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On $KS_{gp}(KS_{gsp})$-Irresolute Functions in Kasaj Topological Spaces

[1]P. Sathishmohan, [2] M. Sindhu, [2]K.Lavanya and [3]K.Rajalakshmi

[1]Assistant Professor, [2]Research Scholar, Department of Mathematics,
Kongunadu Arts and Science College (Autonomous),
Coimbatore-641 029.
[3]Assistant Professor, Department of Science and Humanities,Sri Krishna
College of Engineering and Technology , Coimbatore-641008.
Email: sindhujosna@gmail.com [2]

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Abstract
The focus of this article is to introduce a new category of generalized-pre-continuous and generalized-semi-pre-continuous processes in Kasaj topological spaces, represented KS_{gp} -continuous function and KS_{gsp}-continuous function, respectively, and to examine the relationships between such features and other capabilities within those spaces. For both KS_{gp} -continuous function and indeed the KS_{gsp}-continuous function are provided. In Kasaj topological spaces, the KS_{gp}-irresolute function and the KS_{gsp}-irresolute function are the subjects of this paper's exposition and exploration.

Keywords: $KS_R(X)$, KS_{gp} -continuous functions, KS_{gsp} -continuous functions, KS_{gp}-irresolute functions, KS_{gsp}-irresolute functions.

1 INTRODUCTION AND PRELIMINARIES

Kashyap G.Rachchh and Sajeed introduced Kashyap G.Rachchh and Sajeed topological spaces as a partial extension of Micro topological space. Additionally, we'll define the Kasaj-generalized-precontinuous function and the Kasaj-generalized-semi-pre continuous function, as well as look into their fundamental characteristics and determine how they relate to one another. In Kasaj topological spaces, P. Sathishmohan et al. introduced the new type of closed sets known as KS_{gp}-closed sets and KS_{gsp}-closed sets. In Kasaj topological spaces, E. Prakash et al.introduced $KS_{sg}(KS_{gs})$-continuous and irresolute functions. A function υ : $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is said to be KS-continuous(KS-semi-continuous,KS-pre-continuous,KS-α-continuous,KS-β-continuous)[3] if the inverse image of every KS-closed set (KS-semi-closed,KS-pre-closed,KS-α-closed,KS-β-closed)in $\mathcal{H}$, is KS-closed in $\mathcal{G}$.

2 $KS_{gp}(KS_{gsp})$-Irresolute Functions

In Kasaj topological spaces, the KS_{gp} -continuous and irresolute functions and the KS_{gsp}-continuous and irresolute functions are extensively examined in this article.

Definition: A function $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is said to be KS_{gp}-continuous, if the inverse image of every KS-closed set in $\mathcal{H}$ is KS_{gp}-closed in $\mathcal{G}$.

Definition: A function $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is said to be KS_{gsp}-continuous, if the inverse image of every KS-closed set in $\mathcal{H}$ is KS_{gsp}-closed in $\mathcal{G}$.

Definition: A function $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is said to be KS_{gp}-irresolute, if the inverse image of every KS_{gp}-closed set in $\mathcal{H}$ is KS_{gp}-closed in $\mathcal{G}$.

Definition: A function $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is said to be KS_{gsp}-irresolute, if the inverse image of every KS_{gsp}-closed set in $\mathcal{H}$ is KS_{gsp}-closed in $\mathcal{G}$.

Theorem: If a map has the following coordinates $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$. If υ is KS-continuous, then it is KS_{gp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS-continuous. Let any KS-closed set in $\mathcal{H}$ represent $\mathfrak{S}$. The $\upsilon^{-1}(\mathfrak{S})$ inverted image is then KS-closed in $\mathcal{G}$. Ever KS_{gp}-closed is closed for every KS-closed set, $\upsilon^{-1}(\mathfrak{S})$ is closed for KS_{gp} in $\mathcal{G}$. υ is thus KS_{gp}-continuous.

The following example demonstrates how the flipside of the aforementioned theorem need not be true. Let $\mathcal{G} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{G} \backslash \mathcal{R} = \{\{\gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $\mathbb{X} = \{\vartheta, \aleph, \gimel\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathcal{G}, \phi, \{\gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$. $\mathbb{S} = \{\gimel, \mathrm{F}\}$, $\mathbb{S}' = \{\vartheta, \varkappa, \aleph\}$. Then $KS_{\mathcal{R}}(\mathbb{X})$ -open sets are $\{\mathcal{G}, \phi, \{\gimel\}, \{\mathrm{F}\}, \{\gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{X})$ -closed sets are $\{\mathcal{G}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \varkappa, \aleph\}, \{\gimel, \mathrm{F}\}, \{\gimel\}, \{\mathrm{F}\}\}$. Let $\mathcal{H} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{H} \backslash \mathcal{R} = \{\{\vartheta, \gimel\}, \{\varkappa, \mathrm{F}\}, \{\aleph\}\}$ and $\mathbb{Y} = \{\vartheta, \varkappa, \aleph, \gimel\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{Y}) = \{\mathcal{H}, \phi, \{\vartheta, \aleph, \gimel\}, \{\varkappa, \mathrm{F}\}\}$. $\mathbb{S} = \{\varkappa\}$, $\mathbb{S}' = \{\vartheta, \aleph, \gimel, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{Y})$ -open sets are $\{\mathcal{H}, \phi, \{\varkappa\}, \{\mathrm{F}\}, \{\varkappa, \mathrm{F}\}, \{\vartheta, \aleph, \gimel\}, \{\vartheta, \aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \gimel\}\}$ and $KS_{\mathcal{R}}(\mathbb{Y})$ -closed sets are $\{\mathcal{H}, \phi, \{\vartheta, \aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \aleph, \gimel\}, \{\varkappa, \mathrm{F}\}, \{\varkappa\}, \{\mathrm{F}\}\}$. Then υ is KS_{gp}-continuous but not KS-closed. Since $(\vartheta, \aleph, \gimel, \mathrm{F})$ is KS-closed in $\mathcal{H}$, but $\upsilon^{-1}(\vartheta, \aleph, \gimel, \mathrm{F}) = \{\vartheta, \varkappa, \gimel, \mathrm{F}\}$ is KS_{gp}-closed but not KS-closed set in $\mathcal{G}$.

Theorem: If a map υ is constructed as follows: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$. If υ is KS pre-continuous, then it is KS_{gp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS-pre-continuous. In $\mathcal{H}$, let $\mathfrak{S}$ represent any KS-pre-closed set. When this occurs, KS-pre-closed in $\mathcal{G}$ is the inverse image of $\upsilon^{-1}(\mathfrak{S})$. Every set that is KS-pre-closed is KS_{gp}-closed, hence $\upsilon^{-1}(\mathfrak{S})$ is KS_{gp}-closed in $\mathcal{G}$. υ is thus KS_{gp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS-α-continuous, then it is KS_{gp}-continuous.

Proof: Let $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS-α-continuous. Let $\mathfrak{S}$ be any KS-α-closed set in $\mathcal{H}$. Then the inverse image of $\upsilon^{-1}(\mathfrak{S})$ is KS-α-closed in $\mathcal{G}$. Even before every KS-α-closed set is KS_{gp}-Closed, $\upsilon^{-1}(\mathfrak{S})$ is KS_{gp}-closed in $\mathcal{G}$. Therefore υ is

KS_{gp}-continuous.

The following example demonstrates how the flipside of the aforementioned theorem need not be true. Let $\mathcal{G} = \{\vartheta, \varkappa, ℵ, ℷ, \mathrm{F}\}$, with $\mathcal{G}\backslash\mathcal{R} = \{\{\vartheta, \varkappa\}, \{ℵ, ℷ\}, \{\mathrm{F}\}\}$ and $\mathbb{X} = \{\vartheta, \mathrm{F}\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathcal{G}, \phi, \{\mathrm{F}\}, \{\vartheta, \varkappa, \mathrm{F}\}, \{\vartheta, \varkappa\}\}$. $\mathbb{S} = \{\vartheta\}$, $\mathbb{S}' = \{\varkappa, ℵ, ℷ, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{X})$ -open sets are $\{\mathcal{G}, \phi, \{\vartheta\}, \{\varkappa\}, \{\mathrm{F}\}, \{\vartheta, \varkappa\}, \{\vartheta, \mathrm{F}\}, \{\varkappa, \mathrm{F}\}, \{\vartheta, \varkappa, \mathrm{F}\}, \{\varkappa, ℵ, ℷ, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{X})$ -closed sets are $\{\mathcal{G}, \phi, \{\varkappa, ℵ, ℷ, \mathrm{F}\}, \{\vartheta, ℵ, ℷ, \mathrm{F}\}, \{\vartheta, \varkappa, ℵ, ℷ\}, \{ℵ, ℷ, \mathrm{F}\}, \{\varkappa, ℵ, ℷ\}, \{\vartheta, ℵ, ℷ\}, \{ℵ, ℷ\}, \{\vartheta\}\}$. Let $\mathcal{H} = \{\vartheta, \varkappa, ℵ, ℷ, \mathrm{F}\}$, with $\mathcal{H}\backslash\mathcal{R} = \{\{\varkappa, ℵ\}, \{ℷ, \mathrm{F}\}, \{\vartheta\}\}$ and $\mathbb{Y} = \{\vartheta, \varkappa, ℵ, ℷ\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{Y}) = \{\mathcal{H}, \phi, \{\vartheta, \varkappa, ℵ\}, \{ℷ, \mathrm{F}\}\}$. $\mathbb{S} = \{ℷ\}$, $\mathbb{S}' = \{\vartheta, \varkappa, ℵ, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{Y})$-open sets are $\{\mathcal{H}, \phi, \{ℷ\}, \{\mathrm{F}\}, \{ℷ, \mathrm{F}\}, \{\vartheta, \varkappa, ℵ\}, \{\vartheta, \varkappa, ℵ, ℷ\}, \{\vartheta, \varkappa, ℵ, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{Y})$ -closed sets are $\{\mathcal{H}, \phi, \{\vartheta, \varkappa, ℵ, \mathrm{F}\}, \{\vartheta, \varkappa, ℵ, ℷ\}, \{\vartheta, ℵ, ℷ\}, \{ℷ, \mathrm{F}\}, \{ℷ\}, \{\mathrm{F}\}\}$. Then υ is KS_{gp}-continuous but not KS-α-closed. Since $(ℷ)$ is KS-closed in $\mathcal{H}$, but $\upsilon^{-1}(ℷ) = \{ℵ\}$ is KS_{gp}-closed but not KS-α-closed set in $\mathcal{G}$.

Theorem: If a map υ is KS_{gp}-continuous and $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$, then $\upsilon(KS_{gpcl}(P)) \subseteq KS_{cl}\upsilon(P))$ for each subset P of $\mathcal{G}$.

Proof: If υ is KS_{gp}-continuous, then $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$. P should be a subset of $\mathcal{G}$. Then Assuming that $\upsilon^{-1}(KS_{cl}\upsilon(P))$ is KS_{gp}-closed in $\mathcal{G}$, $KS_{cl}\upsilon(P)$ is KS-closed in $\mathcal{H}$. Additionally, $\mathrm{P} \subseteq \upsilon^{-1}(KS_{cl}\upsilon(P)$ and $\upsilon(P) \subseteq KS_{cl}\upsilon(P)$. Consequently, $KS_{gpcl}(P)) \subseteq \upsilon^{-1}(KS_{cl}\upsilon(P))$. In light of this, $\upsilon(KS_{gpcl}(P)) \subseteq KS_{cl}\upsilon(P))$.

Theorem: If a map υ is a function, then $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$. The assertions that follow are then equivalent.

1. υ is KS_{gp}-continuous.
2. The inverse image of every KS-closed set in $\mathcal{H}$ is KS_{gp}-closed in $\mathcal{G}$.

Proof: Suppose that υ is KS_{gp}-continuous. In $\mathcal{H}$, let $\mathbb{F}$ be a KS-closed set. Then, in $\mathcal{H}$, $\mathbb{F}^c$ is KS-open. By (1), KS_{gp}-open in $\mathcal{G}$ is $\upsilon^{-1}(\mathbb{F})^c = \mathcal{G} - \upsilon^{-1}(\mathbb{F})$. Because of this, $\upsilon^{-1}(\mathbb{F})$ is KS_{gp}-closed in $\mathcal{G}$. This suggests that (1)⟹(2).

Let's demonstrate (2)⟹ (1) now. Assume that each KS-closed set in $\mathcal{H}$ has an inverse image in $\mathcal{G}$ that is KS_{gp}-closed. let $\mathcal{H}$ be any KS-closed set in $\mathcal{H}$. Then, in $\mathcal{H}$, $\mathcal{H}^c$ is KS-open. By (2), KS_{gp} is open for $\upsilon^{-1}(\mathcal{H}^c)$. However, $\upsilon^{-1}(\mathcal{H}^c) = \mathcal{G} - \upsilon^{-1}(\mathcal{H})$ is KS_{gp} -open in the $\mathcal{G}$. Because of this, $\upsilon^{-1}(\mathcal{H})$ is KS_{gp} -closed in $\mathcal{G}$. υ is thus KS_{gp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-pre-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_{g}-continuous.

Proof: Let $\mathfrak{S}$ be any KS-closed set in $\mathcal{I}$. Even before $\hbar$ is KS-pre-continuous, $\hbar^{-1}(\mathfrak{S})$ is

KS-pre-closed in $\mathcal{H}$. Ever υ is KS_{gp}-continuous, $\upsilon^{-1}(\hbar^{-1}(\mathfrak{S})) = (\hbar \circ \upsilon)^{-1}(\mathfrak{S})$ is KS_g-closed in $\mathcal{G}$. Therefore $(\hbar \circ \upsilon)$ is KS_g-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_g-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_{gp}-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-pre-continuous.

Proof: Let $\mathfrak{S}$ be any KS-closed set in $\mathcal{I}$. Even before $\hbar$ is KS_{gp}-continuous, $\hbar^{-1}(\mathfrak{S})$ is KS_{gp}-closed in $\mathcal{H}$. Ever υ is KS_g-continuous, $\upsilon^{-1}(\hbar^{-1}(\mathfrak{S})) = (\hbar \circ \upsilon)^{-1}(\mathfrak{S})$ is KS-pre-closed in $\mathcal{G}$. Therefore $(\hbar \circ \upsilon)$ is KS-pre-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-α-continuous.

Proof: Let $\mathfrak{S}$ be any KS-closed set in $\mathcal{I}$. Even before $\hbar$ is KS-continuous, $\hbar^{-1}(\mathfrak{S})$ is KS-closed in $\mathcal{H}$. Ever υ is KS_{gp}-continuous, $\upsilon^{-1}(\hbar^{-1}(\mathfrak{S})) = (\hbar \circ \upsilon)^{-1}(\mathfrak{S})$ is KS-α-closed in $\mathcal{G}$. Therefore $(\hbar \circ \upsilon)$ is KS-α-continuous.

Remark: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_{gp}-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_{gp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-irresolute, then it is KS_{gp}-continuous.

Proof: Take υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS_{gp}-irresolute. Let $\mathfrak{S}$ be any KS_{gp}-closed set in $\mathcal{H}$ that seems to be KS-closed. The set $\upsilon^{-1}(\mathcal{H})$ is KS_{gp}-closed in $\mathcal{G}$ even though υ is KS_{gp}-irresolute. υ thus becomes KS_{gp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-irresolute, then $\upsilon(KS_{gpcl}(P)) \subseteq KS_{cl}\upsilon(P))$ for every subset P of $\mathcal{G}$.

Proof: P should be a subset of $\mathcal{G}$. Then, in $\mathcal{H}$, $KS_{gpcl}\upsilon(P)$ occurs. Due to the fact that υ is KS_{gp}-irresolute, $\upsilon^{-1}(KS_{gpcl}(P))$ is KS_{gp}-closed. Even more P $\subseteq \upsilon^{-1}(\upsilon(P)) \subseteq \upsilon^{-1}(KS_{cl}(\upsilon(P))$. Consequently, $KS_{gpcl}(P) \subseteq \upsilon^{-1}(KS_{cl}\upsilon(P))$. Because of this, $\upsilon(KS_{gpcl}(P)) \subseteq KS_{cl}\upsilon(P))$.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-irresolute, iff $\upsilon^{-1}(\mathcal{H})$ is KS_{gp}-closed set in $\mathcal{G}$ for every KS_{gp}-closed set in $\mathcal{H}$.

Proof: Consider the following scenario υ: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is a KS_{gp}-irresolute, and $\mathbb{F}$ is KS_{gp}-closed in $\mathcal{H}$. Then, in $\mathcal{H}$, $\mathbb{F}^c$ is KS_{gp}-open. $\upsilon^{-1}(\mathbb{F}^c)$ is KS_{gp}-closed in $\mathcal{G}$, by the definition of KS_{gp}-irresolute. But $\upsilon^{-1}(\mathbb{F}^c) = \mathcal{G} - \upsilon^{-1}(\mathbb{F})$. As a result, in $\mathcal{G}$, $\upsilon(\mathbb{F})$ is KS_{gp}-closed. On the other hand, imagine that $\upsilon^{-1}(\mathbb{F})$ is KS_{gp}-closed in $\mathcal{G}$. For every KS_{gp}-closed set G in $\mathcal{H}$. Let G be any KS_{gp}-closed set in $\mathcal{H}$. By definition, $\upsilon^{-1}(\mathbb{F}^c)$ is KS_{gp}-closed in the $\mathcal{G}$. However, $\upsilon^{-1}(\mathbb{F}^c) = \mathcal{G} - \upsilon^{-1}(\mathbb{F})$. Because $\upsilon^{-1}(\mathbb{F})$ is a KS_{gp}-closed set in $\mathcal{G}$, $\mathcal{G} - \upsilon^{-1}(\mathbb{F})$ is also a KS_{gp}-closed set in $\mathcal{G}$. υ is thus KS_{gp}-irresolute.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{R}(\mathbb{Y}))$ be KS-continuous. Let any KS-closed set in H represent $\mathfrak{S}$. Then $\upsilon^{-1}(\mathfrak{S})$ inverted image is then KS-closed in $\mathcal{G}$. Every set that is KS-closed is also KS_{gsp}-closed, so $\upsilon^{-1}(\mathfrak{S})$ is also KS_{gsp}-closed in $\mathcal{G}$. υ is thus KS_{gsp}-continuous.

The following example demonstrates how the flipside of the aforementioned theorem need not be true. Let $\mathcal{G} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{G}\backslash\mathcal{R} = \{\{\vartheta, \varkappa\}, \{\aleph, \gimel\}, \{\mathrm{F}\}\}$ and $\mathbb{X} = \{\vartheta, \mathrm{F}\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathcal{G}, \phi, \{\mathrm{F}\}, \{\vartheta, \varkappa, \mathrm{F}\}, \{\vartheta, \varkappa\}\}$. $\mathcal{S} = \{\vartheta\}$, $\mathcal{S}' = \{\varkappa, \aleph, \gimel, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{X})$ -open sets are $\{\mathcal{G}, \phi, \{\gimel\}, \{\vartheta, \varkappa\}, \{\aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \gimel\}, \{\aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{X})$ -closed sets are $\{\mathcal{G}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \gimel\}, \{\aleph, \mathrm{F}\}, \{\vartheta, \varkappa\}, \{\gimel\}\}$. Let $\mathcal{H} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{H}\backslash\mathcal{R} = \{\{\vartheta, \gimel\}, \{\varkappa, \mathrm{F}\}, \{\aleph\}\}$ and $\mathbb{Y} = \{\vartheta, \varkappa, \aleph, \gimel\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{Y}) = \{\mathcal{H}, \phi, \{\vartheta, \varkappa, \aleph\}, \{\gimel, \mathrm{F}\}\}$. $\mathcal{S} = \{\gimel\}$, $\mathcal{S}' = \{\vartheta, \varkappa, \aleph, \mathrm{F}\}$. Then $KS_{R}(\mathbb{Y})$-open sets are $\{\mathcal{H}, \phi, \{\gimel\}, \{\mathrm{F}\}, \{\gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{Y})$ -closed sets are $\{\mathcal{H}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \aleph, \gimel\}, \{\gimel, \mathrm{F}\}, \{\gimel\}, \{\mathrm{F}\}\}$. Then υ is KS_{gsp}-continuous but not KS-closed. Since $(\vartheta, \aleph, \gimel)$ is KS-closed in $\mathcal{H}$, but $\upsilon^{-1}(\vartheta, \aleph, \gimel) = \{\vartheta, \aleph, \gimel\}$ is KS_{gsp}-closed but not KS-closed set in $\mathcal{G}$.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS-semi-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS-semi-continuous. let $\mathfrak{S}$ be any KS-semi-closed set in $\mathcal{H}$. Then, $\upsilon^{-1}(\mathfrak{S})$ is image is KS-semi-closed in $\mathcal{G}$. Since KS_{gsp}-closed is closed for any KS-semi-closed set, $\upsilon^{-1}(\mathfrak{S})$ is KS_{gsp}-closed in $\mathcal{G}$. υ is thus KS_{gsp}-continuous. The following example demonstrates how the flipside of the aforementioned theorem need not be true. Let $\mathcal{G} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{G}\backslash\mathcal{R} = \{\{\vartheta, \varkappa\}, \{\aleph, \gimel\}, \{\mathrm{F}\}\}$ and $\mathbb{X} = \{\vartheta, \mathrm{F}\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathcal{G}, \phi, \{\mathrm{F}\}, \{\vartheta, \varkappa, \mathrm{F}\}, \{\vartheta, \varkappa\}\}$. $\mathbb{S} = \{\vartheta\}$, $\mathbb{S}' = \{\varkappa, \aleph, \gimel, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{X})$ -open sets are $\{\mathcal{G}, \phi, \{\gimel\}, \{\vartheta, \varkappa\}, \{\aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \gimel\}, \{\aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{X})$ -closed sets are $\{\mathcal{G}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \gimel\}, \{\aleph, \gimel\}, \{\vartheta, \varkappa\}, \{\gimel\}\}$. Let $\mathcal{H} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{H}\backslash R = \{\{\varkappa, \aleph\}, \{\gimel, \mathrm{F}\}, \{\vartheta\}\}$ and $\mathbb{Y} = \{\vartheta, \varkappa, \aleph, \gimel\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{Y}) = \{\mathcal{H}, \phi, \{\vartheta, \varkappa, \aleph\}, \{\gimel, \mathrm{F}\}\}$. $\mathbb{S} = \{\gimel\}$, $\mathbb{S}' = \{\vartheta, \varkappa, \aleph, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{Y})$ -open sets are $\{\mathcal{H}, \phi, \{\gimel\}, \{\mathrm{F}\}, \{\gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{Y})$ -closed sets are $\{\mathcal{H}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \aleph, \gimel\}, \{\gimel, \mathrm{F}\}, \{\gimel\}, \{\mathrm{F}\}\}$. Then f is KS_{gsp}-continuous but not KS-semi-closed. Since $\{\{\vartheta, \varkappa, \aleph\}\}$ is KS-closed in $\mathcal{H}$, but $\upsilon^{-1}(\vartheta, \varkappa, \aleph) = \{\varkappa, \aleph, \gimel\}$ is KS_{gsp}-closed but not KS-semi-closed set in $\mathcal{G}$.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS-pre-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS-pre-continuous. In $\mathcal{H}$, let $\mathfrak{S}$ represent any KS-pre-closed set. When this occurs, KS-pre-closed in $\mathcal{G}$ is the inverse image of $\upsilon^{-1}(\mathfrak{S})$. In $\mathcal{G}$, $\upsilon^{-1}(\mathfrak{S})$ is KS_{gsp}-closed. Even before KS-pre-closed set is KS_{gsp}-closed. υ is thus KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS-α-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS-alpha-continuous. Let $\mathfrak{S}$ represent any $\mathcal{H}$-based KS-α-closed set. Then, $\upsilon^{-1}(\mathfrak{S})$ is image is KS-α-closed in $\mathcal{G}$. In $\mathcal{G}$, $\upsilon^{-1}(\mathfrak{S})$ is KS_{gsp}-closed. Even before every KS-α-closed set is KS_{gsp}-closed. υ is thus KS_{gsp}-continuous. The following example demonstrates how the flipside of the aforementioned theorem need not be true. Let $\mathcal{G} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{G} \backslash \mathcal{R} = \{\{\vartheta, \varkappa\}, \{\aleph, \gimel\}, \{\mathrm{F}\}\}$ and $\mathbb{X} = \{\vartheta, \mathrm{F}\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathcal{G}, \phi, \{\mathrm{F}\}, \{\vartheta, \varkappa, \mathrm{F}\}, \{\vartheta, \varkappa\}\}$. $\mathbb{S} = \{\vartheta\}$, $\mathbb{S}' = \{\varkappa, \aleph, \gimel, \mathrm{F}\}$. Then $KS_{\mathcal{R}}(\mathbb{X})$-open sets are $\{\mathcal{G}, \phi, \{\gimel\}, \{\vartheta, \varkappa\}, \{\aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \gimel\}, \{\aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{X})$-closed sets are $(\mathcal{G}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\aleph, \gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \gimel\}, \{\aleph, \gimel\}, \{\vartheta, \varkappa\}, \{\gimel\}\}$. Let $\mathcal{H} = \{\vartheta, \varkappa, \aleph, \gimel, \mathrm{F}\}$, with $\mathcal{G} \backslash \mathcal{R} = \{\{\gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $\mathbb{Y} = \{\vartheta, \aleph, \gimel\}$. Then the nano topology, $\tau_{\mathcal{R}}(\mathbb{Y}) = \{\mathcal{H}, \phi, \{\gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$. $\mathbb{S} = \{\gimel, \mathrm{F}\}$, $\mathbb{S}' = \{\vartheta, \varkappa, \aleph\}$. Then $KS_{\mathcal{R}}(Y)$-open sets are $\{\mathcal{H}, \phi, \{\gimel\}, \{\mathrm{F}\}, \{\gimel, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}\}$ and $KS_{\mathcal{R}}(\mathbb{Y})$-closed sets are $\{\mathcal{H}, \phi, \{\vartheta, \varkappa, \aleph, \mathrm{F}\}, \{\vartheta, \varkappa, \aleph, \gimel\}, \{\vartheta, \varkappa, \aleph\}, \{\gimel, \mathrm{F}\}, \{\gimel\}, \{\mathrm{F}\}\}$. Then υ is KS_{gsp}-continuous but not KS-α-closed. Since $\{\gimel\}$ is KS-closed in $\mathcal{H}$, but $\upsilon^{-1}(\gimel) = \{\aleph\}$ is KS_{gsp}-closed but not KS-α-closed set in G.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_R(\mathbb{X}), KS_R(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_R(\mathbb{Y}), KS_R(\mathbb{Y}))$ is KS-β-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_R(\mathbb{X}), KS_R(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_R(\mathbb{Y}), KS_R(\mathbb{Y}))$ must be KS-β-continuous. Let $\mathfrak{S}$ represent any $\mathcal{H}$-based KS-β-closed set. Then, $\upsilon^{-1}(\mathfrak{S})$ is image is KS-β-closed in $\mathcal{G}$. Ever any set that is KS-β-closed is also KS_{gsp}-closed, $\upsilon^{-1}(\mathfrak{S})$ is KS_{gsp}-closed in $\mathcal{G}$. υ is thus KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_g-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS_g-continuous. let $\mathfrak{S}$ be any KS_g-closed set in $\mathcal{H}$. The $\upsilon^{-1}(\mathfrak{S})$ inverse image is then KS_g-closed in $\mathcal{G}$. Even before every set that is KS_g-closed is also KS_{gsp}-closed, $\upsilon^{-1}(\mathfrak{S})$ is closed in $\mathcal{G}$. υ is thus KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gp}-continuous, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS_{gp}-continuous. let $\mathfrak{S}$ be any KS_{gp}-closed set in $\mathcal{H}$. In $\mathcal{G}$, the inverse image of υ^{-1}(S) is then closed by KS_{gp}. Even before every set that is KS_{gp}-closed is also KS_{gsp}-closed, $\upsilon^{-1}(\mathfrak{S})$ is closed in $\mathcal{G}$. υ is thus KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gsp}-continuous, then $\upsilon(KS_{gspcl}(P)) \subseteq KS_{cl}\upsilon(P))$ for every subset P of $\mathcal{G}$.

Proof: Let υ be the following: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS_{gsp}-continuous. P should be a subset of $\mathcal{G}$. Then $\upsilon^{-1}(KS_{cl}\upsilon(P))$ is implied since $KS_{cl}\upsilon(P))$ is KS-closed in $\mathcal{H}$. In the $\mathcal{G}$, is KS_{gsp}-closed. Additionally, P $\subseteq \upsilon^{-1}(KS_{cl}\upsilon(P))$ and $\upsilon(P) \subseteq KS_{cl}\upsilon(P)$. In light of this, $KS_{gspcl}(P) \subseteq \upsilon^{-1}(KS_{cl}\upsilon(P))$. In light of this, $\upsilon(KS_{gspcl}(P)) \subseteq KS_{cl}\upsilon(P))$.

Theorem:If a map υ is a function, then $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$. The assertions that follow are then equivalent.

1. υ is KS_{gsp}-continuous.
2. The inverse image of every KS-closed set in $\mathcal{H}$ is KS_{gsp}-closed in $\mathcal{G}$.

Proof: Suppose that υ is KS_{gsp}-continuous. In $\mathcal{H}$, let $\mathbb{F}$ be a KS-closed set. Then, in $\mathcal{H}$, $\mathbb{F}^c$ is KS-open. By (1), KS_{gsp}-open in $\mathcal{G}$ is $\upsilon^{-1}(\mathbb{F})^c = \mathcal{G} - \upsilon^{-1}(\mathbb{F})$. Because of this, $\upsilon^{-1}(\mathbb{F})$ is KS_{gsp}-closed in $\mathcal{G}$. This suggests that (1)⟹(2).

Let's demonstrate (2)⟹ (1) now. Assume that each KS-closed set in $\mathcal{H}$ has an inverse image in $\mathcal{G}$ that is KS_{gsp}-closed. let $\mathcal{H}$ be any KS-closed set in $\mathcal{H}$. Then, in $\mathcal{H}$, $\mathcal{H}^c$ is KS-open. By (2), KS_{gsp} is open for $\upsilon^{-1}(\mathcal{H}^c)$. However, $\upsilon^{-1}(\mathcal{H}^c) = \mathcal{G} - \upsilon^{-1}(\mathcal{H})$ is KS_{gp}-open in the $\mathcal{G}$. Because of this, $\upsilon^{-1}(\mathcal{H})$ is KS_{gsp}-closed in $\mathcal{G}$. υ is thus KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gsp}-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-pre-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_g-continuous.

Proof: let $\mathfrak{S}$ be,any KS-closed set in $\mathcal{I}$. Ever $\hbar^{-1}(\mathfrak{S})$ is KS-pre-closed in $\mathcal{H}$, $\hbar$ is KS-pre-continuous. Even before υ is KS_{gsp}-continuous function, $\upsilon^{-1}(\hbar^{-1}(\mathfrak{S})) = (\hbar \circ \upsilon)^{-1}(\mathfrak{S})$. In the $\mathcal{G}$, is KS_g-closed. Because of this, $(\hbar \circ \upsilon)$ is hence KS_g-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_g-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_{gsp}-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \to (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-pre-continuous.

Proof: let $\mathfrak{S}$ be any KS-closed set in $\mathcal{I}$. Ever $\hbar$ is KS_{gsp}-continuous, $\hbar^{-1}(\mathfrak{S})$ is KS_{gsp}-closed in $\mathcal{H}$. Even before υ is KS_g-continuous, $\upsilon^{-1}(\hbar^{-1}(\mathfrak{S})) = (\hbar \circ \upsilon)^{-1}(\mathfrak{S})$ is

KS-pre-closed in $\mathcal{G}$. Because of this,$(\hbar \circ \upsilon)$ is KS-pre-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gsp}-continuous and $\hbar : (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \rightarrow (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-continuous, then $\hbar \circ \upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS-α-continuous.

Proof: let $\mathfrak{S}$ be any KS-closed set in $\mathcal{I}$. Ever $\hbar$ is KS-continuous, which means that $\hbar^{-1}(\mathfrak{S})$ is KS-closed in $\mathcal{H}$. Even before υ is a KS_{gsp}-continuous function, $\upsilon^{-1}(\hbar^{-1}(\mathfrak{S})) = (\hbar \circ \upsilon)^{-1}(\mathfrak{S})$ is KS-α-closed in $\mathcal{G}$. Because of this, $(\hbar \circ \upsilon)$ is KS-α-continuous.

Remark: If a map υ is KS_{gsp}-continuous and υ: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{G}), KS_{\mathcal{R}}(\mathbb{G})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ and $\hbar$: $(\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y})) \rightarrow (\mathcal{I}, \tau_{\mathcal{R}}(\mathbb{Z}), KS_{\mathcal{R}}(\mathbb{Z}))$ is KS_{gsp}-continuous, then $(\hbar \circ \upsilon)$: $(\mathcal{G}, \tau_R(X), KS_R(X)) \rightarrow \left(\mathcal{I}, \tau_R(\mathbb{Z}), KS_R(\mathbb{Z})\right)$ is KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y})), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gsp}-irresolute, then it is KS_{gsp}-continuous.

Proof: Let υ be the following: $(\mathcal{G}, \tau\mathcal{R}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ be KS_{gsp}-irresolute. Any KS-closed set in $\mathcal{H}$, and hence any set that is KS_{gsp}-closed in $\mathcal{H}$, is $\mathfrak{S}$. Given that υ is KS_{gsp}-irresolute, $\upsilon^{-1}(\mathcal{H})$ is KS_{gsp}-closed set in $\mathcal{G}$. υ is thus KS_{gsp}-continuous.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gsp}-irresolute, then $\upsilon(KS_{gspcl}(P)) \subseteq KS_{cl}\upsilon(P))$ for every subset P of $\mathcal{G}$.

Proof: P should be a subset of $\mathcal{G}$. Then, in $\mathcal{H}$, KS_{gsp}-closed on $KS_{cl}\upsilon(P)$. Considering that υ is KS_{gsp} -irresolute, $\upsilon^{-1}(KS_{gsp}cl(P))$ is KS_{gsp} -closed in $\mathcal{G}$. Additionally, P $\subseteq \upsilon^{-1}(\upsilon(P)) \subseteq \upsilon^{-1}(KS_{cl}\upsilon(P)$. Consequently, $KS_{gspcl}(P) \subseteq \upsilon^{-1}(KS_{cl}\upsilon(P))$. In light of this, $\upsilon(KS_{gsp}cl(P)) \subseteq KS_{cl}\upsilon(P))$.

Theorem: If a map $\upsilon : (\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is KS_{gsp}-irresolute, iff $\upsilon^{-1}(\mathcal{H})$ is KS_{gsp}-closed set in $\mathcal{G}$ for every KS_{gsp}-closed set in $\mathcal{H}$.

Proof: Consider the following scenario υ: $(\mathcal{G}, \tau_{\mathcal{R}}(\mathbb{X}), KS_{\mathcal{R}}(\mathbb{X})) \rightarrow (\mathcal{H}, \tau_{\mathcal{R}}(\mathbb{Y}), KS_{\mathcal{R}}(\mathbb{Y}))$ is a KS_{gsp}-irresolute, and $\mathbb{F}$ is KS_{gsp}-closed in $\mathcal{H}$. Then, in $\mathcal{H}$, F^c is KS_{gsp}-open. $\upsilon^{-1}(\mathbb{F}^c)$ is KS_{gsp}-closed in $\mathcal{G}$, by the definition of KS_{gsp}-irresolute. But $\upsilon^{-1}(\mathbb{F}^c) = \mathcal{G} - \upsilon^{-1}(\mathbb{F})$. As a result, in $\mathcal{G}$, $\upsilon(\mathbb{F})$ is KS_{gsp}-closed. On the other hand, imagine that $\upsilon^{-1}(\mathbb{F})$ is KS_{gsp}-closed in $\mathcal{G}$. For every KS_{gsp}-closed set G in $\mathcal{H}$. Let G be any KS_{gsp}-closed set in $\mathcal{H}$. By definition, $\upsilon^{-1}(\mathbb{F}^c)$ is KS_{gsp}-$\mathbb{C}$losed in the $\mathcal{G}$. However, $\upsilon^{-1}(\mathbb{F}^c) = \mathcal{G} - \upsilon^{-1}(\mathbb{F})$. Because $\upsilon^{-1}(\mathbb{F})$ is a KS_{gsp}-closed set in $\mathcal{G}$, $\mathcal{G} - \upsilon^{-1}(\mathbb{F})$ is also a KS_{gsp}-closed set in $\mathcal{G}$. υ is thus KS_{gsp}-irresolute.

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