \circ f$ is $\mathcal{N}g\omega\alpha$-continuous.

**Remark 3.17.**The composition of two $\mathcal{N}g\omega\alpha$-continuous functions need not be $\mathcal{N}g\omega\alpha$-continuous function and this is shown by the following example.

**Example 3.18.** Let$U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$and $X = \{a, c\} \subseteq U$. Then$\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ . Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{y\}, \{z, w\}\}$and $Y = \{x\} \subseteq V$. Then $\tau_R(Y) = \{U, \emptyset, \{x\}\}$ and $W = \{p, q, r, s\}$ with $W/R = \{\{p\}, \{r\}, \{q, s\}\}$ and $Z = \{p, r\} \subseteq W$. Then$\tau_R(Z) = \{U, \emptyset, \{p, r\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = z$, $f(d) = w$ and $g: (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ as $g(x) = r$, $g(y) = s$, $g(z) = q$, $g(w) = p$. Here $f$ and $g$ are $\mathcal{N}g\omega\alpha$-continuous but $g \circ f$ is not $\mathcal{N}g\omega\alpha$-continuous, because $F = \{q, s\}$ is nano closed set in $W$ but $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F)) = f^{-1}(g^{-1}(\{q, s\})) = f^{-1}(\{y, z\}) = \{b, c\}$ which is not $\mathcal{N}g\omega\alpha$-closed in $U$. Hence the composition of two $\mathcal{N}g\omega\alpha$-continuous functions need not be $\mathcal{N}g\omega\alpha$-continuous.

## 4. $\mathcal{N}g\omega\alpha$-irresolute function

In this section, we define and study the concepts of $\mathcal{N}g\omega\alpha$-irresolute functionwhich is included in the class of $\mathcal{N}g\omega\alpha$-continuous functionin nano topological spaces. Further, some of its basic properties and condition are investigated.

**Definition 4.1.** A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be $\mathcal{N}g\omega\alpha$-irresolute function, if the inverse image of every $\mathcal{N}g\omega\alpha$-closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed set in $U$.

**Theorem 4.2.** A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute if and only if the inverse image of every $\mathcal{N}g\omega\alpha$-closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed set in $U$.

**Proof.** Assume that$f$ is$\mathcal{N}g\omega\alpha$-irresolute and and $B$ be any $\mathcal{N}g\omega\alpha$-closed set in $V$, then $f^{-1}(B^c)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. Since $f^{-1}(B^c) = U - f^{-1}(B)$, $f^{-1}(B)$is $\mathcal{N}g\omega\alpha$-closed set in $U$. Hence, the inverse image of every $\mathcal{N}g\omega\alpha$-closed set in $V$ is $\mathcal{N}g\omega\alpha$-closed set in $U$.

Conversely, let $G$ be any $\mathcal{N}g\omega\alpha$-closed set in $V$, then $f^{-1}(G^c)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. Since $f^{-1}(G^c) = U - f^{-1}(G)$, and so, $f^{-1}(G)$is $\mathcal{N}g\omega\alpha$-closed set in $U$. Therefore, $f$ is $\mathcal{N}g\omega\alpha$-irresolute.

**Theorem 4.2.** If a function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute, then it is $\mathcal{N}g\omega\alpha$-continuous.

**Proof.** Let$f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute function. Let $B$be anynano open set in $V$. Since every nano open set is $\mathcal{N}g\omega\alpha$-open. This implies that,$B$ is $\mathcal{N}g\omega\alpha$-open set in $V$. Since $f$ is $\mathcal{N}g\omega\alpha$-irresolute function, $f^{-1}(B)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. Therefore ,$f$ is $\mathcal{N}g\omega\alpha$-continuous.

**Remark 4.3.**The converse of the above theorem need not be true as seen from the following example.

**Example 4.4.** Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$and $X = \{a, c\} \subseteq U$. Then$\tau_R(X) = \{U, \emptyset, \{a, c\}\}$ and $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{y\}, \{z, w\}\}$ and $Y = \{x\} \subseteq V$. Then $\tau_R(Y) = \{U, \emptyset, \{x\}\}$ Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$as $f(a) = x$, $f(b) = y$, $f(c) = w$, $f(d) = z$. Here $f\mathcal{N}g\omega\alpha$-continuous but not $\mathcal{N}g\omega\alpha$-irresolute, because $F = \{z, w\}$ is nano closed set in $V$ but $f^{-1}(\{z, w\}) = \{c, d\}$ which is not $\mathcal{N}g\omega\alpha$-closed in $U$.

**Theroem 4.5.**Let $U$, $V$ and $W$ be a nano topological spaces. If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute and $g: (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous, then the composition $g \circ f: (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$is$\mathcal{N}g\omega\alpha$-continuous.

**Proof.** Let $B$be a nano open set in $W$. Since $g$ is $\mathcal{N}g\omega\alpha$-continuous, $g^{-1}(B)$ is $\mathcal{N}g\omega\alpha$-open set in $V$.

Since $f$ is $\mathcal{N}g\omega\alpha$-irresolute, $f^{-1}(g^{-1}(B))$ is $\mathcal{N}g\omega\alpha$-open set in $U$. But $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$. Therefore $g \circ f: (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Theroem 4.6.**Let $U$, $V$ and $W$ be a nano topological spaces. If $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$ and $g:\left(V,\tau_{R'}(Y)\right)\to(W,\tau_{R''}(Z))$ are $\mathcal{N}g\omega\alpha$-irresolute, then the composition $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-irresolute.

**Proof.** Let $H$be nano$\mathcal{N}g\omega\alpha$-open open set in $W$. Since $g$ is $\mathcal{N}g\omega\alpha$-irresolute, $g^{-1}(H)$ is $\mathcal{N}g\omega\alpha$-open set in $V$. Since $f$ is $\mathcal{N}g\omega\alpha$-irresolute, $f^{-1}\left(g^{-1}(H)\right)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. But $f^{-1}\left(g^{-1}(H)\right)=(g\circ f)^{-1}(H)$. Therefore $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-irresolute.

**Theroem 4.7.**Let $U$, $V$ and $W$ be a nano topological spaces. If $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute and $g:\left(V,\tau_{R'}(Y)\right)\to(W,\tau_{R''}(Z))$ is nano continuous, then the composition $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Proof.** Let $G$be a nano open set in $W$. Since $g$ is nano continuous, $g^{-1}(G)$ is nano open set in $V$. Since every nano open set is $\mathcal{N}g\omega\alpha$-open set, this implies that,$g^{-1}(G)$ is $\mathcal{N}g\omega\alpha$-open set in $V$.

Since $f$ is $\mathcal{N}g\omega\alpha$-irresolute, $f^{-1}\left(g^{-1}(G)\right)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. But $f^{-1}\left(g^{-1}(G)\right)=(g\circ f)^{-1}(G)$. Therefore $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Theroem 4.8.**Let $U$, $V$ and $W$ be a nano topological spaces. If $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute and $g:\left(V,\tau_{R'}(Y)\right)\to(W,\tau_{R''}(Z))$ is nano $\alpha$-continuous, then the composition $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Proof.** Let $A$be a nano open set in $W$. Since $g$ is nano $\alpha$-continuous, $g^{-1}(A)$ is nano $\alpha$-open set in $V$. Since every nano $\alpha$-open set is $\mathcal{N}g\omega\alpha$-open set, this implies that, $g^{-1}(A)$ is $\mathcal{N}g\omega\alpha$-open set in $V$.

Since $f$ is $\mathcal{N}g\omega\alpha$-irresolute, $f^{-1}\left(g^{-1}(A)\right)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. But $f^{-1}\left(g^{-1}(A)\right)=(g\circ f)^{-1}(A)$. Therefore $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Theroem 4.9.**Let $U$, $V$ and $W$ be a nano topological spaces. If $f:\left(U,\tau_R(X)\right)\to(V,\tau_{R'}(Y))$ is $\mathcal{N}g\omega\alpha$-irresolute and $g:\left(V,\tau_{R'}(Y)\right)\to(W,\tau_{R''}(Z))$ is nano $g$-continuous, then the composition $g\circ f:\left(U,\tau_R(X)\right)\to(W,\tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

**Proof.** Let $O$be a nano open set in $W$. Since $g$ is nano $g$-continuous, $g^{-1}(O)$ is nano $g$-open set in $V$. Since every nano $g$-open set is $\mathcal{N}g\omega\alpha$-open set, this implies that, $g^{-1}(O)$ is $\mathcal{N}g\omega\alpha$-open set in $V$.

Since $f$ is $\mathcal{N}g\omega\alpha$-irresolute, $f^{-1}\big(g^{-1}(O)\big)$ is $\mathcal{N}g\omega\alpha$-open set in $U$. But $f^{-1}\big(g^{-1}(O)\big) = (g \circ f)^{-1}(O)$. Therefore $g \circ f: \big(U, \tau_R(X)\big) \to (W, \tau_{R''}(Z))$ is $\mathcal{N}g\omega\alpha$-continuous.

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