Generalized Nonagonal Fuzzy Number and its Application in Assignment Problem

M. Suba[1*], Shanmugapriya R[2], M.L. Suresh[3] and Karthik S[4]

[1,2,3,4]Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, Tamil Nadu, India

Emil: [1]suba.hari87@gmail.com, [2]spriyasathish11@gmail.com, [3]mukunthusuresh@gmail.com and [4]karthik.kaka123@gmail.com

*Corresponding author

Article Info
Page Number: 3506-3514
Publication Issue:
Vol. 71 No. 4 (2022)

Article History
Article Received: 25 March 2022
Revised: 30 April 2022
Accepted: 15 June 2022
Publication: 19 August 2022

Abstract

Fuzzy number plays a dynamic part in decision makings because it is used to represent uncertain terms. Researchers employed triangular and trapezoidal fuzzy numbers for solving problems in uncertain environment. Fuzzy numbers have also been extended recently such as pentagonal, hexagonal, heptagonal and so on. This paper aims to generalize the nonagonal fuzzy number which contains set of 9-tuples. Membership functions and alpha cuts of linear and nonlinear generalized nonagonal fuzzy number with symmetry and asymmetry have also been derived. Finally, fuzzy assignment problem has been solved to demonstrate the efficacy of the proposed fuzzy number.

Keywords: Fuzzy number, Nonagonal fuzzy number, membership function, alpha cut.

1. Introduction:

Zadeh, L.A introduced fuzzy sets in 1965 [19] to offer a rational way of solving difficulties when vagueness and imprecision arises. The application of fuzzy logic includes artificial intelligence, control system, expert system, decision makings etc. Arithmetic operations on fuzzy numbers are useful in computational models, diagnostic system, forecasting models and so on [16, 17, 18]. Fuzzy number is multi-valued quantity which has been proposed by Dubois and Prade as a subset of the real number [5].

Triangular and trapezoidal fuzzy numbers are familiar among researchers to handle fuzziness in real life situations [3, 4]. Triangular fuzzy number was used by Karthik et.al., to study the impact of pesticides on human health [8]. Hexagonal, heptagonal, nonagonal, decagonal fuzzy numbers were proposed to tackle vagueness [6, 7, 13]. Fuzzy equations were solved by Sankar and Manimohan using pentagonal fuzzy number [14]. Karthik et. al., proposed linear and nonlinear membership functions for the generalized heptagonal fuzzy number and introduced Haar ranking method for hexagonal fuzzy number [9]. Malini and Kennedy solved fuzzy transportation problem through octagonal fuzzy numbers [10]. Felix et.al., proposed nonagonal fuzzy number, arithmetic operations and alpha cuts [7]. Bindu and Govindarajan used nonagonal fuzzy number to evaluate the performance of single server queuing model [1]. A novel ranking method was proposed for nonagonal fuzzy number by Deepika and Rekha to solve fuzzy transportation problem [2]. Venkatesh and Britto introduced a ranking method using decagonal fuzzy number for diet control [15]. Nagadevi and Rosario solved transportation problem, in which decagonal fuzzy numbers are used to represent transportation costs to find minimum transportation cost [11]. Naveena and Rajkumar introduced arithmetic operations of reverse order pentadecagonal, nonagonal and decagonal fuzzy numbers [12].

It is observed that linear membership functions paid more attention in defining fuzzy numbers. Hence, this study proposes both linear and nonlinear membership functions for generalized nonagonal fuzzy number.

This paper is organized in the following way. A brief review of literatures that are motivating to carry out this research is discussed in Section 1. Nonagonal Fuzzy Number and Its Variations in the shapes of membership curves such as linear, nonlinear and alpha cuts are derived for generalized nonagonal fuzzy number in Section 2. Finally, the conclusion and application of nonagonal fuzzy number are discussed.

2. Preliminaries

2.1 Fuzzy set:

Fuzzy set is a subset of a universe of discourse X, where each element in a fuzzy set has some degrees of belongingness that is characterised by a mapping from X to [0, 1].

2.2 Fuzzy numbers

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number when it satisfies the following conditions

(i) $\tilde{A}$ is convex.
$\mu_{\tilde{A}}\left(\lambda\theta_1 + (1-\lambda)\theta_2\right) \geq \min(\mu_{\tilde{A}}(\theta_1), \mu_{\tilde{A}}(\theta_2)),\ \forall\,\theta \in \left[\theta_1, \theta_2\right], \lambda \in \left[0,1\right].$

(ii) $\tilde{A}$ is normal if $\max \mu_{\tilde{A}}(\theta) = 1$.

(iii) $\tilde{A}$ is piecewise continuous.

2.3 Alpha Cut

The α-cut of the fuzzy set is the set of all elements whose membership value is greater than or equal to α, that is $\tilde{A}_\alpha = \{\theta \in X \,/\, \mu_{\tilde{A}}(\theta) \geq \alpha\}$, where $\alpha \in \left[0,1\right]$.

3. Nonagonal Fuzzy Number and It's Variation

Linear Nonagonal Fuzzy Number with Symmetry (LNFNS)

A LNFNS is given as $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9; m, n, o)$ where $m, n, o \in (0,1)$

$$\mu(x)=\begin{cases} m\left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1\le x\le a_2\\ m-(m-n)\left(\dfrac{x-a_2}{a_3-a_2}\right) & a_2\le x\le a_3\\ n-(n-o)\left(\dfrac{x-a_3}{a_4-a_3}\right) & a_3\le x\le a_4\\ o-(o-1)\left(\dfrac{x-a_4}{a_5-a_4}\right) & a_4\le x\le a_5\\ 1 & x=a_5\\ o-(o-1)\left(\dfrac{a_6-x}{a_6-a_5}\right) & a_5\le x\le a_6\\ n-(n-o)\left(\dfrac{a_7-x}{a_7-a_6}\right) & a_6\le x\le a_7\\ m-(m-n)\left(\dfrac{a_8-x}{a_8-a_7}\right) & a_7\le x\le a_8\\ m\left(\dfrac{a_9-x}{a_9-a_8}\right) & a_8\le x\le a_9\\ 0 & x\le a_1\ \&\ x\ge a_9 \end{cases}$$

α-cut of LNFNS

The α-cut for LNFNS is derived and is given by

$$A_\alpha=\begin{cases} A_{1L}(\alpha)=a_1+\left(\dfrac{\alpha}{m}\right)(a_2-a_1)\ \ for\ \ \alpha\in[0,m]\\ A_{2L}(\alpha)=a_2+\left(\dfrac{\alpha-m}{n-m}\right)(a_3-a_2)\ \ for\ \ \alpha\in[m,n]\\ A_{3L}(\alpha)=a_3+\left(\dfrac{\alpha-n}{o-n}\right)(a_4-a_3)\ \ for\ \ \alpha\in[n,o]\\ A_{4L}(\alpha)=a_4+\left(\dfrac{\alpha-o}{1-o}\right)(a_5-a_4)\ \ for\ \ \alpha\in[o,1]\\ A_{4R}(\alpha)=a_6-\left(\dfrac{\alpha-o}{1-o}\right)(a_6-a_5)\ \ for\ \ \alpha\in[o,1]\\ A_{3R}(\alpha)=a_7-\left(\dfrac{\alpha-n}{o-n}\right)(a_7-a_6)\ \ for\ \ \alpha\in[n,o]\\ A_{2R}(\alpha)=a_8-\left(\dfrac{\alpha-m}{n-m}\right)(a_8-a_7)\ \ for\ \ \alpha\in[m,n]\\ A_{1R}(\alpha)=a_9-\left(\dfrac{\alpha}{m}\right)(a_9-a_8)\ \ for\ \ \alpha\in[0,m] \end{cases}$$

Linear Nonagonal Fuzzy Number with Asymmetry (LNFNAS)

A LNFNAS is given as

$\tilde{A}_{LAS}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9;m,q,n,p,o,r)$ where $m,n,p,q,o,r\in(0,1)$

$$\mu(x)=\begin{cases} m\left(\frac{x-a_1}{a_2-a_1}\right) & a_1\le x\le a_2\\ m-(m-n)\left(\frac{x-a_2}{a_3-a_2}\right) & a_2\le x\le a_3\\ n-(n-o)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3\le x\le a_4\\ o-(o-1)\left(\frac{x-a_4}{a_5-a_4}\right) & a_4\le x\le a_5\\ 1 & x=a_5\\ r-(r-1)\left(\frac{a_6-x}{a_6-a_5}\right) & a_5\le x\le a_6\\ q-(q-r)\left(\frac{a_7-x}{a_7-a_6}\right) & a_6\le x\le a_7\\ p-(p-q)\left(\frac{a_8-x}{a_8-a_7}\right) & a_7\le x\le a_8\\ p\left(\frac{a_9-x}{a_9-a_8}\right) & a_8\le x\le a_9\\ 0 & x\le a_1\ \&\ x\ge a_9 \end{cases}$$

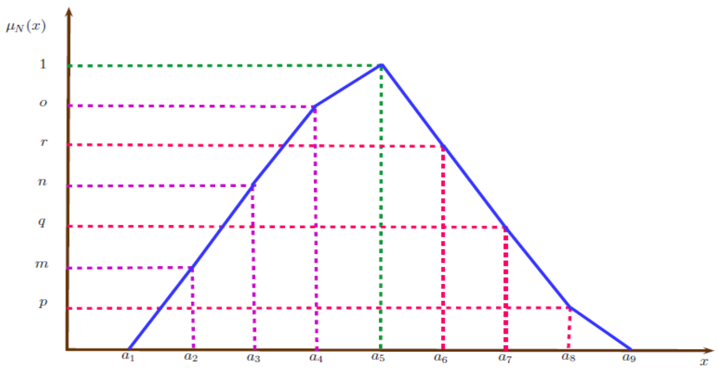

Figure 1: Generalized Nonagonal Fuzzy Number

α-cut of LNFNAS

The α-cut for LNFNAS is derived and is given by

$$A_\alpha=\begin{cases} A_{1L}(\alpha)=a_1+\left(\frac{\alpha}{m}\right)(a_2-a_1)\ \ for\ \ \alpha\in[0,m]\\ A_{2L}(\alpha)=a_2+\left(\frac{\alpha-m}{n-m}\right)(a_3-a_2)\ \ for\ \ \alpha\in[m,n]\\ A_{3L}(\alpha)=a_3+\left(\frac{\alpha-n}{o-n}\right)(a_4-a_3)\ \ for\ \ \alpha\in[n,o]\\ A_{4L}(\alpha)=a_4+\left(\frac{\alpha-o}{1-o}\right)(a_5-a_4)\ \ for\ \ \alpha\in[o,1]\\ A_{4R}(\alpha)=a_6-\left(\frac{\alpha-r}{1-r}\right)(a_6-a_5)\ \ for\ \ \alpha\in[r,1]\\ A_{3R}(\alpha)=a_7-\left(\frac{\alpha-q}{r-q}\right)(a_7-a_6)\ \ for\ \ \alpha\in[q,r]\\ A_{2R}(\alpha)=a_8-\left(\frac{\alpha-p}{q-p}\right)(a_8-a_7)\ \ for\ \ \alpha\in[p,q]\\ A_{1R}(\alpha)=a_9-\left(\frac{\alpha}{p}\right)(a_9-a_8)\ \ for\ \ \alpha\in[0,p] \end{cases}$$

Non-Linear Nonagonal Fuzzy Number with Symmetry (NLNFNS)

A NLNFNS is given as

$\tilde{A}_{NLS} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9; m, n, o)_{(k_1,k_2,k_3,k_4;l_1,l_2,l_3,l_4)}$ where $m, n, o \in (0,1)$

$$\mu(x) = \begin{cases} m\left(\dfrac{x-a_1}{a_2-a_1}\right)^{k_1} & a_1 \le x \le a_2 \\ m-(m-n)\left(\dfrac{x-a_2}{a_3-a_2}\right)^{k_2} & a_2 \le x \le a_3 \\ n-(n-o)\left(\dfrac{x-a_3}{a_4-a_3}\right)^{k_3} & a_3 \le x \le a_4 \\ o-(o-1)\left(\dfrac{x-a_4}{a_5-a_4}\right)^{k_4} & a_4 \le x \le a_5 \\ 1 & x = a_5 \\ o-(o-1)\left(\dfrac{a_6-x}{a_6-a_5}\right)^{l_1} & a_5 \le x \le a_6 \\ n-(n-o)\left(\dfrac{a_7-x}{a_7-a_6}\right)^{l_2} & a_6 \le x \le a_7 \\ m-(m-n)\left(\dfrac{a_8-x}{a_8-a_7}\right)^{l_3} & a_7 \le x \le a_8 \\ m\left(\dfrac{a_9-x}{a_9-a_8}\right)^{l_4} & a_8 \le x \le a_9 \\ 0 & x \le a_1 \,\&\, x \ge a_9 \end{cases}$$

α-cut of NLNFNS:

The α-cut for NLNFNS is derived and is given by

$$A_\alpha = \begin{cases} A_{1L}(\alpha) = a_1 + \left(\dfrac{\alpha}{m}\right)^{k_1}(a_2-a_1) \ \ for \ \alpha \in [0,m] \\ A_{2L}(\alpha) = a_2 + \left(\dfrac{\alpha-m}{n-m}\right)^{k_2}(a_3-a_2) \ \ for \ \alpha \in [m,n] \\ A_{3L}(\alpha) = a_3 + \left(\dfrac{\alpha-n}{o-n}\right)^{k_3}(a_4-a_3) \ \ for \ \alpha \in [n,o] \\ A_{4L}(\alpha) = a_4 + \left(\dfrac{\alpha-o}{1-o}\right)^{k_4}(a_5-a_4) \ \ for \ \alpha \in [o,1] \\ A_{4R}(\alpha) = a_6 - \left(\dfrac{\alpha-o}{1-o}\right)^{l_1}(a_6-a_5) \ \ for \ \alpha \in [o,1] \\ A_{3R}(\alpha) = a_7 - \left(\dfrac{\alpha-n}{o-n}\right)^{l_2}(a_7-a_6) \ \ for \ \alpha \in [n,o] \\ A_{2R}(\alpha) = a_8 - \left(\dfrac{\alpha-m}{n-m}\right)^{l_3}(a_8-a_7) \ \ for \ \alpha \in [m,n] \\ A_{1R}(\alpha) = a_9 - \left(\dfrac{\alpha}{m}\right)^{l_4}(a_9-a_8) \ \ for \ \alpha \in [0,m] \end{cases}$$

Non-Linear Nonagonal Fuzzy Number with Asymmetry (NLNFNAS)

A NLNFNAS is given as

$$\tilde{A}_{NLAS} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9; m, n, o, p, q, r)_{(k_1,k_2,k_3,k_4;l_1,l_2,l_3,l_4)}$$

where $m, n, p, q, o, r \in (0,1)$

$$\mu(x)=\begin{cases} m\left(\dfrac{x-a_1}{a_2-a_1}\right)^{k_1} & a_1\le x\le a_2\\ m-(m-n)\left(\dfrac{x-a_2}{a_3-a_2}\right)^{k_2} & a_2\le x\le a_3\\ n-(n-o)\left(\dfrac{x-a_3}{a_4-a_3}\right)^{k_3} & a_3\le x\le a_4\\ o-(o-1)\left(\dfrac{x-a_4}{a_5-a_4}\right)^{k_4} & a_4\le x\le a_5\\ 1 & x=a_5\\ r-(r-1)\left(\dfrac{a_6-x}{a_6-a_5}\right)^{l_1} & a_5\le x\le a_6\\ q-(q-r)\left(\dfrac{a_7-x}{a_7-a_6}\right)^{l_2} & a_6\le x\le a_7\\ p-(p-q)\left(\dfrac{a_8-x}{a_8-a_7}\right)^{l_3} & a_7\le x\le a_8\\ p\left(\dfrac{a_9-x}{a_9-a_8}\right)^{l_4} & a_8\le x\le a_9\\ 0 & x\le a_1 \,\&\, x\ge a_9 \end{cases}$$

α-cut of NLNFNAS

The α-cut for NLNFNAS is derived and is given by

$$A_\alpha=\begin{cases} A_{1L}(\alpha)=a_1+\left(\dfrac{\alpha}{m}\right)^{k_1}(a_2-a_1)\ for\ \alpha\in[0,m]\\ A_{2L}(\alpha)=a_2+\left(\dfrac{\alpha-m}{n-m}\right)^{k_2}(a_3-a_2)\ for\ \alpha\in[m,n]\\ A_{3L}(\alpha)=a_3+\left(\dfrac{\alpha-n}{o-n}\right)^{k_3}(a_4-a_3)\ for\ \alpha\in[n,o]\\ A_{4L}(\alpha)=a_4+\left(\dfrac{\alpha-o}{1-o}\right)^{k_4}(a_5-a_4)\ for\ \alpha\in[o,1]\\ A_{4R}(\alpha)=a_6-\left(\dfrac{\alpha-r}{1-r}\right)^{l_1}(a_6-a_5)\ for\ \alpha\in[r,1]\\ A_{3R}(\alpha)=a_7-\left(\dfrac{\alpha-q}{r-q}\right)^{l_2}(a_7-a_6)\ for\ \alpha\in[q,r]\\ A_{2R}(\alpha)=a_8-\left(\dfrac{\alpha-p}{q-p}\right)^{l_3}(a_8-a_7)\ for\ \alpha\in[p,q]\\ A_{1R}(\alpha)=a_9-\left(\dfrac{\alpha}{p}\right)^{l_4}(a_9-a_8)\ for\ \alpha\in[0,p] \end{cases}$$

4. Fuzzy Assignment Problem

Suppose there are m jobs which are to be executed by m persons. Assume that each person can do one job at a time. Let $\tilde{P}_{ij}$ be a fuzzy cost of allocating the i^{th} person to the j^{th} job. Let the decision parameter y_{ij} denote the allotment of the i^{th} person to the j^{th} job.

The problem is to determine an assignment (which job should be given to which person on one- one basis) so that the total cost of executing all jobs is optimum. These kinds of problems are said to be assignment problem. Mathematically, the problem is defined as

$$\text{min (or) max } X = \sum_{j=1}^{m}\sum_{i=1}^{m} \tilde{P}_{ij} y_{ij}$$

Subject to

$$\sum_{i=1}^{m} y_{ij} = 1 \quad for\ i = 1, 2, \ldots, m.$$
$$\sum_{j=1}^{m} y_{ij} = 1 \quad for\ j = 1, 2, \ldots, m.$$

y_{ij}=1, if the ith job is allotted to jth person

0, if the ith job is not allotted to jth person

Example 4.1:

A Fuzzy Assignment Problem with 4 machines and 4 jobs is studied. The cost matrix Cij * whose values are represented by nonagonal fuzzy number. The problem is to find the minimum assignment cost. Here, the Hungarian method is used to compute the optimal allocation.

A=

$$\begin{bmatrix} (4,5,2,3,6,7,1,9,8) & (4,5,21,3,6,7,11,9,8) & (14,5,12,3,26,7,1,9,8) & (14,5,2,3,6,7,11,9,8) \\ (14,5,2,3,6,7,21,9,8) & (14,5,2,3,6,7,10,9,8) & (4,5,52,33,6,7,12,9,8) & (4,5,2,3,6,17,21,9,8) \\ (34,5,2,3,6,7,21,9,8) & (24,5,22,3,6,7,1,9,8) & (4,5,2,53,6,7,1,9,18) & (41,5,2,3,6,7,15,9,8) \\ (44,5,2,3,6,7,41,9,8) & (34,5,2,3,6,7,1,9,8) & (34,5,22,3,6,7,19,9,8) & (4,15,2,3,6,7,14,9,8) \end{bmatrix}$$

After taking average,

$$A = \begin{bmatrix} 5 & 8.2 & 9.4 & 7.2 \\ 8.3 & 7.1 & 15.1 & 8.3 \\ 10.5 & 9.4 & 10.5 & 10.6 \\ 13.8 & 8.3 & 12.5 & 7.5 \end{bmatrix}$$

Row wise subtraction,

$$A = \begin{bmatrix} 0 & 3.2 & 4.4 & 2.2 \\ 1.2 & 0 & 8 & 1.2 \\ 1.1 & 0 & 1.1 & 1.2 \\ 6.3 & 0.8 & 5 & 0 \end{bmatrix}$$

Column wise subtraction,

$$A = \begin{bmatrix} 0 & 3.2 & 3.3 & 2.2 \\ 1.2 & 0 & 6.9 & 1.2 \\ 1.1 & 0 & 0 & 1.2 \\ 6.3 & 0.8 & 3.9 & 0 \end{bmatrix}$$

Number of rows = Number of squares. The assignment cost is = 5 + 7.1 + 10.5 + 7.5 = 30.1.

Nonagonal fuzzy number facilitates to solve assignment problem whenever uncertainty arises. The advantage of nonagonal fuzzy numbers over existing fuzzy numbers is it is used to clearly describe the problem and get the most appropriate solution.

5. Conclusion

In this present study, the generalized nonagonal fuzzy numbers have been derived under fuzzy environment which may help to handle uncertainties in the decision-making problems. These kinds of fuzzy numbers are helpful when decision maker needs to represent a parameter at 9 different points. The following important outcomes have been attained in this research,

- Generalized nonagonal fuzzy number with symmetry and asymmetry in the membership curve have been derived.
- Alpha cut for generalized nonagonal fuzzy number has also been derived.
- Assignment problem for optimal allocation is solved by facilitating nonagonal fuzzy numbers.

These numbers are helpful in solving fuzzy assignment problems and fuzzy transportation problems to obtain optimal solution.

References:

1) Bindu, V. R., & Govindarajan, R. (2019). Fuzzy queues in nonagonal fuzzy number through DSW algorithm, EPRA International Journal of Multidisciplinary Research, 5(9), 176-180.
2) Deepika, K., & Rekha, S. (2017). A New Ranking Function of Nonagonal Fuzzy Numbers, International Journal for Modern Trends in Science and Technology, 3(9), 149-151.
3) Devadoss, A. V., & Felix, A. (2013). A new Fuzzy DEMATEL method in an Uncertain Linguistic Environment, Advances in Fuzzy Sets and Systems, 16(2), 93-123.
4) Devadoss, A. V., & Felix, A. (2013). A Fuzzy DEMATEL approach to Study cause and effect relationship of Youth Violence, International Journal of Computing Algorithm, 2, 363-372.
5) Dubois, D., & Prade, H. (1978). Operations on Fuzzy Numbers, International Journal of Systems Science, 9(6), 613-626.
6) Felix, A., & Devadoss, A. V. (2015). A new Decagonal Fuzzy Number under Uncertain Linguistic Environment, International Journal of Mathematics and Its Applications, 3(1), 89-97.

7) Felix, A., Christopher, S., & Devadoss, A. V. (2015). A Nonagonal Fuzzy Number and Its Arithmetic Operation, International Journal of Mathematics And its Applications, 3(2), 185-195.
8) Karthik, S., Saroj Kumar Dash., & Punithavelan, N. (2020). A Fuzzy Decision-Making System for the Impact of Pesticides Applied in Agricultural Fields on Human Health, International Journal of Fuzzy System Applications, 9(3), 42-62.
9) Karthik, S., Saroj Kumar Dash., Punithavelan, N. (2019). Haar Ranking of Linear and Non-Linear Heptagonal Fuzzy Number and Its Application, International Journal of Innovative Technology and Exploring Engineering, 8(6), 1212-1220.
10) Malini, S. U., & Kennedy, C. (2013). An Approach for Solving Fuzzy Transportation Problem Using Octagonal Fuzzy Numbers, Applied Mathematical Sciences, 7(54), 2661-2673.
11) Nagadevi, S., & Rosario, M. (2019). A study on fuzzy transportation problem using decagonal fuzzy number, Advances and Applications in Mathematical Sciences, 18(10), 1209-1225.
12) Naveena, N., & Rajkumar, A. (2019). A New Reverse Order Pentadecagonal, Nanogonal and Decagonal Fuzzy Number with Arithmetic Operations, International Journal of Recent Technology and Engineering, 8(3), 7937-7943.
13) Rathi, K., & Balamohan, S. (2014). Representation and Ranking of Fuzzy Numbers with Heptagonal Membership Function Using value and Ambiguity Index, Applied Mathematical Sciences, 8(87), 4309 – 4321.
14) Sankar Prasad Mondal., & Manimohan Mandal. (2017). Pentagonal fuzzy number, its properties and application in fuzzy equation, Future Computing and Informatics Journal 2, 110- 117.
15) Venkatesh, A., & Britto, M. (2020). A Mathematical Model for Diet control Using Ranking of Decagonal Fuzzy Number, Journal of Xi'an University of Architecture & Technology, 12(7), 945-951.
16) Wang, Y. (2007). The Cognitive Processes of Formal Inferences}, International Journal of Cognitive Informatics and Natural Intelligence, 1(4), 75-86
17) Wang, Y., & Berwick, R. C. (2012a). Towards a Formal Framework of Cognitive Linguistics, Journal of Advanced Mathematics and Applications, 1(2), 250-263.
18) Zadeh, L. A. (1975). The concept of a Linguistic variable and its Application to Approximate Reasoning (Part II), Information Science, 8, 301-357.
19) Zadeh, L. A. (1975b). Fuzzy Logic and Approximate Reasoning, Syntheses, 30, 407-428.