Study Of M/M/1 Feedback Retrial queueing Network with Three Nodes When Catastrophe Occurs

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Abstract

In this article, we concentrate on M/M/1 feedback retrial queueing network with three nodes when catastrophes occur. We take three nodes, with feedback on third node with probability w and 1-w, derive the probability of the framework is idle (no customer), the likelihood of n number of the customers in the system and queue length, system length, queue time, system time using Little's formula. The mathematical models are given to test the rightness of this model.

Keywords: Queueing Network,Feedback,Retrial,Catastrophes

I .Introduction:

The ongoing interest for a specific help surpasses the ability to offer the support is known as queue. Queue is framed on the off chance that the help expected by the client isn't quickly accessible. Waiting for service is a part of our daily life, for instance stand by at lodgings, line up at the rail route reservation counter and line up for administration in banks. The waiting phenomenon isn't just an encounter restricted to people alone but also the jobs wait to be processed on a machine, airplanes circle in air before given permission to land at an airport, vehicles stand by at the traffic signals and so forth…

In general, we can't abstain from holding up without causing unreasonable costs or postponement. As a matter of fact, we can expect to diminish the unfavorable effect of holding up as far as possible. The investigation of queuesdetermines the measures of performance of a queue situation, including the average waiting time and the average queue length.

Queueing theory was first presented by AgnerKrarupErlang [2], a Danish Mathematician and engineer, which served as groundwork of queueing theory.

When the operating characteristics of a queueing system depend on time, the framework is supposed to be in transient state. While the working qualities of a queueing framework are free of time, the framework is supposed to be in consistent state.

A queuing network is an assortment of nodes, with every nodes addressing a service facility. James. R. Jackson [5] presented the utilization of queueing networks in 1957. Jackson's organization [6] has made the main commitment to the queueingnetwork. Queueing network models have many applications, including service centres, computer networks, correspondence organizations, creation and adaptable assembling frameworks, air terminal terminals, and medical care frameworks, among others. Queueing networks are distributed into three types: open, closed, and mixed networks. External arrivals are received by the open network and routed to an external departure. Closed networks have a fixed number of customers who move between queues but never leave the system. Mixed networks are open to some types of customers but closed to others.

Retrial queueing models depend on the way that, when the server is in the busy, showing up clients don't arrange or move the framework immediately, however rather go to a virtual area known as orbit and take a luck again after some random time. Retrial queues are broadly and effectively utilized as numerical models of several computer systems and telecommunication networks. The Retrial queue is first introduced by Kosten [7] in 1947.Falin G.I [3]has analyzed a survey of the main results and methods of the theory of retrial queues.

Feedback queue plays a vigorous role in everyday service systems wherever tasks may demand repeated services. Queueing structures that comprise the ability for a customer to return to the service due to dissatisfaction with the service or the need for additional service are referred to as queues with feedback. The phenomenon of feedback can be seen in many practical situations, such as reworking in the production system, communication networks, and supermarkets, among others. Takacs [12] has introduced queues with feedback mechanism in 1963.

Catastrophes are unexpected calamities that occur in a service facility. Gelenbe [4] introduced the concept of catastrophes, which has gained significant scholarly attention in recent decades due to its wide application in service structures, processor systems, and industrial organizations. The thought of catastrophe has been important in the fields of science and technology. It happens at random, resulting in the loss of all units and the activation of the service capabilitytill a new arrival is not rare in most real-life situations. When a disaster occurs in the structure, all available customers are immediately shattered, and the server is deactivated. When a new arrival occurs, the server is ready to serve. Furthermore, Chao [1] have been motivated to evaluate migration process with catastrophes and computer network with virus infections.

B.KrishnaKumar, D.Arivudainambi [8] have discussed the transient explanation of an M/M/1queue with catastrophes.S.Shanmugasundaram and S. Chitra [10] have focused on the time reliant solution of M/M/1retrial queue and feedback on non retrial customers with catastrophes.Thangarajet.al[13] has studied the M/M/1 queue with catastrophes using continued fractions. S.Shanmugasundaram , S.Vanitha [11] has focused on Steady state Analysis of M/M/1 Retrial Queueing Network with Catastrophes.

II. Depiction of the Model:

We study a three-node open queueing network with retrial, feedback and catastrophe. External customers arriveat the system using a Poisson process with arrival rate λ .If the server is idle, customers who arrive from outside will most likely link the queue with probability r. Customers join the retrial queue with probability 1-r , if the server is busy. It is supposed that only the customer at the front of the queue is permitted to receive facility on a first-come, first-served basis, and that the capacity is infinite. If the server is busy during the retrial, the customer rejoins the orbit. This procedure is repeated until the customer observes that the server in node 1 is idle and receives the request and necessary service during the retrial. If the server is full, customers are enforced to wait in an infinitely large orbit and repeat their request after an exponential time. After receiving service in node 1, they can either proceed to node 2 with probability u or proceed to node 3 with probability 1-u. Customers can leave the system with probability v after receiving service at node 2, or they can proceed to node 3 with probability 1-v . The customer exits the system after completing the service at node 3 with probability w or enters node 2 as a feedback with probability 1-w. Each node operates on an M/M/1 basis. Node 1, node 2, and node 3 are exponentially distributed with service rates μ_1, μ_2 and μ_3 .Catastrophes occur as a result of the arrival and service processes, which follow the Poisson process with rate γ. When a catastrophic event occurs in the system, all available customers are immediately destroyed, and the server is deactivated. When a new arrival occurs, the server is ready to serve. The system is shown in Fig.1.

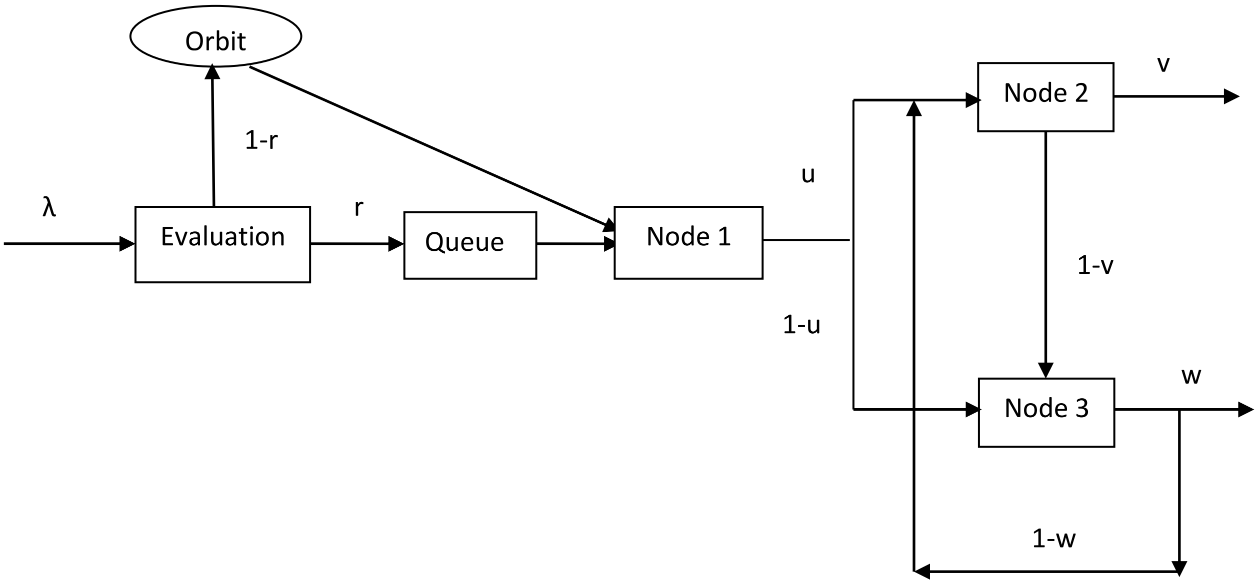


Fig.1 Classical Retrial Feedback Queueing System

Let P(x,t) = $\sum_{n=0}^{\infty} P_n(t)x^n$, where $P_n(t)$denote the transient state probability with n customers in the system at time t.

If time t = 0, then$P_0(0) = 1$.

From above, $P_n(t)$constitutes the system of differential – difference equations:

$$
\begin{aligned}
P_0{}'(t) = {} & -\lambda P_0(t) \\
& + \big[[(r+u)\mu_1 + (r+v)\mu_2] + [(r+u)\mu_1 + (r+1-v)\mu_2 + (r+w)\mu_3] \\
& + [(r+u)\mu_1 + (r+1-v)\mu_2 + (r+1-w)\mu_3] \\
& + [(r+1-u)\mu_1 + (r+w)\mu_3] + [(r+1-u)\mu_1 + (r+1-w)\mu_3] \\
& + [(1-r+u)\mu_1 + (1-r+v)\mu_2] \\
& + [(1-r+u)\mu_1 + (1-r+1-v)\mu_2 + (1-r+w)\mu_3] \\
& + [(1-r+u)\mu_1 + (1-r+1-v)\mu_2 + (1-r+1-w)\mu_3] \\
& + [(1-r+1-u)\mu_1 + (1-r+w)\mu_3] \\
& + [(1-r+1-u)\mu_1 + (1-r+1-w)\mu_3]\big] P_1(t) + \gamma[1 - P_0(t)]
\end{aligned}
$$

$$= -\lambda P_0(t) + [(4u+9)\mu_1 + (7-2v)\mu_2 + 8\mu_3]P_1(t) + \gamma[1 - P_0(t)]$$
(1)

Then for n = 1,2,3 . . . .

$\mathrm{P_n}'(\mathrm{t}) = \lambda \mathrm{P_{n-1}}(\mathrm{t}) - \big[\lambda + \gamma + [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\big]\mathrm{P_n}(\mathrm{t}) + [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\mathrm{P_{n+1}}(\mathrm{t})$ (2)

In Steady state , $\lim_{n\to\infty} P_n(t) = P_n$ and $P_n{}'(t) = 0$ $as t \to \infty$

Steady state equations corresponding to equations (1) and (2) are as follows

$$0 = -\lambda P_0 + [(4u+9)\mu_1 + (7-2v)\mu_2 + 8\mu_3]P_1 + \gamma[1 - P_0]$$

$$(\lambda + \gamma)P_0 = \gamma + [(4u+9)\mu_1 + (7-2v)\mu_2 + 8\mu_3]P_1$$
(3)

$$
\begin{aligned}
0 = {} & \lambda \mathrm{P_{n-1}} - \big[\lambda + \gamma + [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\big]\mathrm{P_n} \\
& + [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\mathrm{P_{n+1}}
\end{aligned}
$$

$\big[\lambda + \gamma + [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\big]\mathrm{P_n} = \lambda \mathrm{P_{n-1}} + [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\mathrm{P_{n+1}}$ (4)

Equation (3) $\Longrightarrow$

$$\gamma = (\lambda + \gamma)\mathrm{P_0} - [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\mathrm{P_1}$$

$$= \left[(\lambda + \gamma) - [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\frac{\mathrm{P_1}}{\mathrm{P_0}}\right]\mathrm{P_0}$$

$$\mathrm{P_0} = \frac{\gamma}{(\lambda+\gamma) - [(4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3]\frac{\mathrm{P_1}}{\mathrm{P_0}}}$$
(5)

$$\frac{\mathrm{P_n}}{\mathrm{P_{n-1}}} = \frac{\lambda}{(\lambda+\gamma+(4\mathrm{u}+9)\mu_1+(7-2\mathrm{v})\mu_2+8\mu_3) - [(4\mathrm{u}+9)\mu_1+(7-2\mathrm{v})\mu_2+8\mu_3]\frac{\mathrm{P_{n+1}}}{\mathrm{P_n}}}$$
(6)

Using the above two equations and continued fraction.

$$\mathrm{P}_0 = \frac{\gamma}{(\lambda+\gamma) - \frac{\delta\lambda}{(\lambda+\delta+\gamma)-\delta\frac{\mathrm{P}_2}{\mathrm{P}_1}}}$$

Where $\delta = (4\mathrm{u}+9)\mu_1 + (7-2\mathrm{v})\mu_2 + 8\mu_3$

$$\mathrm{P}_0 = \frac{\gamma}{(\lambda+\gamma) - \frac{\delta\lambda}{(\lambda+\delta+\gamma)-\frac{\lambda\delta}{\lambda+\delta+\gamma}- \dots}}$$

$$\mathrm{P}_0 = \frac{\gamma}{(\lambda+\gamma)-\alpha}$$

(7)

Where $\alpha = \frac{\delta\lambda}{(\lambda+\delta+\gamma)-\frac{\lambda\delta}{\lambda+\delta+\gamma}- \dots}$

Equation (7) satisfies the quadratic equation

$$\alpha^2 - (\lambda+\delta+\gamma)\alpha + \lambda\delta = 0$$

The roots of the above equation are $\frac{\beta\pm\sqrt{\beta^2-4\lambda\delta}}{2}$, where $\beta = \lambda+\delta+\gamma$

Let the roots be α_1 and α_2 , we take the unique real root which lies within [0 , 1).

Substituting α_2 in equation (7), we get

$$\mathrm{P}_0 = \frac{\gamma}{(\lambda+\gamma)-\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2}}$$

(8)

$$\mathrm{P}_0 = \frac{\gamma}{(\beta-\delta)-\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2}}$$

Multiply and divide by $\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2\lambda\delta}$

After some algebraic calculation, we get

$$\mathrm{P}_0 = \frac{\gamma\left(\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2\lambda\delta}\right)}{1-\delta\left(\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2\lambda\delta}\right)}$$

(9)

After expanding binomially,

$$P_0 = \sum_{n=0}^{\infty} \delta^n \left(\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2\lambda\delta}\right)^{n+1} + \gamma \sum_{n=0}^{\infty} \delta^n \left(\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2\lambda\delta}\right)^{n+1} \quad (10)$$

From equation (6) we get

$$\frac{P_n}{P_{n-1}} = \frac{\gamma}{(\lambda+\gamma+\delta) - \frac{\lambda\delta}{(\lambda+\delta+\gamma)-\frac{\lambda\delta}{\lambda+\delta+\gamma}- \ldots}}$$

By similar argument as earlier, the above reduces to

$$\frac{P_n}{P_{n-1}} = \frac{\lambda}{\frac{\beta+\sqrt{\beta^2-4\lambda\delta}}{2}}, \quad n = 1, 2 \ldots\ldots \quad (11)$$

Multiply and divide by $\beta-\sqrt{\beta^2-4\lambda\delta}$

After some calculation we get

$$P_n = \left(\frac{\beta-\sqrt{\beta^2-4\lambda\delta}}{2\delta}\right)^n P_0, \quad n = 1, 2 \ldots\ldots\ldots \quad (12)$$

Where $\beta = \lambda + \gamma + \delta$

We get the steady state probabilities P_0 and P_n for n = 1 ,2 . . . . from the equations (10) and (11)

Theorem 1:

If $\gamma > 0$, at steady state the asymptotic behavior of the average queue length L_q is

$L_q = \frac{\lambda-\delta}{\gamma} + \frac{2\delta}{2(\lambda+\gamma)-\left[(\lambda+\delta+\gamma)-\sqrt{(\lambda+\delta+\gamma)^2-4\lambda\delta}\right]}$, Where$\delta = (4u+9)\mu_1 + (7-2v)\mu_2 + 8\mu_3$

Proof :

Consider the equations (1) an (2) with initial condition $P_0(0) = 1$

$$\frac{\partial P(x,t)}{\partial t} = \left[\lambda x + \frac{\delta}{x} - (\lambda+\gamma+\delta)\right] P(x,t) + \delta\left(1-\frac{1}{x}\right) P_0(x) + \delta \quad (13)$$

Mean is $m(t) = \sum_{n=1}^{\infty} nP_n(t) = \frac{\partial P(x,t)}{\partial t}$ at x=1

Differentiating the equation (13) with respect to x at x=1 , we get

$$\frac{dm(t)}{dx} + \gamma m(t) = \lambda - (1 - P_0)\delta$$

Solving form(t) with $m(0) = \sum_{n=1}^{\infty} nP_n(t) = 0$, we get

$$m(t) = \frac{\lambda}{\gamma}(1 - e^{-\gamma t}) - \frac{\delta}{\gamma}(1 - e^{-\gamma t}) + \delta \int_0^t P_0(u)\, e^{-\gamma(t-u)} du \qquad (14)$$

Taking Laplace Transform for Equation (1) and (2) ,we get

$$P_0^*(x) = \frac{1 + \frac{\gamma}{x}}{(x + \lambda + \gamma) - \left[\frac{\beta - \sqrt{\beta^2 - 4\lambda\delta}}{2}\right]}$$

(15)

Taking Laplace Transforms for equation (14), if$m^*(x)$ is the laplace transform of $m(t)$

$$m^*(x) = \frac{\lambda - \delta}{x(x + \gamma)} + \frac{\delta}{(x + \gamma)} P_0^*(x)$$

(16)

$$\lim_{t \to \infty} \mathrm{m(t)} = \lim_{x \to 0} \mathrm{x}\, m^*(x)$$

Using equation (15) and the above concept for the equation (16) , we get

$$\mathrm{L_q} = \mathrm{m(t)} = \frac{\lambda - \delta}{\gamma} + \frac{2\delta}{2(\lambda + \gamma) - \left[(\lambda + \delta + \gamma) - \sqrt{(\lambda + \delta + \gamma)^2 - 4\lambda\delta}\right]}$$

III .Balance Equations:

By using Little's formula, we get the following equations

$\mathbf{L_q}$of the system

$$\mathrm{L_q} = \frac{\lambda - \delta}{\gamma} + \frac{2\delta}{2(\lambda + \gamma) - \left[(\lambda + \delta + \gamma) - \sqrt{(\lambda + \delta + \gamma)^2 - 4\lambda\delta}\right]}$$

Where $\delta = (4\mathrm{u} + 9)\mu_1 + (7 - 2\mathrm{v})\mu_2 + 8\mu_3$

$\mathbf{W_q}$of a customer in all three queues

$$\mathrm{W_q} = \frac{\mathrm{L_q}}{\lambda} = \left(\frac{\lambda - \delta}{\gamma} + \frac{2\delta}{2(\lambda + \gamma) - \left[(\lambda + \delta + \gamma) - \sqrt{(\lambda + \delta + \gamma)^2 - 4\lambda\delta}\right]}\right)\frac{1}{\lambda}$$

$\mathbf{L_s}$ofa customer

$$L_s = L_q + \frac{\lambda}{\mu} = \left(\frac{\lambda-\delta}{\gamma} + \frac{2\delta}{2(\lambda+\gamma) - \left[(\lambda+\delta+\gamma) - \sqrt{(\lambda+\delta+\gamma)^2 - 4\lambda\delta}\right]}\right) + \frac{\lambda}{\mu}$$

W_s of a customer

$$W_s = \frac{L_s}{\lambda} = \left[\frac{\lambda-\delta}{\gamma} + \frac{2\delta}{2(\lambda+\gamma) - \left[(\lambda+\delta+\gamma) - \sqrt{(\lambda+\delta+\gamma)^2 - 4\lambda\delta}\right]} + \frac{\lambda}{\mu}\right]\frac{1}{\lambda}$$

IV. PARTICULAR CASE :

When $\mu_1 = \mu$ and $\mu_2 = \mu_3 = 0$,if u = 0, then there will be only one node, when r = 0 and w = 0 there is no retrial and feedback , so we get $\delta = \mu$

Asymptotic behavior of L_q, when $\gamma > 0$ is

$$L_q = \frac{\lambda-\delta}{\gamma} + \frac{2\delta}{2(\lambda+\gamma) - \left[(\lambda+\delta+\gamma) - \sqrt{(\lambda+\delta+\gamma)^2 - 4\lambda\delta}\right]} \tag{17}$$

Case 1:

The equation (17) coincides with Shanmugasundaram and Vanitha [11] , when $\gamma > 0$ and w = 0 , i.e there is no feedback ,asymptotic behavior of L_q, when $\gamma > 0$ is

$$L_q = \frac{\lambda-\mu}{\gamma} + \frac{2\mu}{2(\lambda+\gamma) - \left[(\lambda+\mu+\gamma) - \sqrt{(\lambda+\mu+\gamma)^2 - 4\lambda\mu}\right]}$$

Case 2 :

The equation (17) coincides with Thangaraj and Vanitha [13] by taking q = 1, asymptotic behavior of $m(t)$ when $\gamma > 0$ is

$$m(t) = \frac{\lambda-\mu q}{\gamma} + \frac{2\mu q}{2(\lambda+\gamma) - \left[(\lambda+\mu q+\gamma) - \sqrt{(\lambda+\mu q+\gamma)^2 - 4\lambda\mu q}\right]} \qquad \text{as } t \to 0$$

V. Numerical Examples:

Number of customers in all the three queues for the system

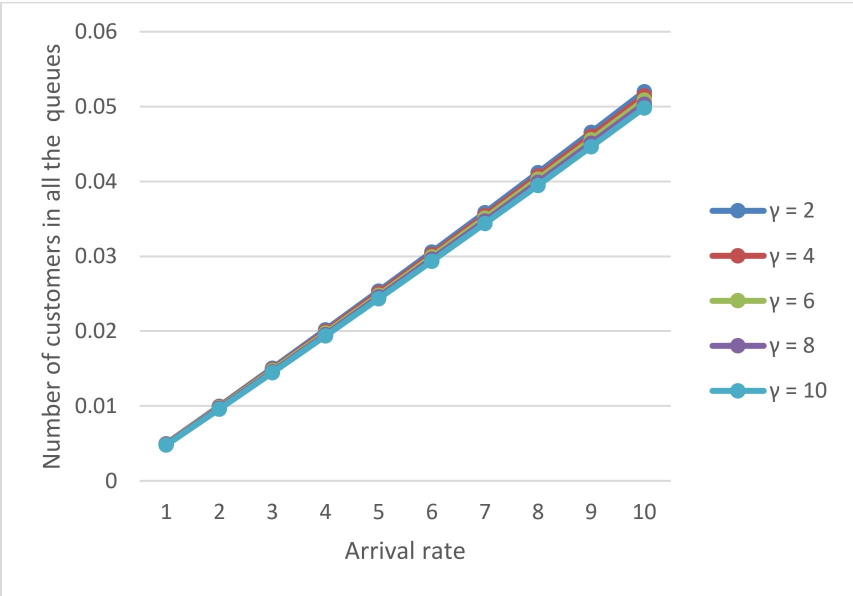
For u=0.4, v=0.6, w=0.8, $\mu_1 = 6$, $\mu_2 = 7$,$\mu_3 = 12$, $\lambda = 1,2,3,4,5,6,7,8,9,10$, $\gamma = 2,4,6,8,10$ (catastrophe effect), the customers in each of the queues for the system is measured in table 1.

In Fig. 2, as λ rises, the customer in the three queues grows and as γ value increases, the customer in each of the queues declines.

Table : 1

	2	4	6	8	10
1	0.00497	0.00492	0.00487	0.00483	0.00478
2	0.009989	0.00989	0.00979	0.0097	0.0096
3	0.015058	0.01491	0.01476	0.01461	0.01447
4	0.020178	0.01997	0.01977	0.01957	0.01938
5	0.025348	0.02509	0.02483	0.02458	0.02434
6	0.030572	0.03025	0.02994	0.02964	0.02934
7	0.035847	0.03547	0.0351	0.03474	0.03439
8	0.041177	0.04074	0.04031	0.0399	0.03949
9	0.046561	0.04606	0.04558	0.0451	0.04463
10	0.052001	0.05144	0.05089	0.05035	0.04983

Fig : 2 Number of customers in all the Queues



W_q of a customer in all the three queues

For u=0.4, v=0.6, w=0.8, $\mu_1 = 6$, $\mu_2 = 7$,$\mu_3 = 12$, $\lambda = 1,2,3,4,5,6,7,8,9,10$, $\gamma = 2,4,6,8,10$ (catastrophe effect), the waiting time of a customer in each of the queues for the system is measured in table 2.

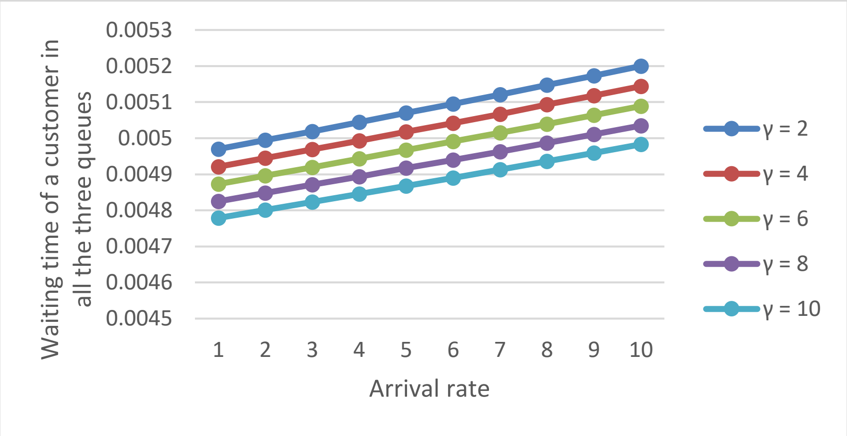
In Fig 3, as λ rises , W_q in each of the queues increases and as γ value rises, W_q in each of three queues declines.

Table :2

	2	4	6	8	10
1	0.00497	0.004921	0.004873	0.004825	0.004779
2	0.004995	0.004945	0.004896	0.004848	0.004801

3	0.005019	0.004969	0.004919	0.004871	0.004823
4	0.005044	0.004993	0.004943	0.004893	0.004845
5	0.00507	0.005018	0.004967	0.004917	0.004868
6	0.005095	0.005042	0.004991	0.00494	0.00489
7	0.005121	0.005067	0.005015	0.004963	0.004913
8	0.005147	0.005093	0.005039	0.004987	0.004936
9	0.005173	0.005118	0.005064	0.005011	0.004959
10	0.0052	0.005144	0.005089	0.005035	0.004983

Fig : 3 W_q of a customer in all the three queues



Number of customers inthesystem

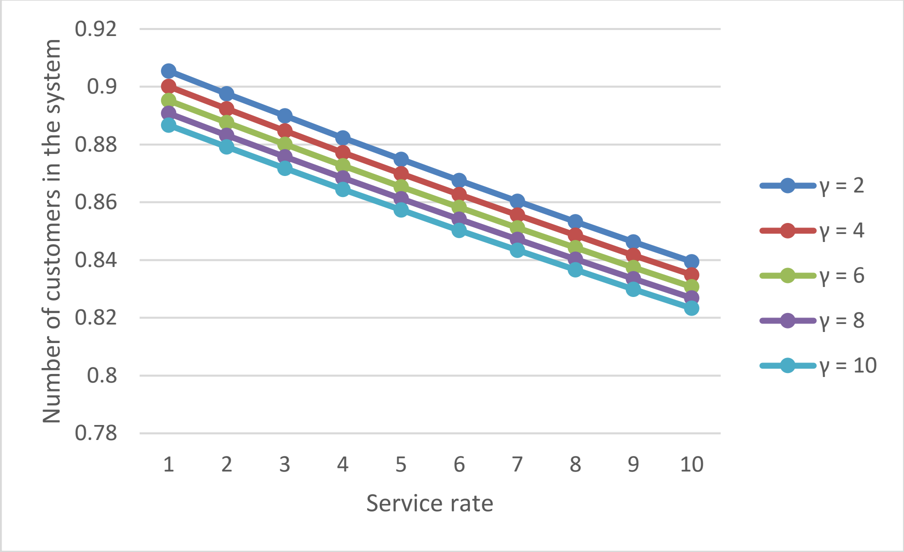
For u=0.4, v=0.6, w=0.8, $\mu_1 = 4, 4.2, 4.4, 4.6, 4.8, 5, 5.2, 5.4, 5.6, 5.8$, $\mu_2 = 7$,$\mu_3 = 12$, $\lambda = 6$, $\gamma = 2,4,6,8,10$ (catastrophe effect), the number of customers in the system is calculated in table 3.

In Fig 4, as the service rate rises and for various values of γ, the number of customers in the system falls.

Table: 3

	2	4	6	8	10
4	0.905445	0.900095	0.89523	0.890784	0.886701
4.2	0.897607	0.892353	0.887573	0.883199	0.87918
4.4	0.889903	0.884744	0.880045	0.875742	0.871785
4.6	0.882331	0.877263	0.872643	0.86841	0.864514
4.8	0.874886	0.869908	0.865365	0.861199	0.857363
5	0.867566	0.862675	0.858207	0.854107	0.850329
5.2	0.860367	0.85556	0.851167	0.847131	0.84341
5.4	0.853287	0.848563	0.84424	0.840268	0.836602
5.6	0.846322	0.841678	0.837426	0.833515	0.829904
5.8	0.83947	0.834905	0.830721	0.82687	0.823312

Fig :4 Number of the customers in the system



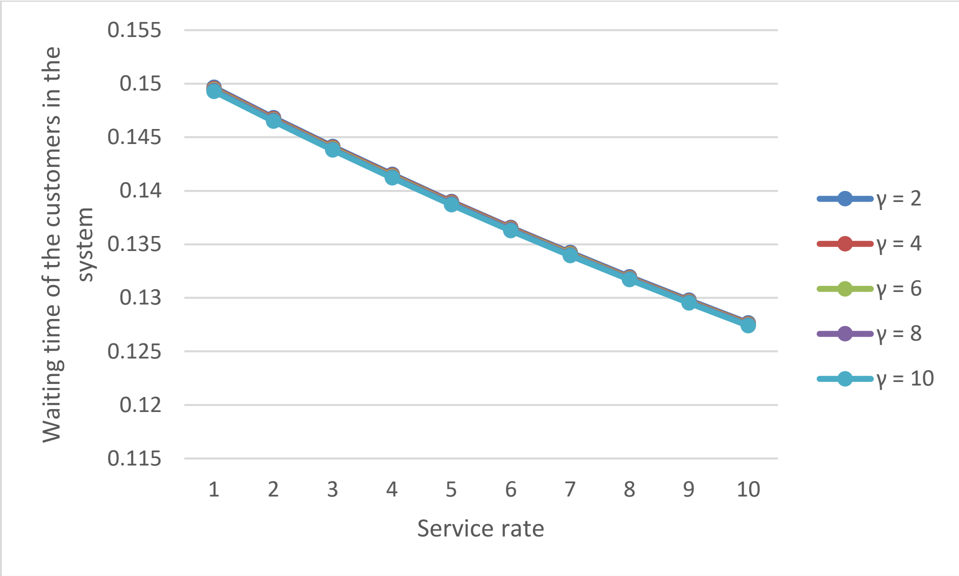
W_s of a customer

For u=0.4, v=0.6, w=0.8, $\mu_1 = 4, 4.2, 4.4, 4.6, 4.8, 5, 5.2, 5.4, 5.6, 5.8$, $\mu_2 = 7, \mu_3 = 12$, $\lambda = 6$, $\gamma = 2,4,6,8,10$ (catastrophe effect), $W_s$is measured in table 4. In Fig 5, as the service rate rises and for various values of γ, $W_s$declines.

Table : 4

	2	4	6	8	10
4	0.149656	0.149561	0.149469	0.14938	0.149293
4.2	0.146845	0.146754	0.146666	0.14658	0.146496
4.4	0.144137	0.14405	0.143965	0.143883	0.143802
4.6	0.141527	0.141444	0.141362	0.141283	0.141206
4.8	0.139011	0.13893	0.138852	0.138776	0.138702
5	0.136582	0.136505	0.136429	0.136356	0.136284
5.2	0.134237	0.134162	0.13409	0.134019	0.13395
5.4	0.131971	0.131899	0.131829	0.131761	0.131694
5.6	0.12978	0.129711	0.129643	0.129578	0.129513
5.8	0.127661	0.127594	0.127529	0.127466	0.127404

Fig : 5 W_s of a customer



VI. Conclusion :

Here we have determined the probability of the system is idle (no client), the likelihood of n number of the clients in the framework and queue length, system length, queue time, system time using Little's formula. The mathematical models shows as the appearance rate builds, the quantity of clients and the waiting time for every one of the three queues increments and as γ value expands, the quantity of clients and holding up season of a client in every one of the three queues diminishes and as the service rate increments and for different values of γ, the quantity of clients and the holding up season of a client in the system diminishes. The specific cases are additionally determined. This shows the achievability of the model.

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