Single Server Queueing Model with Catastrophe, Restoration and Partial Breakdown

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Abstract

In this article, we have considered an M/M/1/N model with catastrophe, restoration and breakdown. The server works with the slower rate of service during partial breakdown. The number of times the system attains its capacity has been analyzed through matrix geometric method and some performance measures are obtained and numerical illustrations are also presented.

Keywords: Catastrophe, Restoration, Working Breakdown, Repair, Matrix Geometric Method.

1. Introduction:

In many real life situations, queues are often seen and it has various applications in different fields. The term catastrophe is a sudden destruction. When a queue undergoes catastrophe, all the customers are removed from the system. So, the system is in the position to regain its state. Moreover, the system takes its time to accept new customers which is referred as restoration time. Chao(1995) had studied A queueing network model with catastrophes and product from solution. Balasubramanian (2015) has derived finite markovian queue with catastrophe and bulk service. Rakesh Kumar (2008) have procured the solution for the queueing system with catastrophe and restoration along with batch arrivals in A catastrophic-cum-restorative queueing system with correlated batch arrivals and variable capacity. Jain, And Kumar, (2005) worked out the catastrophe in transient solution of a catastrophiccum-restorative queueing problem with correlated arrivals and variable service capacity. Seenivasan et al (2021) has obtained M/M/2 heterogeneous queueing system having unreliable server with catastrophes and restoration. Seenivasan and Abinaya(2021) has studied Markovian queueing model with single working vacation and catastrophe.

Dicrescenzo et al (2003) who analyzed well about the catastrophic queues in the m/m/1 queue with catastrophes and its continuous approximation. Kumar, And Arivudainambi,(2000) acquired the solution for single server queue in Transient Solution of an M/M/1 Queue with Catastrophes. Danesh Garg (2013) has found out the result by employing probability generating function in performance analysis of number of times a system reaches its capacity with catastrophe and restoration.

The mechanism of service is subject to partial breakdown. Perhaps the server still keeps working even in breakdown with a slower rate since it is considered as partial. After the process of repair, the server will switch over to normal working rate. Many authors have been analyzed breakdown since it's applicable in many areas like communication networks, manufacturing industries and so on. Kalidass, Kasturi (2012) had perused in queue with working breakdowns. Kalidas, Pavithra(2016) investigated a research in an M/M/1/N queue with working breakdowns and Bernoulli feedbacks. Kim, Lee.(2014) established his work in working breakdown in the M/G/1 queue with disasters and working breakdowns. Kim et al(2017) had studied about breakdown in Analysis of unreliable BMAP/PH/n type queue with Markovian flow of breakdowns. Yang & Wu (2017) had derived the Analysis of a finite-capacity system with working breakdowns and retention of impatient customers. Ye , Liu .(2018) accomplished his work in Analysis of MAP/M/1 queue with working breakdowns". Wartenhosrt (1995) done his research in N parallel Queueing systems with server breakdowns and repair. Neuts,.(1981)derived the Matrix-Geometric solutions in stochastic models. In this paper, we have four sections viz., introduction, model description, numerical illustrations and graphical representations.

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2. Model Description:

We consider a Markovian queueing model with single server . The arrival process follows Poisson distribution with mean rate λ. The service time follows exponential distribution with μ during busy period and with μ_1 during breakdown(refer FIGURE .1). The system becomes empty when it experiences catastrophe.

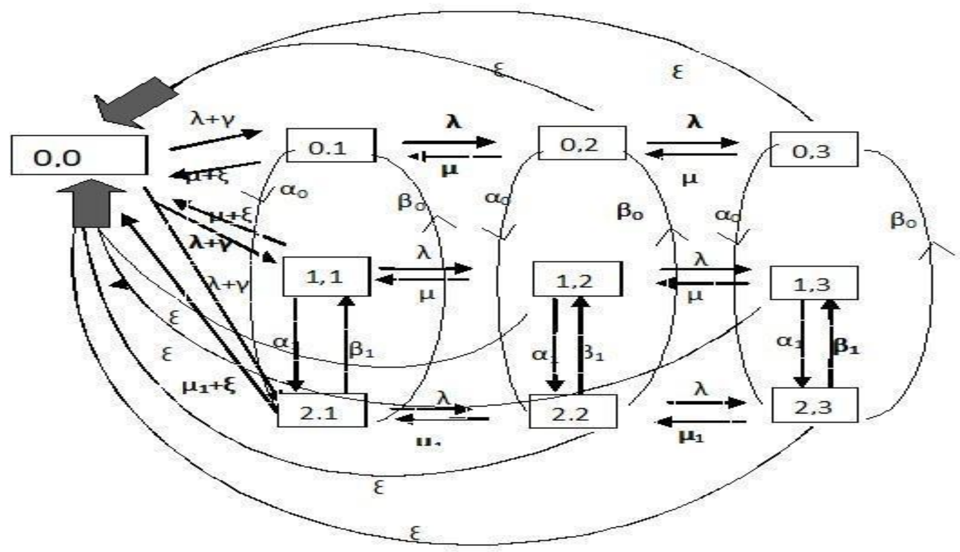


Fig 1. The system of transition diagram

However the catastrophe does not occur while system is not empty. The catastrophe occurs at the rate of ξ. After the occurrence of catastrophe, system takes its time to get ready to accept new customers. This time is restoration time which is identically independently distributed with parameter γ. The partial breakdown takes place at the rate of α_0 while the system is attaining its capacity first time and α_1 for second time. The repair time follows exponential distribution with β_0 and β_1.

Define, $P_{k,n}(t) = \text{Prob.}[K(t) = k, N(t) = n]$, $0 \le n \le N$

where $$K(t) = \begin{cases} 0\text{ , } \textit{when} \text{ the system attains its capacity one time} \\ 1\text{ , } \textit{when} \text{ the system attains its capacity two time} \\ 2\text{ , } \textit{when} \text{ the server is partial breakdown} \end{cases}$$

$N(t)$ = the number of customers in the system at time t.

The Quasi-Birth Death process along with the state space as follow: $\{0,0\}\ Q\{(i,j);i\ge 1,j=0,1\}$.
A QBD process with Infinitesimal generator matrix Q is considered and presented below:

$$Q=\begin{pmatrix} B_{00} & B_{01} & \dots & \dots & \dots & \dots \\ B_{10} & A_1 & A_0 & \dots & \dots & \dots \\ B_{20} & A_2 & A_1 & A_0 & \dots & \dots \\ B_{20} & \dots & A_2 & A_1 & A_0 & \dots \\ B_{20} & \dots & \dots & A_2 & A_1 & A_0 \end{pmatrix}$$

$B_{00}=-(3\lambda+3\gamma)$ $B_{01}=(\lambda+\gamma,\lambda+\gamma,\lambda+\gamma)$

$$B_{10}=\begin{pmatrix} \mu+\xi \\ \mu+\xi \\ \mu_1+\xi \end{pmatrix} \quad B_{20}=\begin{pmatrix} \xi \\ \xi \\ \xi \end{pmatrix}$$

$$A_0=\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$A_1=\begin{pmatrix} -(\lambda+\mu+\xi+\alpha_0) & 0 & \alpha_0 \\ 0 & -(\lambda+\mu+\xi+\alpha_1) & \alpha_1 \\ \beta_0 & \beta_1 & -(\lambda+\mu_1+\xi+\beta_0+\beta_1) \end{pmatrix}$$

$$A_2=\begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_1 \end{pmatrix}$$

2.1. Matrix Geometric Method and Stability Condition

$P_{k,n}(t) = \text{Prob.}[K(t) = k, N(t) = n], 0\leq n\leq N$

where
$$K(t)=\begin{cases} 0\,, \textit{when} \text{ the system attains its capacity one time} \\ 1\,, \textit{when} \text{ the system attains its capacity two time} \\ 2\,, \textit{when} \text{ the server is partial breakdown} \end{cases}$$

and N (t) = the number of customers in the system at time t.
The static probability row matrix is PQ=0 ----1

Where $P= (P_0, P_1, P_2, \dots)$, $P_0=P_{00}$ & $P_i=(P_{0i},P_{1i},P_{2i})$

From the equation 1, we have

$$B_{00}P_0+B_{10}P_1+B_{20}(P_2+P_3+\dots)=0$$

$$B_{01}P_0+A_1P_1+A_2P_2=0$$

$$A_0 P_1+A_1 P_2+A_2P_3=0$$

$$A_0 P_2+A_1 P_3+A_2P_4=0$$

In general, we have,

$$A_0 P_{i-1}+A_1 P_i+A_2P_{i+1}=0 \quad\text{--- 2}$$

We arrive at the geometric relation

$$P_i=P_1R_{i-1} \quad\text{---- 3}$$

By using the relation 3 in the equation 2,

$$R_{n+1} = -A_1^{-1}(A_0 + A_2R_n^2)$$

The condition of the normality is

$$P_0e+P_1(I-R)^{-1}e=1 \quad\text{---- 4}$$

where e is the column vector with all its elements equal to one.

The static condition of such a QBD, (See Neuts (1981)) can be obtained by the drift condition P A_0e < P A_2e

Where P==(P_0,P_1,P_2) is got from the generator A and A is given by A = A_0 +A_1+A_2 and so

$$A=\begin{pmatrix} -L & 0 & \alpha_0 \\ 0 & -M & \alpha_1 \\ \beta_0 & \beta_1 & -N \end{pmatrix}$$

$$where\ L=(\xi+\alpha_0), M=(\xi+\alpha_1)\ \&\ N=(\xi+\beta_0+\beta_1)$$

A is irreducible and P can be shown to be unique such that PA=0 and Pe=1 ---- 6

By using the equation 6, we have

$$P_0=\left(1+\frac{LN}{\alpha_0\beta_1}-\frac{\alpha_0}{\alpha_1}+\frac{L}{\beta_0}\right)^{-1}$$

$$P_1=\left(\frac{LN}{\alpha_0\beta_1}-\frac{\alpha_0}{\alpha_1}\right)P_0$$

$$P_2=\frac{L}{\beta_0}P_0$$

The static condition takes the format

$$\lambda\left(P_0+P_1+P_2\right)<\mu\left(P_0+P_1\right)+\mu_1P_2$$

The equation 7 is the static probability of A and the probability vectors are obtained by utilizing the equations 3 & 4 and the rate matrix.

3. Numerical Illustrations

The numerical illustrations are done by the mathematical concepts explained above. By varying the values of arrival rate λ, the service rate μ, the partial breakdown while the system reached its capacity zero time α_0 and the corresponding repair time β_0 , we have arrived at eighteen illustrations.

3.1 Illustration I:

It is assumed that $\lambda=0.1$, $\mu=0.7$, $\mu_1=0.3$, $\xi=0.05$, $\gamma=0.06$, $\alpha_0=0.01$, $\alpha_1=0.02$, $\beta_0=0.06$ and $\beta_1=0.07$.

The rate matrix is $\begin{pmatrix} 0.130336 & 0.000334 & 0.003043 \\ 0.000572 & 0.128813 & 0.005979 \\ 0.016278 & 0.018693 & 0.192751 \end{pmatrix}$

Table I.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.507802			0.507802
P_1	0.120998	0.121716	0.164149	0.406863
P_2	0.018512	0.018787	0.032736	0.070035
P_3	0.002956	0.003038	0.006478	0.012472
P_4	0.000492	0.000513	0.001276	0.002281
P_5	0.000085	0.000090	0.000250	0.000425
P_6	0.000015	0.000016	0.000049	0.000080
P_7	0.000003	0.000003	0.000009	0.000015
P_8	0.000000	0.000001	0.000002	0.000003
			Total	0.999976

The probability vectors are given by $P = (P_0, P_1, P_2, \ldots)$. The vectors $P_0 = 0.507802$ and $P_1 = (0.120988, 0.121716, 0.164149)$ are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be $0.999976 \cong 1$

3.2 Illustration II:

It is assumed that λ=0.2, μ=0.7, μ_1=0.3, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.256951 & 0.000866 & 0.006569 \\ 0.001477 & 0.253703 & 0.012827 \\ 0.031212 & 0.035729 & 0.350513 \end{pmatrix}$

Table II.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.348010			0.348010
P_1	0.133947	0.134614	0.169518	0.438079
P_2	0.039907	0.040325	0.062025	0.142257
P_3	0.012250	0.012481	0.022520	0.047251
P_4	0.003869	0.003982	0.008134	0.015985
P_5	0.001254	0.001304	0.002927	0.005485
P_6	0.000415	0.000436	0.001051	0.001902
P_7	0.000140	0.000149	0.000377	0.000666
P_8	0.000048	0.000051	0.000135	0.000234
P_9	0.000016	0.000018	0.000048	0.000082
P_{10}	0.000006	0.000006	0.000017	0.000029
P_{11}	0.000002	0.000002	0.000006	0.000010
P_{12}	0.000001	0.000001	0.000002	0.000004
P_{13}	0.000000	0.000000	0.000001	0.000001
			Total	0.999995

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.348010 and P_1 = (0.133947, 0.134614, 0.169518) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be $0.999995 \cong 1$

3.3 Illustration III:

It is assumed that λ=0.3, μ=0.7, μ_1=0.3, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.378177 & 0.001606 & 0.010285 \\ 0.002727 & 0.372957 & 0.019928 \\ 0.043722 & 0.049872 & 0.474253 \end{pmatrix}$

Table III.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.242676			0.242676
P_1	0.127445	0.127852	0.151719	0.407016
P_2	0.055179	0.055454	0.075812	0.186445
P_3	0.024333	0.024551	0.037626	0.086510
P_4	0.010914	0.011072	0.018584	0.040570
P_5	0.004970	0.005074	0.009146	0.019190
P_6	0.002293	0.002356	0.004490	0.009139
P_7	0.001070	0.001106	0.002200	0.004376
P_8	0.000504	0.000524	0.001076	0.002104
P_9	0.000239	0.000250	0.000526	0.001015
P_{10}	0.000114	0.000120	0.000257	0.000491
P_{11}	0.000055	0.000058	0.000125	0.000238
P_{12}	0.000026	0.000028	0.000061	0.000115
P_{13}	0.000013	0.000013	0.000030	0.000056
P_{14}	0.000006	0.000006	0.000014	0.000026
P_{15}	0.000003	0.000003	0.000007	0.000013
P_{16}	0.000001	0.000001	0.000003	0.000005
P_{17}	0.000001	0.000001	0.000001	0.000003
P_{18}	0.000000	0.000000	0.000001	0.000001
			Total	0.999989

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.242676 and P1 = (0.127445, 0.127852, 0.151719) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999989 ≅ 1

3.4 Illustration IV:

It is assumed that λ=0.4, μ=0.7, μ_1=0.3, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.490986 & 0.001820 & 0.008671 \\ 0.003111 & 0.482830 & 0.016778 \\ 0.047874 & 0.054178 & 0.439155 \end{pmatrix}$

Table IV.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.171289			0.171289
P_1	0.112060	0.112143	0.126669	0.350872
P_2	0.062280	0.062207	0.076512	0.200999
P_3	0.034938	0.034884	0.045996	0.115818
P_4	0.019763	0.019749	0.027550	0.067062
P_5	0.011261	0.011271	0.016456	0.038988
P_6	0.006456	0.006477	0.009808	0.022741
P_7	0.003722	0.003743	0.005835	0.013300
P_8	0.002155	0.002173	0.003467	0.007795
P_9	0.001252	0.001267	0.002058	0.004577
P_{10}	0.000730	0.000740	0.001221	0.002691
P_{11}	0.000427	0.000434	0.000724	0.001585
P_{12}	0.000250	0.000255	0.000429	0.000934
P_{13}	0.000147	0.000150	0.000254	0.000551
P_{14}	0.000086	0.000088	0.000150	0.000324
P_{15}	0.000051	0.000052	0.000089	0.000192
P_{16}	0.000030	0.000031	0.000053	0.000114
P_{17}	0.000017	0.000018	0.000031	0.000066
P_{18}	0.000010	0.000011	0.000018	0.000039
P_{19}	0.000006	0.000006	0.000011	0.000023
P_{20}	0.000003	0.000004	0.000006	0.000013
P_{21}	0.000002	0.000002	0.000004	0.000008
P_{22}	0.000001	0.000001	0.000002	0.000004
P_{23}	0.000001	0.000001	0.000001	0.000003
P_{24}	0.000000	0.000000	0.000001	0.000001

			Total	0.999989

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.171289 and P1 = (0.112060, 0.112143, 0.126669) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999989 $\cong$ 1

3.5 Illustration V:

It is assumed that λ=0.5, μ=0.7, μ_1=0.3, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.593050 & 0.002676 & 0.010894 \\ 0.004569 & 0.582604 & 0.020943 \\ 0.058943 & 0.066354 & 0.519968 \end{pmatrix}$

Table V.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.122301			0.122301
P_1	0.093734	0.093553	0.101597	0.288884
P_2	0.062173	0.061732	0.069413	0.193318
P_3	0.041357	0.040894	0.047327	0.129578
P_4	0.027577	0.027181	0.032214	0.086972
P_5	0.018427	0.018118	0.021897	0.058442
P_6	0.012335	0.012105	0.014868	0.039308
P_7	0.008269	0.008104	0.010086	0.026459
P_8	0.005550	0.005434	0.006838	0.017822
P_9	0.003730	0.003649	0.004632	0.012011
P_{10}	0.002508	0.002453	0.003137	0.008098
P_{11}	0.001688	0.001650	0.002123	0.005461
P_{12}	0.001137	0.001111	0.001437	0.003685
P_{13}	0.000766	0.000749	0.000972	0.002487
P_{14}	0.000516	0.000505	0.000657	0.001678
P_{15}	0.000348	0.000340	0.000444	0.001132
P_{16}	0.000235	0.000230	0.000300	0.000765

P_{17}	0.000158	0.000155	0.000203	0.000516
P_{18}	0.000107	0.000104	0.000137	0.000348
P_{19}	0.000072	0.000071	0.000093	0.000236
P_{20}	0.000049	0.000048	0.000063	0.000160
P_{21}	0.000033	0.000032	0.000042	0.000107
P_{22}	0.000022	0.000022	0.000029	0.000073
P_{23}	0.000015	0.000015	0.000019	0.000049
P_{24}	0.000010	0.000010	0.000013	0.000033
P_{25}	0.000007	0.000007	0.000009	0.000023
P_{26}	0.000004	0.000004	0.000006	0.000014
P_{27}	0.000003	0.000003	0.000004	0.000010
P_{28}	0.000002	0.000002	0.000003	0.000007
P_{29}	0.000001	0.000001	0.000002	0.000004
P_{30}	0.000001	0.000001	0.000001	0.000003
P_{31}	0.000001	0.000000	0.000001	0.000002
			Total	0.999986

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.122301 and P_1 = (0.093734, 0.093553, 0.101597) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999986 $\cong$ 1

3.6 Illustration VI:

It is assumed that μ=0.8, λ=0.1, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.115337 & 0.000121 & 0.000163 \\ 0.000271 & 0.114078 & 0.003198 \\ 0.009431 & 0.010850 & 0.124590 \end{pmatrix}$

Table VI.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.528156			0.528156
P_1	0.111424	0.112283	0.169856	0.393563
P_2	0.015339	0.015650	0.033609	0.064598

P_3	0.002257	0.002344	0.006597	0.011198
P_4	0.000356	0.000377	0.001289	0.002022
P_5	0.000060	0.000064	0.000251	0.000375
P_6	0.000010	0.000011	0.000049	0.000070
P_7	0.000002	0.000002	0.000009	0.000013
P_8	0.000000	0.000000	0.000002	0.000002
			Total	0.999997

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.528156 and P_1 = (0.111424, 0.112283, 0.169856) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999997$\cong$1

3.7 Illustration VII:

It is assumed that μ=0.9, λ=0.1, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.103530 & 0.000125 & 0.001458 \\ 0.000214 & 0.102499 & 0.002879 \\ 0.008381 & 0.009658 & 0.124573 \end{pmatrix}$

Table VII.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.545022			0.545022
P_1	0.103135	0.104074	0.174576	0.381785
P_2	0.012939	0.013262	0.034346	0.060547
P_3	0.001781	0.001867	0.006705	0.010353
P_4	0.000270	0.000290	0.001303	0.001863
P_5	0.000045	0.000049	0.000253	0.000347
P_6	0.000008	0.000009	0.000049	0.000066
P_7	0.000001	0.000001	0.000009	0.000011
P_{08}	0.000000	0.000000	0.000001	0.000001
			Total	0.999995

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.545022 and P_1 = (0.103135, 0.104074, 0.174576) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999995 ≅ 1

3.8 Illustration VIII:

It is assumed that μ=1, λ=0.1, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.093878 & 0.000101 & 0.001323 \\ 0.000173 & 0.093036 & 0.002615 \\ 0.007538 & 0.008698 & 0.124530 \end{pmatrix}$

Table VIII.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.559226			0.559226
P_1	0.095923	0.096903	0.178544	0.371370
P_2	0.011082	0.011403	0.034975	0.057460
P_3	0.001445	0.001526	0.006801	0.009772
P_4	0.000214	0.000232	0.001318	0.001764
P_5	0.000035	0.000039	0.000255	0.000329
P_6	0.000006	0.000007	0.000049	0.000062
P_7	0.000001	0.000001	0.000009	0.000011
P_8	0.000000	0.000000	0.000002	0.000002
			Total	0.999996

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.559226 and P1 = (0.095923, 0.096903, 0.178544) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999996 ≅ 1

3.9 Illustration IX:

It is assumed that μ=1.1, λ=0.1, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.085863 & 0.000084 & 0.001211 \\ 0.000143 & 0.085163 & 0.002396 \\ 0.006849 & 0.007911 & 0.124496 \end{pmatrix}$

Table IX.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.571347			0.571347
P_1	0.089608	0.090606	0.181925	0.362139
P_2	0.009614	0.009927	0.035518	0.055059
P_3	0.001198	0.001275	0.006887	0.009360
P_4	0.000175	0.000192	0.001332	0.001699
P_5	0.000029	0.000032	0.000257	0.000318
P_6	0.000005	0.000006	0.000049	0.000060
P_7	0.000001	0.000001	0.000009	0.000011
P_{08}	0.000000	0.000000	0.000002	0.000002
			Total	0.999995

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.571347 and P1 = (0.089608, 0.090606, 0.181925) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999995 $\cong$ 1

3.10 Illustration X:

It is assumed that μ=1.2, λ=0.1, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.079103 & 0.000070 & 0.001116 \\ 0.000120 & 0.078512 & 0.002210 \\ 0.006274 & 0.007254 & 0.124467 \end{pmatrix}$

Table X.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.581946			0.581946
P_1	0.084052	0.085037	0.185077	0.354166
P_2	0.008413	0.008695	0.035810	0.052918
P_3	0.001006	0.001074	0.006886	0.008966
P_4	0.000145	0.000160	0.001321	0.001626
P_5	0.000024	0.000027	0.000253	0.000304
P_6	0.000004	0.000005	0.000048	0.000057
P_7	0.000001	0.000001	0.000009	0.000011
P_{08}	0.000000	0.000000	0.000002	0.000002
			Total	0.999996

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.581946 and P1 = (0.084052, 0.085037, 0.185077) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999996 ≅ 1

3.11 Illustration XI:

It is assumed that α_0=0.02, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.128505 & 0.000398 & 0.003613 \\ 0.000512 & 0.126919 & 0.005333 \\ 0.010647 & 0.012226 & 0.125037 \end{pmatrix}$

Table XI.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.506097			0.506097
P_1	0.118542	0.121716	0.167067	0.407325
P_2	0.018282	0.018889	0.033728	0.070899
P_3	0.002949	0.003078	0.006740	0.012767
P_4	0.000497	0.000525	0.001338	0.002360
P_5	0.000087	0.000093	0.000265	0.000445

P_6	0.000016	0.000017	0.000052	0.000085
P_7	0.000003	0.000003	0.000010	0.000016
P_{08}	0.000000	0.000000	0.000002	0.000002
			Total	0.999996

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.506097 and P_1 = (0.118542, 0.121716, 0.167067) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999996 $\cong$ 1

3.12 Illustration XII:

It is assumed that α_0=0.03, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.126836 & 0.000588 & 0.005341 \\ 0.000504 & 0.126920 & 0.005341 \\ 0.010496 & 0.012245 & 0.125195 \end{pmatrix}$

Table XII.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.505631			0.505631
P_1	0.117468	0.121794	0.168337	0.407599
P_2	0.017673	0.018966	0.034358	0.070997
P_3	0.002802	0.003106	0.006921	0.012829
P_4	0.000468	0.000533	0.001383	0.002384
P_5	0.000082	0.000095	0.000275	0.000452
P_6	0.000015	0.000017	0.000054	0.000086
P_7	0.000003	0.000003	0.000011	0.000017
P_{08}	0.000000	0.000000	0.000002	0.000002
			Total	0.999997

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.505631 and P1 = (0.117468, 0.121794, 0.168337) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999997 ≅ 1

3.13 Illustration XIII:

It is assumed that α_0=0.04, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.125213 & 0.000772 & 0.007020 \\ 0.000496 & 0.126921 & 0.005349 \\ 0.010349 & 0.012263 & 0.125347 \end{pmatrix}$

Table XIII.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.504591			0.504591
P_1	0.115786	0.121831	0.170338	0.407955
P_2	0.017278	0.019050	0.035122	0.071450
P_3	0.002731	0.003137	0.007128	0.012996
P_4	0.000456	0.000542	0.001433	0.002431
P_5	0.000080	0.000097	0.000286	0.000463
P_6	0.000014	0.000018	0.000057	0.000089
P_7	0.000003	0.000003	0.000011	0.000017
P_{08}	0.000000	0.000001	0.000002	0.000003
			Total	0.999995

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.504591 and P1 = (0.115786, 0.121831, 0.170338) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999995 ≅ 1

3.14 Illustration XIV:

It is assumed that α_0=0.05, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_1=0.02, β_0=0.06 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.123636 & 0.000950 & 0.008652 \\ 0.000489 & 0.126922 & 0.005357 \\ 0.010206 & 0.012281 & 0.125496 \end{pmatrix}$

Table XIV.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.503585			0.503585
P_1	0.114155	0.121867	0.172282	0.408304
P_2	0.016898	0.019130	0.035857	0.071885
P_3	0.002662	0.003168	0.007327	0.013157
P_4	0.000445	0.000550	0.001481	0.002476
P_5	0.000028	0.000099	0.000297	0.000424
P_6	0.000014	0.000018	0.000059	0.000091
P_7	0.000003	0.000003	0.000012	0.000018
P_{08}	0.000000	0.000001	0.000002	0.000003
			Total	0.999943

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.503585 and P_1 = (0.114155, 0.121867, 0.172282) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999943 ≅ 1

3.15 Illustration XV:

It is assumed that β_0=0.07, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.130250 & 0.000202 & 0.001800 \\ 0.000405 & 0.128555 & 0.003547 \\ 0.012396 & 0.012197 & 0.123029 \end{pmatrix}$

Table XV.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.508901			0.508901
P_1	0.123412	0.121577	0.160748	0.405737
P_2	0.019226	0.018708	0.032099	0.070033
P_3	0.003129	0.003016	0.006360	0.012505
P_4	0.000531	0.000508	0.001254	0.002293
P_5	0.000093	0.000089	0.000246	0.000428
P_6	0.000017	0.000016	0.000048	0.000081
P_7	0.000003	0.000003	0.000009	0.000015
P_{08}	0.000000	0.000000	0.000002	0.000002
			Total	0.999995

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.508901 and P1 = (0.123412, 0.121577, 0.160748) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be $0.999995 \cong 1$

3.16 Illustration XVI:

It is assumed that β_0=0.08, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.130276 & 0.000199 & 0.001776 \\ 0.000455 & 0.128548 & 0.003495 \\ 0.013962 & 0.012021 & 0.121413 \end{pmatrix}$

Table XVI.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.510903			0.510903
P_1	0.125897	0.121615	0.158399	0.405911
P_2	0.019793	0.018532	0.030342	0.068667
P_3	0.003224	0.002935	0.005772	0.011931
P_4	0.000542	0.000482	0.001093	0.002117
P_5	0.000094	0.000082	0.000200	0.000376
P_6	0.000016	0.000014	0.000039	0.000069

P_7	0.000003	0.000002	0.000007	0.000012
P_{08}	0.000000	0.000000	0.000001	0.000001
			Total	0.999987

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.510903 and P_1 = (0.125897, 0.121615, 0.158399) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999987 1

3.17 Illustration XVII:

It is assumed that β_0=0.09, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.130301 & 0.000196 & 0.001750 \\ 0.000504 & 0.128542 & 0.003444 \\ 0.015483 & 0.011850 & 0.119842 \end{pmatrix}$

Table XVII.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.512335			0.512335
P_1	0.128195	0.121566	0.155682	0.405443
P_2	0.020376	0.018416	0.029250	0.068042
P_3	0.003341	0.002889	0.005460	0.011690
P_4	0.000563	0.000468	0.001015	0.002046
P_5	0.000097	0.000078	0.000189	0.000364
P_6	0.000017	0.000013	0.000035	0.000065
P_7	0.000003	0.000002	0.000006	0.000011
P_{08}	0.000000	0.000000	0.000001	0.000001
			Total	0.999997

The probability vectors are given by P = (P0, P1, P2,…). The vectors P0 = 0.512335 and P1 = (0.128195, 0.121566, 0.155682) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999997 $\approx$ 1

3.18 Illustration XVIII:

It is assumed that β_0=0.1, λ=0.1, μ=0.7, μ_1=0.6, ξ=0.05, γ=0.06, α_0=0.01, α_1=0.02 and β_1=0.07

The rate matrix is $\begin{pmatrix} 0.130325 & 0.000193 & 0.001725 \\ 0.000551 & 0.128536 & 0.003395 \\ 0.016962 & 0.011684 & 0.118316 \end{pmatrix}$

Table XVIII.The Probability Vectors

	P_{0i}	P_{1i}	P_{2i}	Total
P_{00}	0.513737			0.513737
P_1	0.130403	0.121522	0.153028	0.404953
P_2	0.020923	0.018306	0.028213	0.067442
P_3	0.003447	0.002846	0.005171	0.011464
P_4	0.000581	0.000456	0.000944	0.001981
P_5	0.000099	0.000075	0.000172	0.000346
P_6	0.000017	0.000012	0.000031	0.000060
P_7	0.000003	0.000002	0.000006	0.000011
P_{08}	0.000000	0.000000	0.000001	0.000001
			Total	0.999995

The probability vectors are given by P = (P_0, P_1, P_2,…). The vectors P_0 = 0.513737 and P_1 = (0.130403, 0.121522, 0.153028) are found by using the condition 4. The balance probability vectors are acquired by using the relation given in the equation 3 and the rate matrix. Therefore, the last column comprises of three columns and the total probability is validated to be 0.999995 $\approx$ 1

4. Performance Measures:

The performance analysis is done by utilizing the probability vectors obtained below.

Probability of the system to be empty P(E)=P_0

Probability of mean number of customers to be present in the system when it is reaching its capacity one time, P (OTR) = $\sum_{i=1}^{\infty} iP_{0i}$

Probability of mean number of customers to be present in the system when it is reaching its capacity second time, P (TTR) = $\sum_{i=1}^{\infty} iP_{1i}$

Probability of mean number of customers to be present in the system when the server is under partial breakdown, P (BD) = $\sum_{i=1}^{\infty} iP_{2i}$

Probability of mean number of customers to be present in the system,

P (N) =P (E) + P (OTR) + P (TTR) + P (BD)

4.1 Performance Analysis of Arrival rate:

λ	P (E)	P (OTR)	P (TTR)	P (BD)	P (N)
0.1	0.505782	0.169394	0.171023	0.255782	1.101981
0.2	0.348010	0.276349	0.279478	0.419029	1.322866
0.3	0.242676	0.409127	0.412896	0.597420	1.662119
0.4	0.171289	0.590176	0.590887	0.790374	2.142726
0.5	0.122301	0.847139	0.835315	0.994113	2.798868

4.2 Performance Analysis of Service rate:

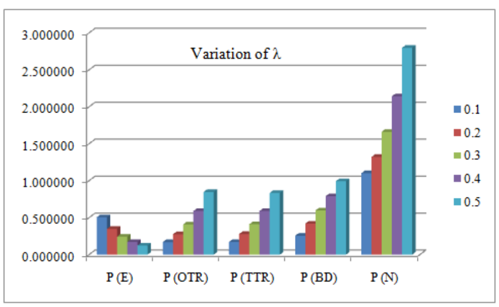
μ	P (E)	P (OTR)	P (TTR)	P (BD)	P (N)
0.8	0.528156	0.150671	0.152523	0.263649	1.094999
0.9	0.545022	0.135716	0.137665	0.270225	1.088628
1.0	0.559226	0.123496	0.125459	0.275817	1.083998
1.1	0.571347	0.113312	0.115256	0.280608	1.080523
1.2	0.581946	0.104627	0.106461	0.284271	1.077305

4.3 Performance Analysis of Breakdown Rate:

α_0	P (E)	P (OTR)	P (TTR)	P (BD)	P (N)
0.02	0.506097	0.166493	0.171416	0.261818	1.105824
0.03	0.505631	0.163613	0.171774	0.265140	1.106158
0.04	0.504591	0.160864	0.172132	0.269563	1.107150
0.05	0.503585	0.157962	0.172463	0.273840	1.107850

4.4 Performance Analysis for Repair Time

β_0	P (E)	P (OTR)	P (TTR)	P (BD)	P (N)
0.07	0.508901	0.173963	0.170635	0.250639	1.104138
0.08	0.510903	0.177910	0.169920	0.242062	1.100795
0.09	0.512335	0.181830	0.169419	0.235827	1.099411
0.10	0.513737	0.185532	0.168957	0.229839	1.098065



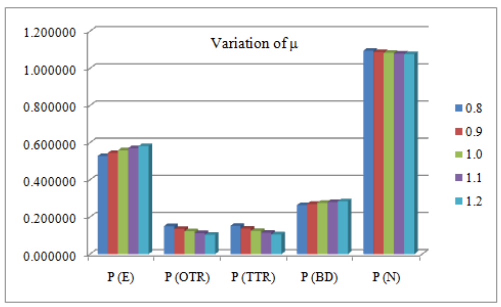
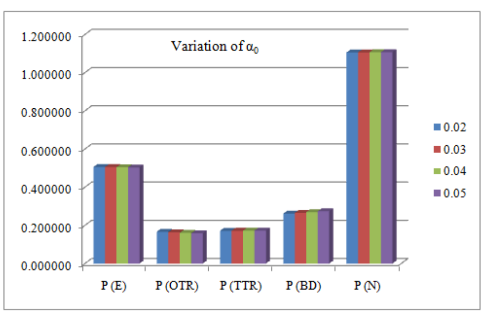


Fig.2. Variation of λ   Fig.3 Variation of μ



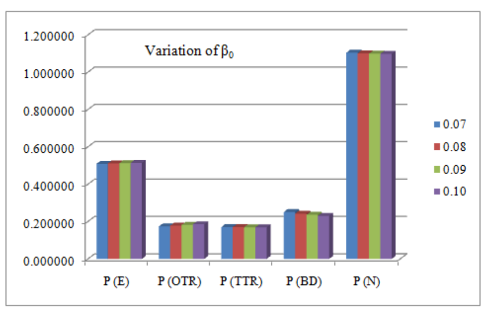


Fig.4. Variation of α_0 Fig.5. Variation of β_0

By using the probability vectors , the performance measures such as Probability of the system to be empty P(E),Probability of mean number of customers to be present in the system when it reaches its capacity one time, P (OTR) ,Probability of mean number of customers to be present in the system when it reaches its capacity two times, P (TTR),Probability of mean number of customers to be present in the system when the server is under partial breakdown, P (BD) , Probability of mean number of customers to be present in the system, P(N) is being obtained. While varying the arrival rate λ from 0.1 to 0.5, the P(E) gradually decreases whereas the other probabilities (P(OTR),P(TTR),P(BD)) are increasing steadily and P(N) increases rapidly. This has been shown in the Fig 2. The probability P(E) slowly increases, while the other probabilities are moderately falling down when varies the service rate from 0.8 to 1.2. The graphical representation of service rate is given in the Fig.3. The variation of partial breakdown shows slight decrease in the probabilities (P(E) & P(OTR)) and increase in the other ones which is shown by the graphical representation in Fig.4 . While varying the repair time, it has been analyzed that the probability P(E) & P(OTR) increases gradually whereas the other ones are falling down slowly.

Conclusion:

We have considered the single server queue with catastrophe, restoration and partial breakdown. The matrix geometric method has been employed to find the probability vectors. Performance analysis such as Probability of the system to be empty ,Probability of mean number of customers to be present in the system when it reaches its capacity one time, Probability of mean number of customers to be present in the system when it reaches its capacity two time, Probability of mean number of customers to be present in the system when the server is under partial breakdown, Probability of mean number of customers to be present in the system is done and numerical results are presented with graphical representation .

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