Binary γ - open sets in Binary Topological Space

K.Muthulakshmi[1#], M.Gilbert Rani [*2]

[1]Assistant Professor in Mathematics,
V.V.Vanniaperumal College for Women(Autonomous), Virudhunagar, India

[2]Assistant Professor in Mathematics,
Arul Anandar College(Autonomous), Karumathur, India

[a]sweetyesther20@gmail.com & [b]gilmathaac@gmail.com

Article Info
Page Number: 3582-3590
Publication Issue:
Vol. 71 No. 4 (2022)

Article History
Article Received: 25 March 2022
Revised: 30 April 2022
Accepted: 15 June 2022
Publication:19 August 2022

Abstract

According to this paper, we bring out the new open set namely as binary γ-open set. And we investigate the concept of binary γ- interior and binary γ-closure

Keywords: Binary open sets, Binary Topological Spaces .

1. Introduction

The introduction of topological space was given by Ryszard Engelking[19] in the year 1977. operation γ of a topology τconcept was put-forth by S. Kasahara [2]. A new idea was imposed by H. Ogata [16]. The concept of new mapping relating the sets are given [1]. G.S.S. Krishnan and K. Balachandran [4] investigated the concept γ- preopen sets. Continuity and separation axioms in [12],[13]. Generalization was introduced byJamal M. Mustafa [7]. R.Seethalakshmi [18] have also said about Nearly Binary Open Sets in this concept.

In this paper, binary γ -open sets in binary topological spaces is introduced and discussed. And here we will also show up with some of the its definitions and properties with examples, which helps to illustrate the binary γ -open sets.

2. BINARY γ-OPEN SETS

To understand the concept, go through [11],[16]. In this section,b$-$O- binary open set, b$-C-$binary $closed\ set,$b-cl– binary closure, BTS -binary topological space,B $\gamma-$TS-binary$\gamma-$ topological space, b γ-binary $\gamma-$open set, b γO- binary $\gamma-$open sets, b $\gamma C-$binary $\gamma-closed\ sets$, $\mathfrak{B}\gamma-$set of all binary $\gamma-$open sets, b-int- binary interior, LP-limit point, LPs- limit points

Now we see the basic definitions

Definition 2.1

For BTS (ζ,η,$\mathfrak{B}$), (i)An operation γ on the BT '$\mathfrak{B}'$ is a mapping γ:$\mathfrak{B}\rightarrow\widetilde{2^X}$ x $\widetilde{2^Y}$ such that (ρ,σ)⊆ $\gamma((\rho,\sigma))$ for every (ρ,σ)∈ $\mathfrak{B}$.(ii)Take a nonempty set $(\lambda,\aleph) \subseteq (\zeta,\eta)$, then $(\lambda,\aleph)$ is called bγ if for all $(\daleth,\beth)$∈ $(\lambda,\aleph)$, there exists $(\rho,\sigma) \in \mathfrak{B}$ such that $(\daleth,\beth) \in (\rho,\sigma)$, $\gamma((\rho,\sigma))$⊆ $(\lambda,\aleph)$. For example,InBTS, ζ={a, b}, η={1,2}

$\mathfrak{B}$ ={$(\emptyset,\emptyset)$, ({a},{1}), (ζ,η)} on $\zeta \times \eta$.

γ: $\mathfrak{B} \rightarrow \widetilde{2^X}x\widetilde{2^Y}$ be an operation defined as follows:

For every $(\lambda,\aleph)$)∈ $\mathfrak{B}$, then $\gamma((\lambda,\aleph))$ =$\begin{cases} (\lambda,\aleph) & \text{if } (\lambda,\aleph) = (\{a\},\{1\}) \\ (\lambda,\aleph) \cup (\{b\},\{2\}) & if (\lambda,\aleph) \neq (\{a\},\{1\}) \end{cases}$

Here $(\emptyset,\emptyset)$, (ζ,η), $(\{a\},\{1\})$ are b γO.

(iii)$(\lambda,\aleph)$ isb γC if $(\lambda,\aleph)^c$∈ $\mathfrak{B}\gamma$.

Result 2.1:

Let $\{(A_\alpha, B_\alpha)$/ $\alpha\ \epsilon\Omega\}$ be any collection of b γO on a BTS(ζ,η, $\mathfrak{B}$). Then $\bigcup_{\alpha\epsilon\Omega}(A_\alpha,B_\alpha)$ is also a b γO.

Proof: $(A_1,B_1),(A_2,B_2),\ldots.(A_n,B_n)$ are b γ0 on a BTS.

Let $(\daleth,\beth) \in \bigcup_{\alpha\epsilon\Omega}(A_\alpha,B_\alpha)$. Then $(\daleth,\beth) \in (A_j,B_j)$, for some j $\epsilon\Omega$.

Since (A_j,B_j) is b γ, ∃b γ (ρ,σ) such that $(\daleth,\beth) \in (\rho,\sigma)$, $\gamma((\rho,\sigma))$⊆ $\left(A_j,B_j\right)$.

It follows that $\gamma((\rho,\sigma))$⊆ $\bigcup_{\alpha\epsilon\Omega}(A_\alpha,B_\alpha)$.

Remark 2.1: Arbitrary intersection of b γO is not binary γ-open set.

Result 2.2:

Let $\{(A_\alpha,B_\alpha)$/ $\alpha\ \epsilon\{1,2,\ldots,$n}} be any finite collection of b γOon a BTS. Then $\bigcap_{i=1}^{n}(A_i,B_i)$ is abγ.

Proof: Let $(A_1,B_1),(A_2,B_2),\ldots.(A_n,B_n)$ be b γ0 in (ζ,η,$\mathfrak{B}$).

By induction on n , put n=2,Let (a, b) ∈ $(A_1 \cap A_2, B_1 \cap B_2)$. Then (a, b) ∈ (A_1,B_1) and

(a, b) ∈ (A_2,B_2).Since (A_1,B_1) is b γO, $\exists(U_1,V_1) \in \mathfrak{B}\gamma$ such that (a, b) ∈ (U_1,V_1) and

$\gamma((U_1,V_1))$⊆ (A_1,B_1). Also (A_2,B_2) is b γO, $\exists(U_2,V_2) \in \mathfrak{B}\gamma$ such that (a, b) ∈ $(U_2,V_2)$and

$\gamma((U_2,V_2))$⊆ (A_2,B_2). Therefore (a, b) $\in (U_1\cap U_2, V_1 \cap V_2)and\gamma((U_1 \cap U_2, V_1 \cap V_2)) \subseteq (A_1 \cap A_2, B_1 \cap B_2)$. Hence $(A_1 \cap A_2, B_1 \cap B_2)$ is b$inary\ \gamma - open$. Let us take this result is true for n-1. $(A_1 \cap A_2 \ldots \cap A_n,\ B_1 \cap B_2 \ldots \cap B_n)$= $(A_1 \cap A_2 \ldots \cap A_{n-1},\ B_1 \cap B_2 \ldots \cap B_{n-1}) \cap\ (A_n,B_n)$. By hypothesis, $\bigcap_{i=1}^{n}(A_i,B_i)$ is a b$inary\ \gamma - open$.

Result 2.3:

Let $\{(A_\alpha, B_\alpha)/\ \alpha\ \epsilon\Omega\}$ be any collection of b γC on a BTS.

Then(i) $\bigcap_{\alpha\ \epsilon\Omega}(A_\alpha, B_\alpha)$is also a b γC and (ii)$\bigcup_{\alpha\epsilon\Omega}(A_\alpha, B_\alpha)$ need not be a b γC.

Proof: Let $(A_1, B_1), (A_2, B_2), \ldots. (A_n, B_n)\ldots$ be b γC. Then $(A_1, B_1)^c, (A_2, B_2)^c, \ldots, (A_n, B_n)^c \ldots$ are b γO.

$({A_1}^c, {B_1}^c)$, $({A_2}^c, {B_2}^c)$, . . . , $({A_n}^c, {B_n}^c)$ are b γC. By result 2.1, $\bigcup_{\alpha\epsilon\Omega}({A_\alpha}^c, {B_\alpha}^c,)$ is b γO.(i.e)$\bigcup_{\alpha\ \epsilon\Omega}(A_\alpha, B_\alpha)^c \in \mathfrak{B}\gamma$. $(\bigcap_{\alpha\ \epsilon\Omega}(A_\alpha, B_\alpha))^c$ is b γO.$\Rightarrow$ $\bigcap_{\alpha\ \epsilon\Omega}(A_\alpha, B_\alpha)$is also a b γC.

(ii) By remark 2.1, $\bigcap_{\alpha\ \epsilon\Omega}(A_\alpha, B_\alpha)^c$ need not be a b γO.So,$\bigcup_{\alpha\epsilon\Omega}(A_\alpha, B_\alpha)$ need not be a binary $\gamma-$ *closed set.*

Result 2.4:

Let $\{(A_\alpha, B_\alpha)/\ \alpha\ \epsilon\{1, 2, \ldots, \mathrm{n}\}\}$ be any finite collection of b γC on a BTS $(\zeta, \eta, \mathfrak{B})$. Then $\bigcup_{i=1}^{n}(A_i, B_i)$ is a b$inary\ \gamma - closed\ set$.

Definition 2.2:

Let (P, Q) be any subset of a BTS $(\zeta, \eta, \mathfrak{B}\ )$ and γ be an operation on $\mathfrak{B}$. Let $(P, Q)_*\ = \cup\{P_1: (P_1, Q_1)\ is$ binary γ-open and $(P_1, Q_1) \subseteq$(P, Q)} and $(P, Q)_{**} = \cup\{Q_1: (P_1, Q_1) is$ binary $\gamma -$ open and $(P_1, Q_1) \subseteq$ (P, Q)} .Then clearly $((P, Q)_*, (P, Q)_{**})$ is binary γ $-$open and also $((P, Q)_*, (P, Q)_{**}) \subseteq$ (P, Q).

Also binary $\gamma -$ interior of (P, Q) is denoted as $\mathfrak{B}\gamma$ - int (P, Q) and is defined by the union of all b γO contained in (P, Q).That is $\mathfrak{B}\gamma$ $-$int (P, Q) = $((P, Q)_*, (P, Q)_{**})$.

In the above example, $\mathfrak{B}\gamma$-int$(\{b\}, \{2\}\ ) = (\emptyset, \emptyset)$, $\mathfrak{B}\gamma$-int $(\{a\}, \{1\}\ ) = (\{a\}, \{1\})$

Proposition 2.1:

(P, Q) of (ζ, η) is b γ in $(\zeta, \eta, \mathfrak{B}\ )$ if and only if (P, Q) = $\mathfrak{B}\gamma$- int (P, Q).

Proof :

Let us take (P, Q) is binary $\gamma -$ open in $(\zeta, \eta, \mathfrak{B})$. By Definition 2.2, P$\subseteq (P, Q)_*$ andQ $\subseteq (P, Q)_{**}$.

Therefore (P, Q) $\subseteq ((P, Q)_*, (P, Q)_{**})$.$((P, Q)_*, (P, Q)_{**}) \subseteq$ (P, Q).

Thus (P, Q) = $((P, Q)_*, (P, Q)_{**})$.

Conversely assume that (P, Q) = $\mathfrak{B}\gamma$- int (P, Q).By the above definition, (P, Q) is b γ.

Definition 2.3:

Let (P,Q)* = $\cap\{P_1: (P_1, Q_1)$ is a binary γ-closed & $(P, Q) \subseteq (P_1, Q_1)\}$ and (P,Q)**= $\cap\{Q_1: (P_1, Q_1)$ is a binary γ-closed *&*$(P, Q) \subseteq (P_1, Q_1)\}$.Then clearly ((P,Q)*,(P,Q)**) is binary γ $-$closed and also (P, Q) $\subseteq$((P,Q)*,(P,Q)**) .

Binary γ-closure of (P,Q) is denoted as $\mathfrak{B}\gamma$-cl(P,Q) and is defined as intersection of all b γC containing (P,Q). That is $\mathfrak{B}\gamma$-cl(P, Q) = ((P,Q)*,(P,Q)**)

Proposition 2.2:

(P, Q) of (ζ, η) is binary $\gamma -$ closed set in $(\zeta, \eta, \mathfrak{B})$ if and only if (P, Q) =$\mathfrak{B}\gamma$- cl (P, Q).

Proposition 2.3:

Let (P, Q) $\subseteq (\zeta, \eta)$. Then in BTS$(\zeta, \eta, \mathfrak{B})$,

(i)$\mathfrak{B}\gamma$-int$(\emptyset, \emptyset)$=$(\emptyset, \emptyset)$

(ii)$\mathfrak{B}\gamma$-int(X, Y)=(ζ, η)

(iii)$\mathfrak{B}\gamma$-int($\mathfrak{B}\gamma$-int (P,Q))= $\mathfrak{B}\gamma$-int(P,Q)

(iv)$\mathfrak{B}\gamma$-cl$(\emptyset, \emptyset)$=$(\emptyset, \emptyset)$

(v)$\mathfrak{B}\gamma$-cl(ζ, η)=(ζ, η)

(vi)$\mathfrak{B}\gamma$-cl($\mathfrak{B}\gamma$-cl (P,Q))= $\mathfrak{B}\gamma$-cl(P,Q)

Definition 2.4:

Let ζ and η be any two non empty sets. A binary γ- topology is a binary γ- structure$\mathfrak{B}\gamma \subset \widetilde{2^X} \times \widetilde{2^Y}$, if it satisfies the following

(i)$(\emptyset, \emptyset) \in \mathfrak{B}\gamma$ (ii) $(\zeta, \eta) \in \mathfrak{B}\gamma$ (iii)finite intersection of b γO is binary $\gamma -$open set.

(iv)If $\{(A_\alpha, B_\alpha)/ \alpha\epsilon\delta\}$ is acollection of members of $\mathfrak{B}\gamma$, then $(\bigcup_{\alpha\epsilon\delta} A_\alpha, \bigcup_{\alpha\epsilon\delta} B_\alpha) \in \mathfrak{B}\gamma$.

Result 2.5:

If $\mathfrak{B}$ and $\mathfrak{B}\gamma$ are binary and binary gamma topology respectively, then $\mathfrak{B}\gamma \subset \mathfrak{B}$. And two topologies are same, if γ is identity function on $\mathfrak{B}$.

Proof:

Let (ג, א)∈ $\mathfrak{B}\gamma$.Therefore there exists (ρ, σ) ∈ $\mathfrak{B}$ such that (ד, ב) ∈ (ρ, σ), γ((ρ, σ))⊆ (ג, א), for each (ד, ב) ∈ (ג, א).

Since (ρ, σ)⊆ γ((ρ, σ)), hence, for every (ד, ב) ∈ (ג, א), there exists b−O(ρ, σ) such that (ד, ב) ∈ (ρ, σ), (ρ, σ) ⊆ (ג, א). Therefore (ג, א)) ∈ $\mathfrak{B}$. Hence $\mathfrak{B}$γ ⊂ $\mathfrak{B}$.

Take γ:$\mathfrak{B}$→$\widetilde{2^X}$ x $\widetilde{2^Y}$ such that (ρ, σ)= γ((ρ, σ))

Let (ג, א)) ∈ $\mathfrak{B}$ and let (ד, ב) ∈ (ג, א)). Since (ג, א) is b−O, there exists (ρ, σ) ∈ $\mathfrak{B}$ such that

(ד, ב) ∈ (ρ, σ), (ρ, σ)⊆ (ג, א). Hence, for each (ד, ב) ∈ (ג, א), there exists (ρ, σ) ∈ $\mathfrak{B}$ such that

(ד, ב) ∈ (ρ, σ)and γ(ρ, σ))⊆ (ג, א), since (ρ, σ)= γ((ρ, σ)).

Therefore (ג, א)∈ 𝔅γ. Hence 𝔅 ⊂ 𝔅γ.

Definition 2.5:

Let $(\zeta, \eta, \mathfrak{B}\gamma$ *) be a* $B\gamma - TS$ *and let* (ד,ב) ∈ (ζ, η)*. The* $b\gamma$ (ג, א) *is a binary* γ - neighbourhood of (ד,ב) if ד ∈A and ב ∈ B.

Definition2.6:

Let (ג, א)⊆ Bγ − TS. A point (ד, ב) ∈ (ζ, η) is said to be a LP of (ג, א) if every b γO (ρ, σ) containing (ד, ב)contains a point of (ג, א) different from (ד, ב). The set of all LPs of (ג, א) in the Bγ − TS is denoted by $(ג, א)_{\gamma}{}'$

For example, Take BTS $(\zeta, \eta, \mathfrak{B})$, where ζ= {a, b, c, d}, η= {1,2, 3, 4, 5}

$\mathfrak{B} = \{(\emptyset, \emptyset), (\{a\},\{1\}), (\{b\},\{2\}), (\{a, b\},\{1, 2\}), (\zeta, \eta)\}$ on $\zeta \times \eta$. *Let*$\gamma: \mathfrak{B} \to \widetilde{2^X} \times \widetilde{2^Y}$ be an operation defined as follows:

For every (ג, א)∈ 𝔅, then γ((ג, א)) = $\begin{cases} (ג, א) & \text{if } (ג, א) = (\{b\}, \{2\}) \\ (ג, א) \cup (\{b\}, \{2\}) & \text{if } (ג, א) \neq (\{b\}, \{2\}) \end{cases}$

$\mathfrak{B}\gamma = \{(\emptyset, \emptyset), (\zeta, \eta), (\{b\},\{2\})\}$.

Let ({a, b},{2, 3})⊂ (ζ, η).

Take ({a},{1}) ∈(ζ, η).

Neighborhood of ({a},{1}) is (ζ, η)

({a, b},{2, 3})∩ (ζ, η) =({a, b},{2, 3})

Therefore ({a},{1}) is a LP.

Theorem 2.1:

Let (ג, א)⊆Bγ − TS.Then (ד, ב) ∈ 𝔅γ -cl(ג, א) iff every b γ*O* (ρ, σ) *containing* (ד, ב) intersects (ג, א).

Proof:

Suppose (ד, ב) ∉ 𝔅γ *-cl*(ג, א)*. Then* $(\rho, \sigma) = (\zeta, \eta)$ - 𝔅γ *-cl*(ג, א) *is an b*γ *O, which containing* (ד, ב)*.*

Since (ג, א)⊆ 𝔅γ *-cl*(ג, א)*,* $(\rho, \sigma) \cap$ (ג, א)$= \phi$*.*

Suppose there exist $(\rho, \sigma) \in \mathfrak{B}\gamma$ *such that* (ד, ב) ∈(ρ, σ) *and* $(\rho, \sigma) \cap$(ג, א)$= \phi$*.*

Let(ζ, η) *-* (ρ, σ)*= (P, Q). Then (P, Q) is* 𝔅γ -closed and (ד, ב) ∉ (P, Q). Therefore (ד, ב) ∉ 𝔅γ -cl(ג, א), since by definition of binary gamma closure.

Theorem 2.2:

$\mathfrak{B}\gamma$ -cl$(\gimel, \aleph)$ =$(\gimel, \aleph)\cup(\gimel, \aleph)_{\gamma}{}'$.

Proof:

Let $(\daleth, \beth) \in(\gimel, \aleph)\cup(\gimel, \aleph)_{\gamma}{}'$. Then $(\daleth, \beth) \in(\gimel, \aleph)$ *or* $(\gimel, \aleph)_{\gamma}{}'$. Let us take$(\daleth, \beth) \in (\gimel, \aleph)_{\gamma}{}'$. Then every b γO containing $(\daleth, \beth)$ intersect $(\gimel, \aleph)$. By theorem 2.1, $(\daleth, \beth) \in \mathfrak{B}\gamma$-cl$(\gimel, \aleph)$. Suppose $(\daleth, \beth) \in(\gimel, \aleph)$.

Then $(\daleth, \beth) \in \mathfrak{B}\gamma$ -cl$(\gimel, \aleph)$. since $(\gimel, \aleph)\subset \mathfrak{B}\gamma$ -cl$(\gimel, \aleph)$.Therefore $(\gimel, \aleph)\cup(\gimel, \aleph)_{\gamma}{}' \subseteq \mathfrak{B}\gamma$ -cl$(\gimel, \aleph)$.

If $(\daleth, \beth) \in(\gimel, \aleph)$, then $(\daleth, \beth) \in(\gimel, \aleph)\cup(\gimel, \aleph)_{\gamma}{}'$.Suppose $(\daleth, \beth) \notin(\gimel, \aleph)$. Then by theorem 2.1, every b γ (ρ, σ) of $(\daleth, \beth)$ intersects $(\gimel, \aleph)$. It follows that (ρ, σ) intersect $(\gimel, \aleph)$ different from $(\daleth, \beth)$,

since $(\daleth, \beth) \notin(\gimel, \aleph)$. Therefore$(\daleth, \beth) \in (\gimel, \aleph)_{\gamma}{}'$. Thus $\mathfrak{B}\gamma$ -cl$(\gimel, \aleph)\subseteq(\gimel, \aleph)\cup(\gimel, \aleph)_{\gamma}{}'$.

Theorem 2.3:

A subset of a B$\gamma-$ TS is binary γ-closed iff it contains all its LPs.

Proof:

By Proposition 2.2, the set $(\gimel, \aleph)$ is binary γ-closed iff $(\gimel, \aleph)$ = $\mathfrak{B}\gamma$ -cl$(\gimel, \aleph)$.Also by theorem 2.2, therefore a subset of a B$\gamma-$ TS is binary γ-closed iff it contains all its LPs.

Proposition 2.4:

Let $(\gimel, \aleph)$ be a subset of the BTS $(\zeta, \eta, \mathfrak{B})$. Then $(\daleth, \beth) \in$ b-cl$(\gimel, \aleph)$ iff every b$-$O(ρ, σ) containing $(\daleth, \beth)$ intersects $(\gimel, \aleph)$.

Proof:

Suppose $(\daleth, \beth) \notin$ b-cl$(\gimel, \aleph)$. Then $(\rho, \sigma) = (\zeta, \eta) \setminus$ b-cl$(\gimel, \aleph)$ is an b$-$O, which containing $(\daleth, \beth)$. Since$(\gimel, \aleph)\subseteq$ b-cl$(\gimel, \aleph)$, $(\rho, \sigma)\cap(\gimel, \aleph)= \phi$.

Suppose there exist $(\rho, \sigma) \in \mathfrak{B}$ such that $(\daleth, \beth) \in(\rho, \sigma)$ and $(\rho, \sigma)\cap(\gimel, \aleph)= \phi$. Take$(\zeta, \eta) \setminus (\rho, \sigma)=$ (P, Q). Then (P, Q) is b$-C$ and $(\daleth, \beth) \notin$ (P, Q). Since by definition of binary closure, $(\daleth, \beth) \notin$ b-cl$(\gimel, \aleph)$.

Proposition 2.5:

b-cl$(\gimel, \aleph)$ =$(\gimel, \aleph)\cup(\gimel, \aleph)'$.

Proof:

Let $(\daleth, \beth) \in(\gimel, \aleph)\cup(\gimel, \aleph)'$*. Then* $(\daleth, \beth) \in(\gimel, \aleph)$*or* $(\gimel, \aleph)'$*. Assume that* $(\daleth, \beth) \in (\gimel, \aleph)'$*. Then every b*$-O$ *containing* $(\daleth, \beth)$ *intersect* $(\gimel, \aleph)$*. By proposition 2.4,* $(\daleth, \beth) \in \mathfrak{B}\gamma$ *-cl*$(\gimel, \aleph)$*. Suppose* $(\daleth, \beth) \in(\gimel, \aleph)$ *.*

Then $(\daleth, \beth) \in \mathfrak{B}\gamma$ *-cl*$(\gimel, \aleph)$ *,since* $(\gimel, \aleph)\subset \mathfrak{B}\gamma$ *-cl*$(\gimel, \aleph)$*.Therefore* $(\gimel, \aleph)\cup(\gimel, \aleph)' \subseteq \mathfrak{B}\gamma$ *-cl*$(\gimel, \aleph)$*.*

If $(\daleth, \beth) \in(\gimel, \aleph)$*, then* $(\daleth, \beth) \in(\gimel, \aleph)\cup(\gimel, \aleph)'$*.Suppose* $(\daleth, \beth) \notin(\gimel, \aleph)$*. Then by same proposition, every b*$-O$ (ρ, σ) *of* $(\daleth, \beth)$ *intersects*$(\gimel, \aleph)$*. Hence* (ρ, σ) *intersect*$(\gimel, \aleph)$ *different from* $(\daleth, \beth)$*,* *since* $(ד, ב) \notin (ג, א)$. *Therefore* $(ד, ב) \in (ג, א)'$. *Thus b-cl*$(ג, א) \subseteq (ג, א) \cup (ג, א)'$.

Proposition2.6:

$(ג, א)' \subseteq (ג, א)_{\gamma}{}'$.

Proof:

Let $(ד, ב) \in (ג, א)'$. Then every b−O intersect with $(ג, א)$ except $(ד, ב)$. By the result 2.5,$(ד, ב) \in (ג, א)_{\gamma}{}'$.

Proposition 2.7:

b-cl$(ג, א) \subseteq \mathfrak{B}\gamma$ -cl$(ג, א)$.

Proof:,

Let $(ד, ב) \in$ b-cl$(ג, א)$. Then b-cl$(ג, א)= (ג, א) \cup (ג, א)' \subseteq (ג, א) \cup (ג, א)_{\gamma}{}'$.

By theorem 2.2, b-cl$(ג, א) \subseteq \mathfrak{B}\gamma$ -cl$(ג, א)$.

Result 2.6:

$\mathfrak{B}\gamma$ -int$(ג, א) \subseteq$ b-int$(ג, א)$.

3. CONCLUSION

Hence, we have discussed new concepts. And the theories approached in this concept are beneficial in mathematical operation.

References

[1] B. Ahmad and S. Hussain,Properties of γ-Operations on topological spaces(Aligarh Bull. Math. 22(1) (2003)),pp. 45-51.

[2] S. Kasahara, Operation- compact spaces(Math.Japon,24 (1979)), pp. 97-105.

[3] G.S.S. Krishnan (2003), A new class of semi open sets in a topological space(Proceedings of NCMCM, Allied Publishers, New Delhi), pp. 305-311.

[4] G.S.S. Krishnan and K. Balachandran, On a class of γ-preopen sets in a topological Space(East Asian Math. J.,22(2) (2006a)), pp. 131-149

[5] G.S.S. Krishnan and K. Balachandran, On a class of γ-semiopen sets in a topological Space(Bull. Cal. Math.Soc.,98(6) (2006a)), pp. 517-530.

[6] G.S.S. Krishnan, M. Ganster and K. Balachandran, Operation approaches on semi-open sets and applications(Kochi. J. Math. 2, (2007)), pp. 21-33.

[7] Jamal M. Mustafa, On Binary Generalized Topological Spaces(Refaad General Letters in Mathematics, 2(3) 2017), pp. 111-116.

[8] F.Nakaoka and N.Oda, Some applications of minimal open sets(International Journal of Math. Math. Sci., 27 no. 8 (2001)), pp. 471-476.

[9] Nazir Ahmad Ahengar and J.K. Maitra, On g-binary continuity(Journal of Emerging Technologies and Inovative Research, 5(7) 2018), pp. 240-244.

[10] S. Nithyanantha Jothi and P.Thangavelu, On binary topological spaces(Pacific- Asian journal of Mathematics 5(2), 2011), pp. 133-138.

[11] S. Nithyanantha Jothi and P.Thangavelu, Topology between two sets(Journal of Mathematical Sciences & Computer Applications, 1(3) (2011)), pp. 95-107.

[12] S. Nithyanantha Jothi and P.Thangavelu, Topology between two sets(Journal of Mathematical Sciences & Computer Applications, 1(3) (2011)), pp. 95-107.

[13] S. Nithyanantha Jothi and P.Thangavelu, On binary continuity and binary separation axioms(Ultra Scientist of Physical Sciences, Vol. 24(1)A, (2012)), pp. 121- 126.

[14] S. Nithyanantha Jothi and P.Thangavelu, Binary semi open sets in binary topological spaces(IJMA-7(9), 2016), pp. 73-76.

[15] S. Nithyanantha Jothi and P.Thangavelu, Generalized Binary Regular Closed sets(International Journal of Applied Sciences (ISSN 2455-4499), 4(2), 2016), pp. 259- 263.

[16] S. Nithyanantha Jothi, Binary Semi Continuous functions(International Journal of Mathematics Trends and Technology -Volume 49 Number 2 September 2017).

[17] H.Ogata, operations on topological spaces and associated topology(Math. Japon., 36(1), (1991)), pp. 175-184

[18] Ryszard Engelking, General Topology(Polish Scientific Publishers, Warszawa (1977)).

[19] R.Seethalakshmi, K. Kamaraj, Nearly Binary Open Sets in Binary Topological Spaces(International Conference on Applied and Computational Mathematics(ICACM-2018)).

[20] O Ravi, R Senthil Kumar, A Hamari Choudhi, Weakly ⊐ g-closed sets, BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE, 4, Vol. 4(2014), 1-9

[21] O Ravi, R Senthil Kumar, Mildly Ig-closed sets, Journal of New Results in Science, Vol3,Issue 5 (2014) page 37-47

[22] O Ravi, A senthil kumar R & Hamari CHOUDHİ, Decompositions of Ï g-Continuity via Idealization, Journal of New Results in Science, Vol 7, Issue 3 (2014), Page 72-80.

[23] O Ravi, A Pandi, R Senthil Kumar, A Muthulakshmi, Some decompositions of πg-continuity, International Journal of Mathematics and its Application, Vol 3 Issue 1 (2015) Page 149-154.

[24] S. Tharmar and R. Senthil Kumar, Soft Locally Closed Sets in Soft Ideal Topological Spaces, Vol 10, issue XXIV(2016) Page No (1593-1600).

[25] K. Prabhavathi, R. Senthilkumar, P.Arul pandy, m-$I_{\pi g}$-Closed Sets and m-$I_{\pi g}$-Continuity, Journal of Advanced Research in Dynamical and Control Systems Vol 10 issue 4 (2018) Page no 112-118

[26] K. Prabhavathi, R. Senthilkumar, I. Athal, M. Karthivel, A Note on Iβ * g Closed Sets, Journal of Advanced Research in Dynamical and Control Systems11(4 Special Issue), pp. 2495-2502.

[27] K PRABHAVATHI, K NIRMALA, R SENTHIL KUMAR, WEAKLY (1, 2)-CG-CLOSED SETS IN BIOTOPOLOGICAL SPACES, Advances in Mathematics: Scientific Journal vol 9 Issue 11(2020) Page 9341-9344