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On $KS_{gs}(KS_{sg})$-Irresolute Functions in Kasaj Topological Spaces

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Abstract
The aim of this paper is to introduce a new class of continuous and irresolute functions called $KS_{gs}(KS_{sg})$ -continuous and $KS_{gs}(KS_{sg})$-irresolute functions in Kasaj Topological Spaces. Also we study some characterizations of $KS_{gs}(KS_{sg})$ -continuous and $KS_{gs}(KS_{sg})$-irresolute functions.

Keywords: $KS_{\mathfrak{R}}(\mathfrak{X})$, $KS_{\mathfrak{R}}(\mathfrak{Y})$, KS-semi-continuous, KS_g -continuous, KS_{gs} -continuous, KS_{sg} -continuous, KS-irresolute, KS-semi-irresolute, KS_g-irresolute, KS_{gs}-irresolute, KS_{sg}-irresolute

1. INTRODUCTION AND PRELIMINARIES

In 2020, Kashyap.G.Rachchh and Sajeed.I.Ghanchi [1] introduced partial extension of Micro Topological Space namely Kasaj Topological Spaces and the Kasaj-continuous functions. Kashyap.G.Rachchh, Sajeed.I.Ghanchi and Asfak A.Soneji [2] define new types of continuous functions namely Kasaj-pre-continuous function, Kasaj-semi-continuous function, Kasaj-α-continuous function and Kasaj-β- continuous function and in above year [2] studied about generalized closed set in Kasaj topological spaces. We defined and studied [4] of KS_{gs}-closed, KS_{sg}-closed sets in Kasaj Topological Space in 2022. A short while ago, Sathishmohan et al.[5] proposed and clarified the ideology of KS_{gp} and KS_{gsp}-closed sets in Kasaj Topological Spaces in 2022. The notions of irresolute functions in various Topological Spaces were investigated by various author in last few decades. This notions we to introduce the concept of irresolute functions in Kasaj Topological Spaces. The objective of this paper is to introduce and study about KS_{gs}-continuous and irresolute functions and KS_{sg}-continuous and irresolute functions and also study their properties. In this paper we use the following symbols Kasaj closed, Kasaj open, Kasaj Generalized Semi Closed, Kasaj Semi Generalized closed, Kasaj Generalized Semi open, Kasaj Semi Generalized open will be denoted as KS-$\mathfrak{C}$, KS-$\mathfrak{O}$, KS_{gs}-$\mathfrak{C}$, KS_{sg}-$\mathfrak{C}$, KS_{gs}-$\mathfrak{O}$ and KS_{sg}-$\mathfrak{O}$.

2. $KS_{gs}(KS_{sg})$-CONTINUOUS FUNCTIONS

In this section, we initiate a new class of functions, namely $KS_{gs}(KS_{sg})$-continuous functions in Kasaj Topological Spaces and study some of their properties. Also we investigate the relationships between the other existing Kasaj continuous functions.

Definition 2.1 *Let* $(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))$ *and* $(\mathfrak{J},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ *be two Kasaj Topological spaces. A function* $f:\mathfrak{I}\to\mathfrak{J}$ *is called*

1. KS_g-Continuous: if $f^{-1}(\mathfrak{D})\in KS_g$-$\mathfrak{C}$ set in $\mathfrak{I}$ whenever $\mathfrak{D}\in$ KS-$\mathfrak{C}$ set in $\mathfrak{J}$.
2. KS_{gs}-Continuous: if $f^{-1}(\mathfrak{D})\in KS_{gs}$-$\mathfrak{C}$ set in $\mathfrak{I}$ whenever $\mathfrak{D}\in$ KS-$\mathfrak{C}$ set in $\mathfrak{J}$.
3. KS_{sg}-Continuous: if $f^{-1}(\mathfrak{D})\in KS_{sg}$-$\mathfrak{C}$ set in $\mathfrak{I}$ whenever $\mathfrak{D}\in$ KS-$\mathfrak{C}$ set in $\mathfrak{J}$.

For a KS topological spaces $(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))$ and $(\mathfrak{J},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ the following conditions are hold

Theorem 2.2 *Every KS-continuous is* KS_{gs}*-continuous but not contrarily.*

Proof: Let $f:\mathfrak{I}\to\mathfrak{J}$ be KS-continuous. Let $\mathfrak{D}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Then $f^{-1}(\mathfrak{D})$ is KS-$\mathfrak{C}$ set in $\mathfrak{I}$. Whereas every KS-$\mathfrak{C}$ set is KS_{gs}-$\mathfrak{C}$ then $f^{-1}(\mathfrak{D})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. Consequently f is KS_{gs}-continuous.

Example 2.3 *Let* $\mathfrak{I}=\mathfrak{J}=\{\varepsilon,\varphi,\varpi,\varrho,\varsigma\}$ *with* $\mathfrak{I}/\mathfrak{R}=\{\{\varepsilon\},\{\varphi,\varpi,\varrho,\varsigma\}\}$ *and* $\mathfrak{X}=\{\varepsilon,\varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\mathfrak{I},\{\varepsilon\},\{\varphi,\varpi,\varrho,\varsigma\}\}$. *If we consider* $\mathfrak{S}=\{\varpi,\varrho\}$ $\mathfrak{S}'=\{\varepsilon,\varphi,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\{\varepsilon\},\{\varphi,\varsigma\},\{\varpi,\varrho\},\{\varepsilon,\varphi,\varsigma\},\{\varepsilon,\varpi,\varrho\},\{\varphi,\varpi,\varrho,\varsigma\},\mathfrak{I}\}$ *and* $\mathfrak{J}/\mathfrak{R}=\{\{\varepsilon,\varrho\},\{\varpi,\varsigma\},\{\varphi\}\}$ *and* $\mathfrak{X}=\{\varphi,\varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\mathfrak{J},\{\varphi\},\{\varepsilon,\varrho\},\{\varepsilon,\varphi,\varrho\},\}$. *If we consider* $\mathfrak{T}=\{\varepsilon,\varrho\}$ $\mathfrak{T}'=\{\varphi,\varpi,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\{\varphi\},\{\varepsilon,\varrho\},\{\varepsilon,\varphi,\varrho\},\{\varphi,\varpi,\varsigma\},\mathfrak{J}\}$. *Define* $f:(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))\to(\mathfrak{J},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon)=\varepsilon,f(\varphi)=\varpi,f(\varpi)=\varphi,f(\varrho)=\varsigma,f(\varsigma)=\varrho$. *Then* $\{\{\varepsilon,\varphi,\varrho,\varsigma\},\{\varpi,\varphi,\varrho\},\{\varphi,\varrho\},\{\varepsilon,\varsigma\}\}$ *is* KS_{gs}*-continuous but not KS-continuous.*

Theorem 2.4 *Every KS-semi continuous is* KS_{gs}*-continuous but not contrarily.*

Proof: Let $f:\mathfrak{I}\to\mathfrak{J}$ be KS-semi continuous. Let $\mathfrak{D}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Then $f^{-1}(\mathfrak{D})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. Whereas every KS-semi $\mathfrak{C}$ set is KS_{gs}-$\mathfrak{C}$ then $f^{-1}(\mathfrak{D})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$.Consequently f is KS_{gs}-continuous.

Example 2.5 *Let* $\mathfrak{I}=\mathfrak{J}=\{\varepsilon,\varphi,\varpi,\varrho,\varsigma\}$ *with* $\mathfrak{I}/\mathfrak{R}=\{\{\varpi,\varrho\},\{\varepsilon,\varphi,\varsigma\}\}$ *and* $\mathfrak{X}=\{\varepsilon,\varphi,\varpi,\varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\mathfrak{I},\{\varpi,\varrho\},\{\varepsilon,\varphi,\varsigma\}\}$. *If we consider* $\mathfrak{S}=\{\varsigma\}$ $\mathfrak{S}'=\{\varepsilon,\varphi,\varpi,\varrho\}$ $KS_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\{\varsigma\},\{\varepsilon,\varphi\},\{\varpi,\varrho\},\{\varepsilon,\varphi,\varsigma\},\{\varpi,\varrho,\varsigma\},\{\varepsilon,\varphi,\varpi,\varrho\},\mathfrak{I}\}$ *and* $\mathfrak{J}/\mathfrak{R}=\{\{\varpi\},\{\varepsilon,\varphi,\varrho,\varsigma\}\}$ *and* $\mathfrak{X}=\{\varepsilon,\varpi\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\mathfrak{J},\{\varpi\},\{\varepsilon,\varphi,\varrho,\varsigma\}\}$. *If we consider* $\mathfrak{T}=\{\varepsilon\}$ $\mathfrak{T}'=\{\varphi,\varpi,\varrho,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\{\varepsilon\},\{\varpi\},\{\varepsilon,\varpi\},\{\varphi,\varrho,\varsigma\},\{\varepsilon,\varphi,\varrho,\varsigma\},\{\varphi,\varpi,\varrho,\varsigma\},\mathfrak{J}\}$. *Define* $f:(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))\to(\mathfrak{J},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon)=\varphi,f(\varphi)=\varepsilon,f(\varpi)=\varpi,f(\varrho)=\varrho,f(\varsigma)=\varsigma$. *Then* $\{\{\varphi\},\{\varpi\},\{\varphi,\varpi\},\{\varepsilon,\varrho,\varsigma\},\{\varepsilon,\varphi,\varrho,\varsigma\},\{\varepsilon,\varpi,\varrho,\varsigma\}\}$ *is* KS_{gs}*-continuous but not KS-semi continuous.*

Theorem 2.6 *Every KS-*α*-continuous is* KS_{gs}*-continuous but not contrarily.*

Proof: Let $f:\mathfrak{I}\to\mathfrak{J}$ be KS-α continuous. Let $\mathfrak{D}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Then $f^{-1}(\mathfrak{D})$ is KS-α $\mathfrak{C}$ set in $\mathfrak{J}$. Whereas every KS-α $\mathfrak{C}$ set is KS_{gs}-$\mathfrak{C}$ then $f^{-1}(\mathfrak{D})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$.

Consequently f is KS_{gs}-continuous.

Example 2.7 *Let* $\mathfrak{J} = \mathfrak{J} = \{\varepsilon, \varphi, \varpi, \varrho, \varsigma\}$ *with* $\mathfrak{J}/\mathfrak{R} = \{\{\varepsilon\}, \{\varphi, \varpi, \varrho, \varsigma\}\}$ *and* $\mathfrak{X} = \{\varepsilon, \varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \mathfrak{J}, \{\varepsilon\}, \{\varphi, \varpi, \varrho, \varsigma\}\}$. *If we consider* $\mathfrak{S} = \{\varpi, \varrho\}$ $\mathfrak{S}' = \{\varepsilon, \varphi, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \{\varepsilon\}, \{\varphi, \varsigma\}, \{\varpi, \varrho\}, \{\varepsilon, \varphi, \varsigma\}, \{\varepsilon, \varpi, \varrho\}, \{\varphi, \varpi, \varrho, \varsigma\}, \mathfrak{J}\}$ *and* $\mathfrak{J}/\mathfrak{R} = \{\{\varphi, \varpi\}, \{\varrho, \varsigma\}, \{\varepsilon\}\}$ *and* $\mathfrak{X} = \{\varepsilon, \varphi, \varpi, \varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \{\varepsilon, \varphi, \varpi\}, \{\varrho, \varsigma\}, \mathfrak{J}\}\}$. *If we consider* $\mathfrak{T} = \{\varrho\}$ $\mathfrak{T}' = \{\varepsilon, \varphi, \varpi, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \{\varrho\}, \{\varsigma\}, \{\varrho, \varsigma\}, \{\varepsilon, \varphi, \varpi\}, \{\varepsilon, \varphi, \varpi, \varrho\}, \{\varepsilon, \varphi, \varpi, \varsigma\}, \mathfrak{J}\}$. *Define* $f: (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{X}), KS_{\mathfrak{R}}(\mathfrak{X})) \to (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{Y}), KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon) = \varepsilon, f(\varphi) = \varrho, f(\varpi) = \varsigma, f(\varrho) = \varpi, f(\varsigma) = \varphi$. *Then* $\{\{\varphi\}, \{\varpi\}, \{\varphi, \varpi\}, \{\varepsilon, \varrho, \varsigma\}, \{\varepsilon, \varphi, \varrho, \varsigma\}, \{\varepsilon, \varpi, \varrho, \varsigma\}\}$ *is* KS_{gs}*-continuous but not KS-*α *continuous.*

Theorem 2.8 *Every KS-continuous is* KS_{sg}*-continuous but not contrarily.*

Proof: Let $f: \mathfrak{J} \to \mathfrak{J}$ be KS-continuous. Let $\mathfrak{D}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Then $f^{-1}(\mathfrak{D})$ is KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Whereas every KS-$\mathfrak{C}$ set is KS_{sg}-$\mathfrak{C}$ then $f^{-1}(\mathfrak{D})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Consequently f is KS_{sg}-continuous.

Example 2.9 *Let* $\mathfrak{J} = \mathfrak{J} = \{\varepsilon, \varphi, \varpi, \varrho, \varsigma\}$ *with* $\mathfrak{J}/\mathfrak{R} = \{\{\varpi, \varsigma\}, \{\varepsilon, \varphi\}, \{\varrho\}\}$ *and* $\mathfrak{X} = \{\varepsilon, \varphi, \varpi\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \mathfrak{J}, \{\varepsilon, \varphi, \varpi, \varsigma\}, \{\varepsilon, \varphi\}, \{\varpi, \varsigma\}\}$. *If we consider* $\mathfrak{S} = \{\varepsilon, \varrho\}$ $\mathfrak{S}' = \{\varphi, \varpi, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \{\varepsilon\}, \{\varphi\}, \{\varepsilon, \varphi\}, \{\varepsilon, \varrho\}, \{\varpi, \varsigma\}, \{\varepsilon, \varphi, \varrho\}, \{\varepsilon, \varpi, \varsigma\}, \{\varphi, \varpi, \varsigma\}, \{\varepsilon, \varphi, \varpi, \varsigma\}, \{\varepsilon, \varpi, \varrho, \varsigma\}, \mathfrak{J}\}$ *and* $\mathfrak{J}/\mathfrak{R} = \{\{\varepsilon, \varrho\}, \{\varpi, \varsigma\}, \{\varphi\}\}$ *and* $\mathfrak{X} = \{\varphi, \varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \mathfrak{J}, \{\varphi\}, \{\varepsilon, \varrho\}, \{\varepsilon, \varphi, \varrho\}, \}$. *If we consider* $\mathfrak{T} = \{\varepsilon, \varrho\}$ $\mathfrak{T}' = \{\varphi, \varpi, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \{\varphi\}, \{\varepsilon, \varrho\}, \{\varepsilon, \varphi, \varrho\}, \{\varphi, \varpi, \varsigma\}, \mathfrak{J}\}$. *Define* $f: (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{X}), KS_{\mathfrak{R}}(\mathfrak{X})) \to (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{Y}), KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon) = \varepsilon, f(\varphi) = \varphi, f(\varpi) = \varpi, f(\varrho) = \varsigma, f(\varsigma) = \varrho$. *Then* $\{\{\varepsilon, \varsigma\}, \{\varpi, \varrho\}, \{\varphi, \varpi, \varrho\}, \{\varepsilon, \varpi, \varrho, \varsigma\}\}$ *is* KS_{sg}*-continuous but not KS-continuous.*

Theorem 2.10 *Every KS-*α*-continuous is* KS_{sg}*-continuous but not contrarily.*

Proof: Let $f: \mathfrak{J} \to \mathfrak{J}$ be KS-α continuous. Let $\mathfrak{D}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Then $f^{-1}(\mathfrak{D})$ is KS-α $\mathfrak{C}$ set in $\mathfrak{J}$. Whereas every KS-α $\mathfrak{C}$ set is KS_{sg}-$\mathfrak{C}$ then $f^{-1}(\mathfrak{D})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Consequently f is KS_{sg}-continuous.

Example 2.11 *Let* $\mathfrak{J} = \mathfrak{J} = \{\varepsilon, \varphi, \varpi, \varrho, \varsigma\}$ *with* $\mathfrak{J}/\mathfrak{R} = \{\{\varpi, \varrho\}, \{\varphi, \varsigma\}, \{\varepsilon\}\}$ *and* $\mathfrak{X} = \{\varepsilon, \varphi, \varpi, \varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \mathfrak{J}, \{\varepsilon, \varpi, \varrho\}, \{\varphi, \varsigma\}\}$. *If we consider* $\mathfrak{S} = \{\varphi\}$ $\mathfrak{S}' = \{\varepsilon, \varpi, \varrho, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \{\varphi\}, \{\varsigma\}, \{\varphi, \varsigma\}, \{\varepsilon, \varpi, \varrho\}, \{\varepsilon, \varphi, \varpi, \varrho\}, \{\varepsilon, \varpi, \varrho, \varsigma\}, \mathfrak{J}\}$ *and* $\mathfrak{J}/\mathfrak{R} = \{\{\varepsilon, \varrho\}, \{\varpi, \varsigma\}, \{\varphi\}\}$ *and* $\mathfrak{X} = \{\varphi, \varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \mathfrak{J}, \{\varphi\}, \{\varepsilon, \varrho\}, \{\varepsilon, \varphi, \varrho\}, \}$. *If we consider* $\mathfrak{T} = \{\varepsilon, \varrho\}$ $\mathfrak{T}' = \{\varphi, \varpi, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \{\varphi\}, \{\varepsilon, \varrho\}, \{\varepsilon, \varphi, \varrho\}, \{\varphi, \varpi, \varsigma\}, \mathfrak{J}\}$. *Define* $f: (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{X}), KS_{\mathfrak{R}}(\mathfrak{X})) \to (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{Y}), KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon) = \varphi, f(\varphi) = \varepsilon, f(\varpi) = \varrho, f(\varrho) = \varsigma, f(\varsigma) = \varpi$. *Then* $\{\{\varphi, \varpi\}, \{\varrho, \varsigma\}, \{\varepsilon, \varrho, \varsigma\}, \{\varphi, \varpi, \varrho, \varsigma\}\}$ *is* KS_{sg} *-continuous but not KS-*α*-continuous.*

Theorem 2.12 *Let* $(\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{X}), KS_{\mathfrak{R}}(\mathfrak{X}))$ *and* $(\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{Y}), KS_{\mathfrak{R}}(\mathfrak{Y}))$ *be two Kasaj topological spaces and* $\mathfrak{X} \subseteq \mathfrak{J}$ *and* $\mathfrak{Y} \subseteq \mathfrak{J}$. *Then* $f: \mathfrak{J} \to \mathfrak{J}$ *is* KS_{gs}*-continuous function iff* $f^{-1}(\mathfrak{D}) \in KS_{gs}$*-*$\mathfrak{C}$ *in* $\mathfrak{J}$ *whenever* $\mathfrak{D} \in$ *KS-*$\mathfrak{C}$ *in* $\mathfrak{J}$.

Proof: Let $f:\mathfrak{I} \to \mathfrak{I}$ is KS_{gs}-continuous function and $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{I}$. Then $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{I}$. Then $\mathfrak{D}^c \in KS_{\mathfrak{R}}(\mathfrak{Y})$. By hypothesis $f^{-1}(\mathfrak{D}^c) \in KS_{gs}$ in $\mathfrak{I}$, ie., $[f^{-1}(\mathfrak{D})]^c \in KS_{gs}$ in $\mathfrak{I}$. Hence $f^{-1}(\mathfrak{D}) \in KS_{gs}$ - $\mathfrak{C}$ in $\mathfrak{I}$. Whenever $\mathfrak{D} \in$ KS- $\mathfrak{C}$ in $\mathfrak{I}$. Contrarily, Suppose $[f^{-1}(\mathfrak{D})]^c \in KS_{gs}$ in $\mathfrak{I}$ whenever $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{I}$. Let $\mathfrak{D} \in KS_{\mathfrak{R}}(\mathfrak{Y})$ then $\mathfrak{D}^c \in$ KS-$\mathfrak{C}$ in $\mathfrak{I}$. By assumption $f^{-1}(\mathfrak{D}^c) \in KS_{gs}$ $\mathfrak{C}$ in $\mathfrak{I}$. Then $[f^{-1}(\mathfrak{D})]^c \in KS_{gs}$ in $\mathfrak{I}$. ie., $f^{-1}(\mathfrak{D}) \in KS_{gs}$-$\mathfrak{C}$ in $\mathfrak{I}$. Hence, f is KS_{gs}-continuous.

Theorem 2.13 *If $f:\mathfrak{I} \to \mathfrak{I}$ is KS_{gs}-continuous then $f(KS_{gs}cl(\mathfrak{F})) \subset cl(f(\mathfrak{F}))$ for every subset $\mathfrak{F}$ of $\mathfrak{I}$.*

Proof: Given $\mathfrak{F} \subset f^{-1}(f(\mathfrak{F}))$, we have $\mathfrak{F} \subset f^{-1}(cl(f(\mathfrak{F}))$ Now, $cl(f(\mathfrak{F}))$ is KS-$\mathfrak{C}$ set in $\mathfrak{I}$ and Hence, $f^{-1}(cl(f(\mathfrak{F}))$ is KS_{gs}-$\mathfrak{C}$ set containing $\mathfrak{F}$. Consequently, $KS_{gs}cl(\mathfrak{F}) \subset cl(f(\mathfrak{F}))$.

Theorem 2.14 *A function $f:\mathfrak{I} \to \mathfrak{I}$ is KS_{gs}-continuous iff the inverse image of every KS-$\mathfrak{C}$ set in $\mathfrak{I}$ is KS_{gs}-$\mathfrak{C}$ set in $\mathfrak{I}$.*

Proof: Let f be KS_{gs}-continuous and $\mathfrak{F}$ be KS-$\mathfrak{C}$ set in $\mathfrak{I}$. ie., $\mathfrak{I}$-$\mathfrak{F}$ is KS-$\mathfrak{O}$ set in $\mathfrak{I}$. Whereas f is KS_{gs}-continuous, $f^{-1}(\mathfrak{I} - \mathfrak{F})$ is KS_{gs}-$\mathfrak{O}$ set in $\mathfrak{I}$. ie., $f^{-1}(\mathfrak{I}) - f^{-1}(\mathfrak{F}) = \mathfrak{I} - f^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{O}$ in $\mathfrak{I}$. Hence, $f^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$, if f is KS_{gs}-continuous on $\mathfrak{I}$. Contrarily, Let the inverse image of every KS-$\mathfrak{C}$ set in $\mathfrak{I}$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. Let $\mathfrak{G}$ be a KS-$\mathfrak{O}$ set in $\mathfrak{I}$. Then $\mathfrak{I}$-$\mathfrak{G}$ is KS-$\mathfrak{C}$ set in $\mathfrak{I}$. Then $f^{-1}(\mathfrak{I} - \mathfrak{G})$ is KS_{gs}-$\mathfrak{C}$ set in $\mathfrak{I}$. i.e., $f^{-1}(\mathfrak{I}) - f^{-1}(\mathfrak{G}) = \mathfrak{I} - f^{-1}(\mathfrak{G})$ is KS_{gs}-$\mathfrak{C}$ set in $\mathfrak{I}$. Consequently $f^{-1}(\mathfrak{G})$ is KS_{gs}-$\mathfrak{O}$ set in $\mathfrak{I}$. Thus the inverse image of every KS-$\mathfrak{O}$ set in $\mathfrak{I}$ is KS_{gs}-$\mathfrak{O}$ set in $\mathfrak{I}$. ie., f is KS_{gs}-continuous on $\mathfrak{I}$.

Theorem 2.15 *If ($\mathfrak{I}$,$\tau_{\mathfrak{R}}(\mathfrak{X})$,$KS_{\mathfrak{R}}(\mathfrak{X})$) and ($\mathfrak{I}$,$\tau_{\mathfrak{R}}(\mathfrak{Y})$, $KS_{\mathfrak{R}}(\mathfrak{Y})$) are KS-topological spaces with respect to $\mathfrak{X} \subseteq \mathfrak{I}$ and $\mathfrak{Y} \subseteq \mathfrak{I}$ respectively, then any function $f:\mathfrak{I} \to \mathfrak{I}$, the following conditions are equivalent.*

1. f is KS_{gs}-continuous.
2. $f(KS_{gs}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$ for every subset $\mathfrak{P}$ of I.
3. $KS_{gs}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q}))$ for every subset $\mathfrak{Q}$ of $\mathfrak{I}$.

Proof: $1 \Rightarrow 2$ Let f be KS_{gs}-continuous and $\mathfrak{P} \subseteq \mathfrak{I}$. Then $f(\mathfrak{P}) \subseteq \mathfrak{I}$. Whereas f is KS_{gs}-continuous and $KScl(f(\mathfrak{P}))$ is KS-$\mathfrak{C}$ in $\mathfrak{I}$, $f^{-1}(KScl(f(\mathfrak{P})))$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. Whereas $f(\mathfrak{P}) \subseteq KScl(f(\mathfrak{P}))$, $f^{-1}(f(\mathfrak{P})) \subseteq f^{-1}(KScl(f(\mathfrak{P})))$, then $KS_{gs}cl(\mathfrak{P}) \subseteq KS_{gs}cl(f^{-1}(KScl(f(\mathfrak{P})))) = f^{-1}(KScl(f(\mathfrak{P})))$. Thus $KS_{gs}cl(\mathfrak{P}) = f^{-1}(KScl(f(\mathfrak{P})))$. Consequently $f(KS_{gs}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$ for every subset $\mathfrak{P}$ of $\mathfrak{I}$.

$2 \Rightarrow 3$ Let $f(KS_{gs}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$ and $\mathfrak{P} = f^{-1}(\mathfrak{Q}) \subseteq \mathfrak{I}$ for every subset $\mathfrak{Q} \subseteq \mathfrak{I}$. Whereas $f(KS_{gs}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$. We have $f(KS_{gs}cl(f^{-1}(\mathfrak{Q}))) \subseteq$

$KScl(f(f^{-1}(\mathfrak{Q}))) \subseteq KScl(f(\mathfrak{Q})$. ie., $f(KS_{gs}(f^{-1}(\mathfrak{Q}))) \subseteq KScl(\mathfrak{Q})$. This implies that $KS_{gs}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q}))$ for every subset $\mathfrak{Q}$ of $\mathfrak{I}$.

$3 \Rightarrow 1$ Let $KS_{gs}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q}))$ for every subset $\mathfrak{Q}$ of $\mathfrak{I}$. If $\mathfrak{Q}$ is KS-$\mathfrak{C}$ in $\mathfrak{I}$, then $KScl(\mathfrak{Q}) = \mathfrak{Q}$. By assumption, $KS_{gs}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q})) = f^{-1}(\mathfrak{Q})$. Thus $KS_{gs}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(\mathfrak{Q})$. But $f^{-1}(\mathfrak{Q}) \subseteq KS_{gs}cl(f^{-1}(\mathfrak{Q}))$. Consequently $KS_{gs}cl(f^{-1}(\mathfrak{Q})) = f^{-1}(\mathfrak{Q})$. ie., $f^{-1}(\mathfrak{Q})$ is KS_{gs}-$\mathfrak{C}$ $\mathfrak{I}$ for every KS-$\mathfrak{C}$ set $\mathfrak{Q}$ in $\mathfrak{I}$. Consequently f is KS_{gs}-continuous on $\mathfrak{I}$.

Remark 2.16 *The composition of two KS_{gs}-continuous functions is again a KS_{gs}-continuous functions.*

Theorem 2.17 *If $f{:}\,\mathfrak{I} \to \mathfrak{J}$ is KS_{gs}-continuous and $g{:}\,\mathfrak{J} \to \mathfrak{K}$ is KS-continuous, then their composition $g \circ f{:}\,\mathfrak{I} \to \mathfrak{K}$ is KS_{gs}-continuous.*

Proof: Let $\mathfrak{F}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{K}$. Whereas g is KS-continuous, $g^{-1}(\mathfrak{F})$ is KS-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{gs}-continuous and $g^{-1}(\mathfrak{F})$ is in $\mathfrak{J}$. But KS-$\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(\mathfrak{F})) = (g \circ f)^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$ and $g \circ f$ is KS_{gs}-continuous.

Theorem 2.18 *If $f{:}\,\mathfrak{I} \to \mathfrak{J}$ is KS_{gs}-continuous and $g{:}\,\mathfrak{J} \to \mathfrak{K}$ is KS-semi continuous, then their composition $g \circ f{:}\,\mathfrak{I} \to \mathfrak{K}$ is KS_{gs}-continuous.*

Proof: Let $\mathfrak{F}$ be any KS-semi $\mathfrak{C}$ set in $\mathfrak{K}$. Whereas g is KS-semi continuous, $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{gs}-continuous and $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(\mathfrak{F})) = (g \circ f)^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$ and $g \circ f$ is KS_{gs}-continuous.

Theorem 2.19 *If $f{:}\,\mathfrak{I} \to \mathfrak{J}$ is KS_{gs}-continuous and $g{:}\,\mathfrak{J} \to \mathfrak{K}$ is KS-semi continuous, then their composition $g \circ f{:}\,\mathfrak{I} \to \mathfrak{K}$ is KS_g-continuous.*

Proof: Let $\mathfrak{F}$ be any KS-semi $\mathfrak{C}$ set in $\mathfrak{K}$. Whereas g is KS-semi continuous, $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{gs}-continuous and $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(\mathfrak{F})) = (g \circ f)^{-1}(\mathfrak{F})$ is KS_g-$\mathfrak{C}$ in $\mathfrak{I}$ and $g \circ f$ is KS_g-continuous.

Theorem 2.20 *Let $(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))$ and $(\mathfrak{J},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ be two Kasaj topological spaces and $\mathfrak{X} \subseteq \mathfrak{I}$ and $\mathfrak{Y} \subseteq \mathfrak{J}$. Then $f{:}\,\mathfrak{I} \to \mathfrak{J}$ is KS_{sg}-continuous function iff $f^{-1}(\mathfrak{D}) \in KS_{sg}$-$\mathfrak{C}$ in $\mathfrak{I}$ whenever $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{J}$.*

Proof: Let $f{:}\,\mathfrak{I} \to \mathfrak{J}$ is KS_{sg}-continuous function and $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{J}$. Then $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{J}$. Then $\mathfrak{D}^c \in KS_{\mathfrak{R}}(\mathfrak{Y})$. By hypothesis $f^{-1}(\mathfrak{D}^c) \in KS_{sg}$ in $\mathfrak{I}$, ie., $[f^{-1}(\mathfrak{D})]^c \in KS_{sg}$ in $\mathfrak{I}$. Hence $f^{-1}(\mathfrak{D}) \in KS_{sg}$-$\mathfrak{C}$ in $\mathfrak{I}$. Whenever $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{J}$. Contrarily, Suppose $[f^{-1}(\mathfrak{D})]^c \in KS_{sg}$ in $\mathfrak{I}$ whenever $\mathfrak{D} \in$ KS-$\mathfrak{C}$ in $\mathfrak{J}$. Let $\mathfrak{D} \in KS_{\mathfrak{R}}(\mathfrak{Y})$ then $\mathfrak{D}^c \in$ KS-$\mathfrak{C}$ in $\mathfrak{J}$. By assumption $f^{-1}(\mathfrak{D}^c) \in KS_{sg}$ $\mathfrak{C}$ in $\mathfrak{I}$. Then $[f^{-1}(\mathfrak{D})]^c \in KS_{sg}$ in $\mathfrak{I}$. ie., $f^{-1}(\mathfrak{D}) \in KS_{sg}$-$\mathfrak{C}$ in $\mathfrak{I}$. Hence,f is KS_{sg}-continuous.

Theorem 2.21 *If $f{:}\,\mathfrak{I} \to \mathfrak{J}$ is KS_{sg}-continuous then $f(KS_{sg}cl(\mathfrak{F})) \subset cl(f(\mathfrak{F}))$ for every* *subset* $\mathfrak{F}$ *of* $\mathfrak{J}$.

Proof: Let $\mathfrak{F}$ be a subset of $\mathfrak{J}$ such that $f(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas $\mathfrak{F} \subset f^{-1}(f(\mathfrak{F}))$, we have $\mathfrak{F} \subset f^{-1}(cl(f(\mathfrak{F}))$. Now, $cl(f(\mathfrak{F}))$ is KS-$\mathfrak{C}$ set in $\mathfrak{J}$ and hence, $f^{-1}(cl(f(\mathfrak{F}))$ is KS_{sg}-$\mathfrak{C}$ set containing $\mathfrak{F}$. Consequently, $KS_{sg}cl(\mathfrak{F}) \subset cl(f(\mathfrak{F}))$.

Theorem 2.22 *A function* $f: \mathfrak{J} \to \mathfrak{J}$ *is* KS_{sg}*-continuous iff the inverse image of every KS-*$\mathfrak{C}$ *set in* $\mathfrak{J}$ *is* KS_{sg}-$\mathfrak{C}$ *set in* $\mathfrak{J}$.

Proof: Let f be KS_{sg}-continuous and $\mathfrak{F}$ be KS-$\mathfrak{C}$ set in $\mathfrak{J}$. ie., $\mathfrak{J}$-$\mathfrak{F}$ is KS-$\mathfrak{O}$ set in $\mathfrak{J}$. Whereas f is KS_{sg}-continuous, $f^{-1}(\mathfrak{J} - \mathfrak{F})$ is KS_{sg}-$\mathfrak{O}$ set in $\mathfrak{J}$. ie., $f^{-1}(\mathfrak{J}) - f^{-1}(\mathfrak{F}) = \mathfrak{J} - f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{O}$ in $\mathfrak{J}$. Hence, $f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$, if f is KS_{sg}-continuous on $\mathfrak{J}$. Contrarily, Let the inverse image of every KS-$\mathfrak{C}$ set in $\mathfrak{J}$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Let $\mathfrak{G}$ be a KS-$\mathfrak{O}$ set in $\mathfrak{J}$. Then $\mathfrak{J}$-$\mathfrak{G}$ is KS-$\mathfrak{C}$ set in $\mathfrak{J}$. Then $f^{-1}(\mathfrak{J} - \mathfrak{G})$ is KS_{sg}-$\mathfrak{C}$ set in $\mathfrak{J}$. i.e., $f^{-1}(\mathfrak{J}) - f^{-1}(\mathfrak{G}) = \mathfrak{J} - f^{-1}(\mathfrak{G})$ is KS_{sg}-$\mathfrak{C}$ set in $\mathfrak{J}$. Consequently $f^{-1}(\mathfrak{G})$ is KS_{sg}-$\mathfrak{O}$ set in $\mathfrak{J}$. Thus the inverse image of every KS-$\mathfrak{O}$ set in $\mathfrak{J}$ is KS_{sg}-$\mathfrak{O}$ set in $\mathfrak{J}$. ie., f is KS_{sg}-continuous on $\mathfrak{J}$.

Theorem 2.23 *If* ($\mathfrak{J}$,$\tau_{\mathfrak{R}}(\mathfrak{X})$,$KS_{\mathfrak{R}}(\mathfrak{X})$) *and* ($\mathfrak{J}$,$\tau_{\mathfrak{R}}(\mathfrak{Y})$,$KS_{\mathfrak{R}}(\mathfrak{Y})$) *are KS-topological spaces with respect to* $\mathfrak{X} \subseteq \mathfrak{J}$ *and* $\mathfrak{Y} \subseteq \mathfrak{J}$ *respectively, then any function* $f: \mathfrak{J} \to \mathfrak{J}$, *the following conditions are equivalent.*

1. f is KS_{sg}-continuous.
2. $f(KS_{sg}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$ for every subset $\mathfrak{P}$ of $\mathfrak{J}$.
3. $KS_{sg}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q}))$ for every subset $\mathfrak{Q}$ of $\mathfrak{J}$.

Proof: $1 \Rightarrow 2$ Let f be KS_{sg}-continuous and $\mathfrak{P} \subseteq \mathfrak{J}$. Then $f(\mathfrak{P}) \subseteq \mathfrak{J}$. Whereas f is KS_{sg}-continuous and $KScl(f(\mathfrak{P}))$ is KS-$\mathfrak{C}$ in $\mathfrak{J}$, $f^{-1}(KScl(f(\mathfrak{P})))$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas $f(\mathfrak{P}) \subseteq KScl(f(\mathfrak{P}))$, $f^{-1}(f(\mathfrak{P})) \subseteq f^{-1}(KScl(f(\mathfrak{P})))$, then $KS_{sg}cl(\mathfrak{P}) \subseteq KS_{sg}cl(f^{-1}(KScl(f(\mathfrak{P})))) = f^{-1}(KScl(f(\mathfrak{P})))$. Thus $KS_{sg}cl(\mathfrak{P}) = f^{-1}(KScl(f(\mathfrak{P})))$. Consequently $f(KS_{sg}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$ for every subset $\mathfrak{P}$ of $\mathfrak{J}$.

$2 \Rightarrow 3$ Let $f(KS_{sg}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$ and $\mathfrak{P} = f^{-1}(\mathfrak{Q}) \subseteq \mathfrak{J}$ for every subset $\mathfrak{Q} \subseteq \mathfrak{J}$. Whereas $f(KS_{sg}cl(\mathfrak{P})) \subseteq KScl(f(\mathfrak{P}))$. We have $f(KS_{sg}cl(f^{-1}(\mathfrak{Q}))) \subseteq KScl(f(f^{-1}(\mathfrak{Q}))) \subseteq KScl(f(\mathfrak{Q})$. ie., $f(KS_{sg}(f^{-1}(\mathfrak{Q}))) \subseteq KScl(\mathfrak{Q})$. This implies that $KS_{sg}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q}))$ for every subset $\mathfrak{Q}$ of $\mathfrak{J}$.

$3 \Rightarrow 1$ Let $KS_{sg}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q}))$ for every subset $\mathfrak{Q}$ of $\mathfrak{J}$. If $\mathfrak{Q}$ is KS-$\mathfrak{C}$ in $\mathfrak{J}$, then $KScl(\mathfrak{Q}) = \mathfrak{Q}$. By assumption,$KS_{sg}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(KScl(\mathfrak{Q})) = f^{-1}(\mathfrak{Q})$. Thus $KS_{sg}cl(f^{-1}(\mathfrak{Q})) \subseteq f^{-1}(\mathfrak{Q})$. But $f^{-1}(\mathfrak{Q}) \subseteq KS_{sg}cl(f^{-1}(\mathfrak{Q}))$. Consequently $KS_{sg}cl(f^{-1}(\mathfrak{Q})) = f^{-1}(\mathfrak{Q})$. ie., $f^{-1}(\mathfrak{Q})$ is KS_{sg}-$\mathfrak{C}$ $\mathfrak{J}$ for every KS-$\mathfrak{C}$ set $\mathfrak{Q}$ in $\mathfrak{J}$. Consequently f is KS_{sg}-continuous on $\mathfrak{J}$.

Remark 2.24 *The composition of two KS_{sg}-continuous functions is again a KS_{sg}-continuous functions.*

Theorem 2.25 *If $f: \mathfrak{J} \to \mathfrak{J}$ is KS_{sg}-continuous and $g: \mathfrak{J} \to \mathfrak{K}$ is KS-continuous, then their composition $g \circ f: \mathfrak{J} \to \mathfrak{K}$ is KS_{sg}-continuous.*

Proof: Let $\mathfrak{F}$ be any KS-$\mathfrak{C}$ set in $\mathfrak{K}$. Whereas g is KS-continuous, $g^{-1}(\mathfrak{F})$ is KS-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{sg}-continuous and $g^{-1}(\mathfrak{F})$ is in $\mathfrak{J}$. But KS-$\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(\mathfrak{F})) = (g \circ f)^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$ and $g \circ f$ is KS_{sg}-continuous.

Theorem 2.26 *If $f: \mathfrak{J} \to \mathfrak{J}$ is KS_{sg}-continuous and $g: \mathfrak{J} \to \mathfrak{K}$ is KS-semi continuous, then their composition $g \circ f: \mathfrak{J} \to \mathfrak{K}$ is KS_{sg}-continuous.*

Proof: Let $\mathfrak{F}$ be any KS-semi $\mathfrak{C}$ set in $\mathfrak{K}$. Whereas g is KS-semi continuous, $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{sg}-continuous and $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$ and $g \circ f$ is KS_{sg}-continuous.

Theorem 2.27 *If $f: \mathfrak{J} \to \mathfrak{J}$ is KS_{sg}-continuous and $g: \mathfrak{J} \to \mathfrak{K}$is KS-semi continuous, then their composition $g \circ f: \mathfrak{J} \to \mathfrak{K}$ is KS_g-continuous.*

Proof: Let $\mathfrak{F}$ be any KS-semi $\mathfrak{C}$ set in $\mathfrak{K}$. Whereas g is KS-semi continuous, $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{sg}-continuous and $g^{-1}(\mathfrak{F})$ is KS-semi $\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is KS_g-$\mathfrak{C}$ in U and $g \circ f$ is KS_g-continuous.

3. $\boldsymbol{KS_{gs}}(\boldsymbol{KS_{sg}})$-IRRESOLUTE FUNCTIONS

In this section, we discuss a new class of functions, namely $KS_{gs}(KS_{sg})$-irresolute functions in Kasaj Topological Spaces and study some of their properties.

Definition 3.1 *A function $f: (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{X}), KS_{\mathfrak{R}}(\mathfrak{X})) \to (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{Y}), KS_{\mathfrak{R}}(\mathfrak{Y}))$ is called*

1. KS-irresolute: if $f^{-1}(\mathfrak{D}) \in$ KS-semi $\mathfrak{C}$ set in $\mathfrak{J}$ whenever $\mathfrak{D} \in$ KS-semi $\mathfrak{C}$ set in $\mathfrak{J}$.
2. KS_g-irresolute: if $f^{-1}(\mathfrak{D}) \in KS_g$-$\mathfrak{C}$ set in $\mathfrak{J}$ whenever $\mathfrak{D} \in KS_g$-$\mathfrak{C}$ set in $\mathfrak{J}$
3. KS_{gs}-irresolute: if $f^{-1}(\mathfrak{D}) \in KS_{gs}$-$\mathfrak{C}$ set in $\mathfrak{J}$ whenever $\mathfrak{D} \in KS_{gs}$-$\mathfrak{C}$ set in $\mathfrak{J}$.
4. KS_{sg}-irresolute: if $f^{-1}(\mathfrak{D}) \in KS_{sg}$-$\mathfrak{C}$ set in $\mathfrak{J}$ whenever $\mathfrak{D} \in KS_{sg}$-$\mathfrak{C}$ set in $\mathfrak{J}$.

Example 3.2 *Let $\mathfrak{J} = \mathfrak{J} = \{\varepsilon, \varphi, \varpi, \varrho, \varsigma\}$ with $\mathfrak{J}/\mathfrak{R} = \{\{\varpi, \varrho\}, \{\varepsilon, \varphi, \varsigma\}\}$ and $\mathfrak{X} = \{\varepsilon, \varphi, \varpi, \varrho\}$. Then $\tau_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \mathfrak{J}, \{\varpi, \varrho\}, \{\varepsilon, \varphi, \varsigma\}\}$. If we consider $\mathfrak{S} = \{\varsigma\}$ $\mathfrak{S}' = \{\varepsilon, \varphi, \varpi, \varrho\}$ $KS_{\mathfrak{R}}(\mathfrak{X}) = \{\phi, \{\varsigma\}, \{\varepsilon, \varphi\}, \{\varpi, \varrho\}, \{\varepsilon, \varphi, \varsigma\}, \{\varpi, \varrho, \varsigma\}, \{\varepsilon, \varphi, \varpi, \varrho\}, \mathfrak{J}\}$. and $\mathfrak{J}/\mathfrak{R} = \{\{\varrho, \varsigma\}, \{\varepsilon, \varphi, \varpi\}\}$ and $\mathfrak{X} = \{\varepsilon, \varphi, \varpi, \varrho\}$. Then $\tau_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \mathfrak{J}, \{\varepsilon, \varphi, \varpi\}, \{\varrho, \varsigma\}\}$. If we consider $\mathfrak{T} = \{\varpi\}$ $\mathfrak{T}' = \{\varepsilon, \varphi, \varrho, \varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y}) = \{\phi, \{\varpi\}, \{\varepsilon, \varphi\}, \{\varrho, \varsigma\}\{\varepsilon, \varphi, \varpi\}, \{\varpi, \varrho, \varsigma\}, \{\varepsilon, \varphi, \varrho, \varsigma\}, \mathfrak{J}\}$. Define $f: (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{X}), KS_{\mathfrak{R}}(\mathfrak{X})) \to (\mathfrak{J}, \tau_{\mathfrak{R}}(\mathfrak{Y}), KS_{\mathfrak{R}}(\mathfrak{Y}))$ as $f(\varepsilon) = \varsigma, f(\varphi) = \varrho, f(\varpi) = \varphi, f(\varrho) = \varepsilon, f(\varsigma) = \varpi$. Then f is KS-irresolute.*

Example 3.3 *Let $\mathfrak{J} = \mathfrak{J} = \{\varepsilon, \varphi, \varpi, \varrho, \varsigma\}$ with $\mathfrak{J}/\mathfrak{R} = \{\{\varepsilon, \varpi\}, \{\varphi, \varrho\}, \{\varsigma\}\}$ and $\mathfrak{X} =$*

$\{\varepsilon,\varphi,\varpi\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\mathfrak{I},\{\varepsilon,\varpi\},\{\varepsilon,\varphi,\varpi,\varrho\},\{\varphi,\varrho\}\}$. *If we consider* $\mathfrak{S}=\{\varpi\}$ $\mathfrak{S}'=\{\varepsilon,\varphi,\varrho,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\{\varepsilon\},\{\varpi\},\{\varepsilon,\varpi\},\{\varphi,\varrho\},\{\varepsilon,\varphi,\varrho\},\{\varphi,\varpi,\varrho\},\{\varepsilon,\varphi,\varpi,\varrho\}\},\{\varepsilon,\varphi,\varrho,\varsigma\},\mathfrak{I}\}$ *and* $\mathfrak{I}/\mathfrak{R}=\{\{\varepsilon,\varrho\},\{\varpi,\varsigma\},\{\varphi\}\}$ *and* $\mathfrak{X}=\{\varphi,\varsigma\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\mathfrak{I},\{\varphi\},\{\varphi,\varpi,\varsigma\},\{\varpi,\varsigma\}\}$. *If we consider* $\mathfrak{T}=\{\varphi,\varpi\}$ $\mathfrak{T}'=\{\varepsilon,\varrho,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\{\varphi\},\{\varpi\},\{\varsigma\},\{\varphi,\varpi\},\{\varphi,\varsigma\},\{\varpi,\varsigma\},\{\varphi,\varpi,\varsigma\},\{\varepsilon,\varrho,\varsigma\},\{\varepsilon,\varphi,\varpi\},\{\varepsilon,\varphi,\varrho,\varsigma\},\{\varepsilon,\varpi,\varrho,\varsigma\},\mathfrak{I}\}$. *Define* $f:(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))\to(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon)=\varphi,f(\varphi)=\varepsilon,f(\varpi)=\varpi,f(\varrho)=\varsigma,f(\varsigma)=\varrho$. *Then f is* KS_g*-irresolute.*

Example 3.4 *Let* $\mathfrak{I}=\mathfrak{I}=\{\varepsilon,\varphi,\varpi,\varrho,\varsigma\}$ *with* $\mathfrak{I}/\mathfrak{R}=\{\{\varpi,\varrho\},\{\varphi,\varsigma\},\{\varepsilon\}\}$ *and* $\mathfrak{X}=\{\varepsilon,\varsigma\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\mathfrak{I},\{\varepsilon\},\{\varepsilon,\varphi,\varsigma\},\{\varphi,\varsigma\}\}$. *If we consider* $\mathfrak{S}=\{\varepsilon,\varsigma\}$ $\mathfrak{S}'=\{\varphi,\varpi,\varrho\}$ $KS_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\{\varepsilon\},\{\varphi\},\{\varepsilon,\varphi\},\{\varepsilon,\varsigma\},\{\varepsilon,\varphi,\varsigma\},\{\varphi,\varpi,\varrho\},\{\varepsilon,\varphi,\varpi,\varrho\},\mathfrak{I}\}$ *and* $\mathfrak{I}/\mathfrak{R}=\{\{\varepsilon,\varrho\},\{\varpi,\varsigma\},\{\varphi\}\}$ *and* $\mathfrak{X}=\{\varphi,\varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\mathfrak{I},\{\varphi\},\{\varepsilon,\varphi,\varrho\},\{\varepsilon,\varrho\}\}$. *If we consider* $\mathfrak{T}=\{\varepsilon,\varrho\}$ $\mathfrak{T}'=\{\varphi,\varpi,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\{\varphi\},\{\varepsilon,\varrho\},\{\varepsilon,\varphi,\varrho\},\{\varphi,\varpi,\varsigma\},\mathfrak{I}\}$. *Define* $f:(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))\to(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon)=\varphi,f(\varphi)=\varepsilon,f(\varpi)=\varpi,f(\varrho)=\varrho,f(\varsigma)=\varsigma$. *Then f is* KS_{gs}*-irresolute.*

Example 3.5 *Let* $\mathfrak{I}=\mathfrak{I}=\{\varepsilon,\varphi,\varpi,\varrho,\varsigma\}$ *with* $\mathfrak{I}/\mathfrak{R}=\{\{\varepsilon,\varpi\},\{\varphi,\varrho\},\{\varsigma\}\}$ *and* $\mathfrak{X}=\{\varepsilon,\varphi,\varpi\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\mathfrak{I},\{\varepsilon,\varpi\},\{\varepsilon,\varphi,\varpi,\varrho\},\{\varphi,\varrho\}\}$. *If we consider* $\mathfrak{S}=\{\varpi\}$ $\mathfrak{S}'=\{\varepsilon,\varphi,\varrho,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{X})=\{\phi,\{\varepsilon\},\{\varpi\},\{\varepsilon,\varpi\},\{\varphi,\varrho\},\{\varepsilon,\varphi,\varrho\},\{\varphi,\varpi,\varrho\},\{\varepsilon,\varphi,\varpi,\varrho\}\},\{\varepsilon,\varphi,\varrho,\varsigma\},\mathfrak{I}\}$ *and* $V/\mathfrak{R}=\{\{\varepsilon,\varrho\},\{\varpi,\varsigma\},\{\varphi\}\}$ *and* $\mathfrak{X}=\{\varphi,\varrho\}$. *Then* $\tau_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\mathfrak{I},\{\varphi\},\{\varepsilon,\varphi,\varrho\},\{\varepsilon,\varrho\}\}$. *If we consider* $\mathfrak{T}=\{\varepsilon,\varrho\}$ $\mathfrak{T}'=\{\varphi,\varpi,\varsigma\}$ $KS_{\mathfrak{R}}(\mathfrak{Y})=\{\phi,\{\varphi\},\{\varepsilon,\varrho\},\{\varepsilon,\varphi,\varrho\},\{\varphi,\varpi,\varsigma\},\mathfrak{I}\}$. *Define* $f:(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{X}),KS_{\mathfrak{R}}(\mathfrak{X}))\to(\mathfrak{I},\tau_{\mathfrak{R}}(\mathfrak{Y}),KS_{\mathfrak{R}}(\mathfrak{Y}))$ *as* $f(\varepsilon)=\varphi,f(\varphi)=\varepsilon,f(\varpi)=\varrho,f(\varrho)=\varpi,f(\varsigma)=\varsigma$. *Then f is* KS_{sg}*-irresolute.*

Theorem 3.6 *If a function* $f:\mathfrak{I}\to\mathfrak{I}$ *is* KS_{gs}*-irresolute, then it is* KS_{gs}*-continuous.*

Proof: Let $\mathfrak{F}$ be a closed set in $\mathfrak{I}$. We have every closed set is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. Whereas f is KS_{gs}-irresolute, $f^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ set in $\mathfrak{I}$. Consequently for a closed set $\mathfrak{F}$ in $\mathfrak{I}$, $f^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ set in $\mathfrak{I}$. So, f is KS_{gs}-continuous.

Theorem 3.7 *A function* $f:\mathfrak{I}\to\mathfrak{I}$ *is* KS_{gs}*-irresolute iff for every* KS_{gs}-$\mathfrak{O}$ *subset* $\mathfrak{F}$ *of* $\mathfrak{I}$, $f^{-1}(\mathfrak{F})$ *is* KS_{gs}-$\mathfrak{O}$ *in* $\mathfrak{I}$.

Proof: Necessity: If $f:\mathfrak{I}\to\mathfrak{I}$ be KS_{gs}-irresolute, then for every KS_{gs}-$\mathfrak{C}$ subset $\mathfrak{G}$ of $\mathfrak{I}$, $f^{-1}(\mathfrak{G})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. If $\mathfrak{F}$ is any KS_{gs}-$\mathfrak{O}$ subset of $\mathfrak{I}$ then $\mathfrak{I}$-$\mathfrak{F}$ is KS_{gs}-$\mathfrak{C}$. Thus $f^{-1}(\mathfrak{I}-\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$, but $f^{-1}(\mathfrak{I}-\mathfrak{F})=\mathfrak{I}-f^{-1}(\mathfrak{F})$ so that $f^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{O}$ in $\mathfrak{I}$.

Sufficiency: If for all KS_{gs}-$\mathfrak{O}$ subsets F of $\mathfrak{I}$, $f^{-1}(F)$ is KS_{gs}-$\mathfrak{O}$ in $\mathfrak{I}$ and if $\mathfrak{G}$ is any KS_{gs}-$\mathfrak{C}$ subset of $\mathfrak{I}$, then $\mathfrak{I}$-$\mathfrak{G}$ is KS_{gs}-$\mathfrak{O}$. Also, $f^{-1}(\mathfrak{I}-\mathfrak{G})=\mathfrak{I}-f^{-1}(\mathfrak{G})$ is KS_{gs}-$\mathfrak{O}$. Thus $f^{-1}(\mathfrak{G})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$.

Theorem 3.8 *Let* $\mathfrak{I}$, $\mathfrak{I}$ *and* $\mathfrak{K}$ *be any KS-topological spaces and* $f:\mathfrak{I}\to\mathfrak{I}$ *and* $g:\mathfrak{I}\to\mathfrak{K}$ *be two functions. Then the following are true;*

1. If f and g are KS_{gs}-irresolute functions, then their composition $g \circ f: \mathfrak{I} \to \mathfrak{K}$ is KS_{gs}-irresolute.

2. If f is KS_{gs}-irresolute and g is KS_{gs}-continuous, then their composition $g \circ f: \mathfrak{I} \to \mathfrak{K}$ is KS_{gs}-continuous.

Proof: 1. Let $\mathfrak{A}$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{K}$ then $g^{-1}(\mathfrak{A})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{J}$ and $f^{-1}(g^{-1}(\mathfrak{A}))$ is KS_{gs}-$\mathfrak{C}$. Whereas f and g are KS_{gs}-irresolute functions. But $f^{-1}(g^{-1}(\mathfrak{A})) = (g \circ f)^{-1}(\mathfrak{A})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. Consequently $(g \circ f)$ is KS_{gs}-irresolute.

2. Let $\mathfrak{F}$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{K}$. Whereas g is KS_{gs}-continuous. $g^{-1}(\mathfrak{F})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas g is KS_{gs}-irresolute, $f^{-1}(g^{-1}(\mathfrak{F}))$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. But $f^{-1}(g^{-1}(\mathfrak{F})) = (g \circ f)^{-1}(\mathfrak{F})$. Consequently $(g \circ f)$ is KS_{gs}-continuous.

Theorem 3.9 *If function $f: \mathfrak{I} \to \mathfrak{J}$ is KS_{gs}-irresolute then for every subset $\mathfrak{A}$ of $\mathfrak{I}$ such that $f(\mathfrak{A})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{J}$, $f(KS_{gs}cl(\mathfrak{A})) \subseteq KS_{gs}cl(f(\mathfrak{A}))$.*

Proof: Let $\mathfrak{A}$ be a subset of $\mathfrak{I}$ such that $f(\mathfrak{A})$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{J}$. Then $KS_{gs}cl(f(\mathfrak{A}))$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{gs}-irresolute, $f^{-1}(KS_{gs}cl(f(\mathfrak{A}))$ is KS_{gs}-$\mathfrak{C}$ in $\mathfrak{I}$. Now $\mathfrak{A} \subseteq f^{-1}(f(\mathfrak{A})) \subseteq f^{-1}(KS_{gs}cl(f(\mathfrak{A})))$. Consequently $KS_{gs}cl(f(\mathfrak{A})) \subseteq f^{-1}(KS_{gs}cl(f(\mathfrak{A})))$ and hence $f(KS_{gs}cl(\mathfrak{A})) \subseteq f(f^{-1}(KS_{gs}cl(f(\mathfrak{A}))) \subseteq KS_{gs}cl(f(\mathfrak{A}))$.

Theorem 3.10 *If a function $f: \mathfrak{I} \to \mathfrak{J}$ is KS_{sg}-irresolute, then it is KS_{sg}-continuous.*

Proof: Let $\mathfrak{F}$ be a closed set in $\mathfrak{J}$. We have every closed set is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{sg}-irresolute, $f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$ set in $\mathfrak{I}$. Consequently for a closed set $\mathfrak{F}$ in $\mathfrak{J}$, $f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$ set in $\mathfrak{I}$. So, f is KS_{sg}-continuous.

Theorem 3.11 *A function $f: \mathfrak{I} \to \mathfrak{J}$ is KS_{sg}-irresolute iff for every KS_{sg}-$\mathfrak{O}$ $\mathfrak{F}$ of $\mathfrak{J}$, $f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{O}$ in $\mathfrak{I}$.*

Proof: Necessity: If $f: \mathfrak{I} \to \mathfrak{J}$ be KS_{sg}-irresolute, then for every KS_{sg}-$\mathfrak{C}$ subset $\mathfrak{G}$ of $\mathfrak{J}$, $f^{-1}(\mathfrak{G})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{I}$. If $\mathfrak{F}$ is any KS_{sg}-$\mathfrak{O}$ subset of $\mathfrak{J}$ then $\mathfrak{J}$-$\mathfrak{F}$ is KS_{sg}-$\mathfrak{C}$. Thus $f^{-1}(\mathfrak{J} - \mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$, but $f^{-1}(\mathfrak{J} - \mathfrak{F}) = \mathfrak{I} - f^{-1}(\mathfrak{F})$ so that $f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{O}$ in $\mathfrak{I}$.

Sufficiency: If for all KS_{sg}-$\mathfrak{O}$ subsets $\mathfrak{F}$ of $\mathfrak{J}$, $f^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{O}$ in $\mathfrak{I}$ and if $\mathfrak{G}$ is any KS_{sg}-$\mathfrak{C}$ subset of $\mathfrak{J}$, then $\mathfrak{J}$-$\mathfrak{G}$ is KS_{sg}-$\mathfrak{O}$. Also, $f^{-1}(\mathfrak{J} - \mathfrak{G}) = \mathfrak{I} - f^{-1}(\mathfrak{G})$ is KS_{sg}-$\mathfrak{O}$. Thus $f^{-1}(\mathfrak{G})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{I}$.

Theorem 3.12 *Let $\mathfrak{I}$,$\mathfrak{J}$ and $\mathfrak{K}$ be any KS-topological spaces and $f: \mathfrak{I} \to \mathfrak{J}$ and $g: \mathfrak{J} \to \mathfrak{K}$ be two functions. Then the following are true;*

1. If f and g are KS_{sg}-irresolute functions, then their composition $g \circ f: \mathfrak{I} \to \mathfrak{K}$ is KS_{sg}-irresolute.

2. If f is KS_{sg}-irresolute and g is KS_{sg}-continuous, then their composition $g \circ$

$f:\mathfrak{J} \rightarrow \mathfrak{K}$ is KS_{sg}-continuous.

Proof: 1. Let $\mathfrak{A}$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{K}$ then $g^{-1}(\mathfrak{A})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$ and $f^{-1}(g^{-1}(\mathfrak{A}))$ is KS_{sg}-$\mathfrak{C}$. Whereas g and f are KS_{sg}-irresolute functions. But $f^{-1}(g^{-1}(\mathfrak{A})) = (g \circ f)^{-1}(\mathfrak{A})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Consequently $(g \circ f)$ is KS_{sg}-irresolute.

2. Let $\mathfrak{F}$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{K}$. Whereas g is KS_{sg}-continuous. $g^{-1}(\mathfrak{F})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{sg}-irresolute, $f^{-1}(g^{-1}(\mathfrak{F}))$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. But $f^{-1}(g^{-1}(\mathfrak{F})) = (g \circ f)^{-1}(\mathfrak{F})$. Consequently $(g \circ f)$ is KS_{sg}-continuous.

Theorem 3.13 *If function $f:\mathfrak{J} \rightarrow \mathfrak{J}$ is KS_{sg}-irresolute then for every subset $\mathfrak{A}$ of $\mathfrak{J}$ such that $f(\mathfrak{A})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$, $f(KS_{sg}cl(\mathfrak{A})) \subseteq KS_{sg}cl(f(\mathfrak{A}))$.*

Proof: Let $\mathfrak{A}$ be a subset of $\mathfrak{J}$ such that $f(\mathfrak{A})$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Then $KS_{sg}cl(f(\mathfrak{A}))$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Whereas f is KS_{sg}-irresolute, $f^{-1}(KS_{sg}cl(f(\mathfrak{A}))$ is KS_{sg}-$\mathfrak{C}$ in $\mathfrak{J}$. Now $\mathfrak{A} \subseteq f^{-1}(f(\mathfrak{A})) \subseteq f^{-1}(KS_{sg}cl(f(\mathfrak{A})))$. Consequently $KS_{sg}cl(f(\mathfrak{A})) \subseteq f^{-1}(KS_{sg}cl(f(\mathfrak{A})))$ and hence $f(KS_{sg}cl(\mathfrak{A})) \subseteq f(f^{-1}(KS_{sg}cl(f(\mathfrak{A}))) \subseteq KS_{sg}cl(f(\mathfrak{A}))$.

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