Intuitionistic Fuzzy Generalized # α- Connectedness in Intuitionsitic Fuzzy Topological Spaces

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Abstract

The at most aim of this prospectus is to initiate the idea of connectedness in Intuitionistic fuzzy $g^{\#}\alpha$ -closed set and explore its properties.

Keywords: IFT , IFG$^{\#}\alpha$ - CS, IFG$^{\#}\alpha$ - connected space.

1. INTRODUCTION

The idea of fuzzy sets was initiated by Zadeh[7]. Atanasov[1] generalized this concept to intuitionistic fuzzy sets using the concept of fuzzy sets. In 1997 Coker[2] initiated intuitionistic fuzzy topology using the notion of intuitionistic fuzzy topological spaces also he interpreted C_5 connected space in intuitionistic fuzzy topology. Later in 2011 intuitionistic fuzzy C_5 connected between two sets was explained by Santhi.R[5] and S.S. Thakur[6], initiated the concept of GO connected space in intuitionistic fuzzy topology. In 2012 Santhi.R[3], interpolated intuitionistic fuzzy generalized semi-pre connected space. Kokilavani. V introduced the concept of IFG$^{\#}\alpha$ - closed sets and IFG$^{\#}\alpha$ - continuous functions. In this prospectus, we initiated the concept of IFG$^{\#}\alpha$ - connected space and explore its properties.

2. PRELIMINARIES

Definition 2.2. [2] An intuitionistic fuzzy topology (*IFT* in short) on *X* is a family τ of *IFSs* in *X* satisfying the following axioms.

(1) $0_{\sim}, 1_{\sim} \in$

(2) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(3) $\cup G_i \in \tau$ for any family $\{G_i | i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space.
Throughout this section we denote connected space by Cntd space.

Definition 2.2. [2]An IFTS (X, τ) is proposed to be an IFC5-Cntd space if the only IFSs which are both IFOS are $0_{\sim}$ and $1_{\sim}$.

Definition 2.3. [6] An IFTS (X, τ) is proposed to be an IFGO-Cntd space if the only IFSs which are both IFGOS and IFGCS are $0_{\sim}$ and $1_{\sim}$.

Definition 2.4. [5] An IFTS (X, τ) is an IFC5-Cntd among two IFSs A and B if there is no IFOS E in (X, τ) such that A⊆E and E_{q^c} B.
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Definition 2.5. [3] An IFTS (X, τ) is proposed to be an intuitionistic fuzzy generalized semi-pre Cntd space if the only IFSs which are both an IFgsp- open sets and an IFgsp-closed set are $0_{\sim}$ and G_1 .

Definition 2.6. An IFS C in (E, τ) is proposed to be IFG$^{\#\#}$α CS if αcl(C) ⊆U, whenever C⊆U and U is an IFGOS in (E, τ).

3. Intuitionistic Fuzzy Generalized $^{\#\#}$α - Cntd in IFTS

In this segment we have initiated IFG#_-Cntd and establish its properties.

Definition 3.1. An IFT S (E, τ) is proposed to be IFG#_(IFG$^{\#\#}$α) Cntd on condition that IFSs has a pair IFG$^{\#\#}$α OS and IFG$^{\#\#}$α CS are $0_{\sim}$ and J_1 .

Theorem 3.2. Every IFG$^{\#\#}$α -Cntd is IFC5-Cntd but, reverse implication is not possible.

Proof: Consider (E, τ) an IFG$^{\#\#}$α - Cntd. Assume (E, τ) by no means IFC5- Cntd, here we obtain a actual pair of IFOS and IFCS of IFS D in (E, τ). Here D is the pair of IFG$^{\#\#}$α and IFG$^{\#\#}$α in (E, τ). This signifies here (E, τ) by no means an IFG$^{\#\#}$α - Cntd. This is a contravention to our assumption. Consequently (E, τ) should be IFC5-Cntd.

Example 3.3. Let E={p,q}, J_1= < e, (0.2_p, 0.2_q), (0.8_p, 0.8_q) > and J_2 = < e, (0.8_p, 0.7_q), (0.2_p, 0.3_q) >. Here τ= {$0_\sim$, J_1 ,J_2, $1_\sim$} a IFTs on E. We have (E, τ) an IFC5-Cntd but by no means IFG$^{\#\#}$α - Cntd, Considering IFS C = < e, (0.7_p, 0.6_q), (0.3_p, 0.6_q) > is both an IFG$^{\#\#}$α OS and an IFG$^{\#\#}$α CS in (E, τ).

Theorem 3.4. Every IFG#_-Cntd is IFGO-Cntd but, reverse implication is not possible.

Proof: Consider (E, τ) an IFG$^{\#\#}$α - Cntd. Assume (E, τ) is by no means IFGO- Cntd, here remains a actual IFS D which has the pair IFGOS and IFGCS in (E, τ). Then D is the pair of IFG$^{\#\#}$α and IFG$^{\#\#}$α in (E, τ). It signifies here (E, τ) by no means an IFG$^{\#\#}$α - Cntd. This is a contravention to our assumption. Consequently (E, τ) should be IFGO-Cntd.

Example 3.5. Consider E={p,q}, J_1= < e, (0.2_p, 0.2_q), (0.8_p, 0.8_q) > and J_2 = < e, (0.8_p, 0.7_q), (0.2_p, 0.3_q) >. Here τ= {$0_\sim$, J_1 ,J_2, $1_\sim$} a IFTs on E. We have (E, τ) an IFGO-Cntd but by no means IFG$^{\#\#}$α - Cntd, Considering IFS C = < e, (0.7_p, 0.6_q), (0.3_p, 0.6_q) > is both an IFG$^{\#\#}$α OS and an IFG$^{\#\#}$α CS in (E, τ).

Theorem 3.6. If h: (E, τ) → (F, σ) is IFG$^{\#\#}$α - continuous and (E, τ) is IFG$^{\#\#}$α - Cntd, then (F, σ) is IFC5-Cntd.

Proof: Consider (E, τ) an IFG$^{\#\#}$α -Cntd. Assume (F, σ) by no means an IFC5- Cntd, then we obtain a actual pair of IFOS and IFCS of IFS C in (F, σ) . Considering h an IFG$^{\#\#}$α -continuous, $h^{-1}(C)$ is a pair of IFG$^{\#\#}$α and IFG$^{\#\#}$α in (E, τ). This is a contravention to our assumption. Consequently (F, σ) is IFC5- Cntd.

Theorem 3.7. If h: (E, τ) → (F, σ) is IFG$^{\#\#}$α -irresolute onto mapping and (E, τ) an IFG$^{\#\#}$α - Cntd, then (F, σ) is also an IFG$^{\#\#}$α -Cntd.

Proof: Assume (F, σ) by no means an IFG$^{\#\#}$α -Cntd, then we obtain a actual pair of IFG$^{\#\#}$α OS and IFG$^{\#\#}$α CS IFS D in (F, σ) . Considering h an IFG$^{\#\#}$α - irresolute, $h^{-1}(D)$ is pair of IFG$^{\#\#}$α OS and

IFG$^{\#\#}$α CS in (E, τ). But this is a contravention to our assumption. Consequently (F, σ) is IFG$^{\#\#}$α -Cntd.

Proposition 3.8. An IFT S (E, τ) an IFG$^{\#\#}$α -Cntd on the condition that we obtain no zero IFG$^{\#\#}$α OSs C and D in (E, τ) in order that C = Dc.

Proof: **Necessity:** Consider C and D be IFG$^{\#\#}$α OSs in (E, τ) in order that C $\neq 0_\sim$, D $\neq 0_\sim$ and C=D^c. ConsequentlyD^c is IFG$^{\#\#}$α CS. Considering D $\neq 0_\sim$, C=$D^c \neq 1_\sim$. Considering C an actual IFS with a pair of IFG$^{\#\#}$α OS and IFG$^{\#\#}$α CS in (E, τ). Consequently (E, τ) by no means an IFG$^{\#\#}$α -Cntd. But this is contravention to our assumption. Thus we obtain no non-zero IFG$^{\#\#}$α OS C and D in (E, τ) in order that C = D^c.

Sufficiency: Let C be both an IFG$^{\#\#}$α OS and IFG$^{\#\#}$α CS in (E, τ) such that $1_\sim \neq$ C $\neq 0_\sim$. Now let D = C^c. Then D is an IFG$^{\#\#}$α OS and D $\neq 1_\sim$. Considering D^c =C $\neq 0_\sim$, which is a contravention to our assumption. Consequently (E, τ) is an IFG$^{\#\#}$α -Cntd.

Proposition 3.9. An IFT S (E, τ) is IFG$^{\#\#}$α - Cntd on the condition that we obtain no non-zero IFG$^{\#\#}$α OSs C and D in (E, τ) in order that D = C^c, D = (αcl(C))^c and C = D^c.

Proof: Necessity: Suppose that we obtain IFSs C and D in order that C $\neq 0_\sim$, D $\neq 0_\sim$, D=C^c, D= (αcl(C))^{c}and C = (αcl(D))^c. Since (αcl(C))^{c}and (αcl(D))^c are IFG$^{\#\#}$α OSs in (E, τ), C and D are IFG$^{\#\#}$α -Cntd, which is a contravention. Consequently we obtain no non-zero IFG$^{\#\#}$α OSs C and D in (E, τ) in order that D = C^c, D= (αcl(C))^{c}and C=(αcl(D))^c.

Sufficiency: Consider C be a pair of IFG$^{\#\#}$α OS and IFG$^{\#\#}$α CS in (E, τ) in order that$1_\sim \neq$ C $\neq 0_\sim$. Here we use D = C^c, thus remains an contravention to our assumption. Consequently (E, τ) an IFG$^{\#\#}$α - Cntd.

Definition 3.10. An IFTS (E, τ) is an IFC$_5$-Cntd among both IFS C and D if there is no IFTS G in (E, τ) in order that C⊆G and GqD.

Definition 3.11. An IFTS () is an IFG$^\#$α-Cntd among two IFS C and D if there is no IFG$^\#$αCS H in (E, τ) in order that C⊆H and HqD.

Example 3.12. Let E = {p, q}, J_1= < e, (0.6_p, 0.4_q), (0.4_p, 0.1_q) >. Here τ = {$0_\sim$, J_1, $1_\sim$} be IFT on E. We have (E, τ) on $IFG^{\#}\alpha$-Cntd amon the IFS C = < e, (0.6_p, 0.5_q), (0.4_p, 0.2_q) >, D = < e, (0.6_p, 0.5_q), (0.4_p, 0.4_q) > is pair of $IFG^{\#}\alpha$OS and $IFG^{\#}\alpha$CS in (E, τ).

Theorem 3.13. If an IFTS (E, τ) is an $IFG^{\#}\alpha$-Cntd among two IFS C and D, then it is an IFC_5-Cntd among C and D but, reverse implication is not possible.

Proof: Assume (E, τ) by no means an IFC_5 Cntd among C and D, then we obtain an IFOS H in (E, τ) in order that C⊆H and HqD. Considering IFOS an $IFG^{\#}\alpha$OS, we obtain an $IFG^{\#}\alpha$ OS H in (E, τ) in order that C⊆H and HqD. This signifies (E, τ) by no means $IFG^{\#}\alpha$-Cntd among C and D. That is we get a contravention to our assumption. Consequently, the IFTS (E, τ) should be IFC_5- Cntd among C and D.

Example 3.14. Consider E = {p, q}, J_1= < e, (0.3_p, 0.3_q), (0.2_p, 0.3_q). τ = {$0_\sim$, J_1, $1_\sim$} be IFT on E. We have (E, τ) an IFC_5-Cntd among the IFS C = < e, (0.3_p, 0.4_q), (0.6_p, 0.6_q) > and D = < e, (0.5_p, 0.4_q), (0.5_p, 0.6_q) >. But (E, τ) is not an $IFG^{\#}\alpha$ -Cntd among C and D, Considering IFS H= < e, (0.4_p, 0.4_q), (0.6_p, 0.5_q) > an $IFG^{\#}\alpha$OS such that C⊆H and H⊆D^c.

Theorem 3.15. An IFT S (E, τ) is $IFG^{\#}\alpha$ -Cntd among two IFSs C and D on condition that there is no $IFG^{\#}\alpha$OS and $IFG^{\#}\alpha$CS H in (E, τ) in order that C⊆ H ⊆D^c.

Proof: Necessity: Consider (E, τ) an $IFG^{\#}\alpha$ -Cntd among C and D. Assume that we obtain $IFG^{\#}\alpha$OS and $IFG^{\#}\alpha$CS H in (E, τ) in order that C⊆H⊆D^c, we have HqD and C⊆H. This signifies (E, τ) by no means IFG#_-Cntd among C and D, by the de_nition, C contravention to our assumption. Therefore, then we obtain no $IFG^{\#}\alpha$OS and $IFG^{\#}\alpha$CS H in (E, τ) in order that C⊆H⊆D^c.

Sufficiency: Assume (E, τ) by no means $IFG^{\#}\alpha$-Cntd among C and D. Then we obtain an $IFG^{\#}\alpha$OS H in (E, τ) in order that C⊆H and HqD. This signifies that we obtain an $IFG^{\#}\alpha$OS H in (E, τ) in order that C⊆H⊆D^c. But a contravention to our assumption. Consequently (E, τ) should be $IFG^{\#}\alpha$-Cntd among C and D.

Theorem 3.16. If an IFT S (E, τ) is $IFG^{\#}\alpha$ -Cntd among C and D and C⊆C_1, D⊆D_1, then (E, τ) is an $IFG^{\#}\alpha$ -Cntd among C_1 and D_1.

Proof : Assume (E, τ) by no means $IFG^{\#}\alpha$ -Cntd among C_1 and D_1, by known Definition, we obtain $IFG^{\#}\alpha OS$ H in (E, τ) in order that $C_1 \subseteq H$ and HqD_1. This signifies $H \subseteq D^c{}_1$ and $C_1 \subseteq H$. That is $C \subseteq C_1 \subseteq H$. Hence $C \subseteq H$. Since $H \subseteq D^c{}_1$, $D_1 \subseteq H^c$. That is $D \subseteq D_1 \subseteq H^c$. Hence $H \subseteq D^c$. Therefore (E, τ) by no means an $IFG^{\#}\alpha$ -Cntd among C and D. Consequently a contravention to our assumption Thus E should be $IFG^{\#}\alpha$ -Cntd among C_1 and D_1.

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