Mathematical Statistician and Engineering Applications
ISSN: 2094-0343
2326-9865

Neutrosophic Beta Omega Closed Sets in Neutrosophic Topological Spaces

S. Pious Missier[1,b)], A. Anusuya[2,a)], A. Nagarajan[3,c)]
[1]Head & Associate Professor, Department of Mathematics, Don Bosco College of Arts and Science,
(Affiliated to Manonmaniam Sundaranar University,Tirunelveli) Keela Eral, Thoothukudi, Tamil Nadu-628 908, India.
[2] Research Scholar(Reg.No-19222232092024), V.O.Chidambaram College,
(Affiliated to Manonmaniam Sundaranar University,Tirunelveli), Thoothukudi-628 003, India.
[3]Head & Associate Professor, V. O. Chidambaram College,
Thoothukudi, Tamilnadu-628 008, India. nagarajan.voc@gmail.com
[b)]spmissier@gmail.com
[a)]anuanishnakul@gmail.com
[c)]nagarajan.voc@gmail.com

Article Info
Page Number: 129 – 137
Publication Issue:
Vol. 71 No. 3s (2022)

Article History
Article Received: *22 April 2022*
Revised: *10 May 2022*
Accepted: *15 June 2022*
Publication: *19 July 2022*

Abstract

We introduce and examine several applications of neutrosophic beta omega closed sets namely $T_{N\beta\omega-sp}$, $PT_{N\beta\omega-sp}$, $\beta T_{N\beta\omega-sp}$, $SGT_{N\beta\omega-sp}$, $GST_{N\beta\omega-sp}$ in this study. We also introduce the concept of neutrosophic betaomega continuous mapping in neutrosophic topology. Furthermore, we study the relation between the neutrosophic betaomega continuous mapping with the already existing neutrosophic continuous mapping. In addition, we discuss the properties of neutrosophic betaomega continuous mapping.

Keywords: $T_{N\beta\omega-sp}$, $PT_{N\beta\omega-sp}$, $\beta T_{N\beta\omega-sp}$, $SGT_{N\beta\omega-sp}$, $GST_{N\beta\omega-sp}$,neutrosophic beta omega continuous mapping.

AMS Mathematics Subject Classification: 18B30,03E72

I. INTRODUCTION

Fuzzy set theory introduced by Zadeh[11] has laid the foundation for the new mathematical theories in the research of mathematics. Later, Attanasov[2] developed the concept of intuitionistic fuzzy sets. Smarandache[7] was the first to coin the term "neutrosophic set.". Neutrosophic operations and neutrosophic continuous functions have been investigated by Salama[10]. Here, we shall introduce the applications of neutrosophic beta omega closed set and neutrosophic continuity of neutrosophic-beta-omega sets. Also we present the implications of neutrosophic –beta-omega-continuous mapping.

II. PRELIMINARIES

Definition 2.1.[7] If Δ_N is a nonempty fixed set. A neutrosophic set (N-Set) G_N is a form factor $G_N = \{\langle \zeta, \mu_{G_N}(\zeta), \sigma_{G_N}(\zeta), \upsilon_{G_N}(\zeta)\rangle : \zeta \in \Delta_N\}$ where $\mu_{G_N}(\zeta)$, $\sigma_{G_N}(\zeta)$ and $\upsilon_{G_N}(\zeta)$represents the membership level, non-determination level and non-membership level respectively of each element $\zeta \in \Delta_N$to the set G_N. A Neutrosophic set $G_N = \{\langle \zeta, \mu_{G_N}(\zeta), \sigma_{G_N}(\zeta), \upsilon_{G_N}(\zeta)\rangle : \zeta \in \Delta_N\}$ can be recognized as a triple order $\langle \mu_{G_N}, \sigma_{G_N}, \upsilon_{G_N} \rangle$ in $]0,1^+[$on Δ_N.

Definition 2.2. [1] For any two sets $G_N$and H_N,

1. $G_N \subseteq H_N \Leftrightarrow \mu_{G_N}(\zeta) \leq \mu_{H_N}(\zeta)$, $\sigma_{G_N}(\zeta) \leq \sigma_{H_N}(\zeta)$ and $\upsilon_{G_N}(\zeta) \geq \upsilon_{H_N}(\zeta)$ $\forall \zeta \in \Delta_N$
2. $G_N \cap H_N = \langle \zeta, \mu_{G_N}(\zeta) \wedge \mu_{H_N}(\zeta), \sigma_{G_N}(\zeta) \wedge \sigma_{H_N}(\zeta), \upsilon_{G_N}(\zeta) \vee \upsilon_{H_N}(\zeta) \rangle$
3. $G_N \cup H_N = \langle \zeta, \mu_{G_N}(\zeta) \vee \mu_{H_N}(\zeta), \sigma_{G_N}(\zeta) \vee \sigma_{H_N}(\zeta), \upsilon_{G_N}(\zeta) \wedge \upsilon_{H_N}(\zeta) \rangle$
4. $G_N{}^C = \{\langle \zeta, \upsilon_{G_N}(\zeta), 1- \sigma_{G_N}(\zeta), \mu_{G_N}(\zeta)\rangle : \zeta \in \Delta_N\}$
5. $0_N = \{\langle \zeta, 0, 0, 1\rangle : \zeta \in \Delta_N\}$
6. $1_N = \{\langle \zeta, 1, 1, 0\rangle : \zeta \in \Delta_N\}$

Definition 2.3. [9] A neutrosophic topology (NT) is a family τ_N of neutrosophic subsets of$\Delta_N$whose member satisfy the following axioms:

1. $0_N, 1_N \in \tau_N$
2. $\wp_N \cap \mathcal{R}_N \in \tau_N$ for any $\wp_N, \mathcal{R}_N \in \tau_N$
3. $\cup\ \mathcal{R}_{N_i} \in \tau_N\ \forall \{\mathcal{R}_{N_i} : i \in J\} \subseteq \tau_N$

Any neutrosophic set in τ_N is a neutrosophic open set (NO-set). If a neutrosophic set G_N is a neutrosophic open set,then its complement $G_N{}^c$ is a neutrosophic closed set(NC-set) in Δ_N.Also, neutrosophic topological space (NTS) is denoted by (Δ_N, τ_N).

Definition 2.4. [8] A N-SetG_N of (Δ_N,τ_N) is called***neutrosophic beta omegaclosed***($N\beta\omega C$) if $\beta cl_N(G_N) \subseteq U_N$ whenever $G_N \subseteq U_N$ and U_N is $N\omega O$ in (Δ_N,τ_N).

Definition 2.5. [8]For every set $G_N \subseteq (\Delta_N, \tau_N)$, we define

1. $\beta\omega cl_N(G_N) = \cap\{V_N : G_N \subseteq V_N, V_N \in N\beta\omega C(\Delta_N, \tau_N)\}$.
2. $\beta\omega int_N(G_N) = \cup\{U_N : U_N \subseteq G_N$ and $U_N \in N\beta\omega O(\Delta_N, \tau_N)\}$

Definition2.6. [6]A mapping$f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$is said to be

1. ***Neutrosophic-continuous***(N_{cont}) if $f^{-1}(G_N)$ is NCset in (Δ_N, τ_N) for eachNC-set G_N in (Γ_N, σ_N).
2. ***Neutrosophic-pre-continuous***(NP_{cont}) if $f^{-1}(G_N)$ is NPCset in (Δ_N,τ_N) for eachNC-set G_N in (Γ_N, σ_N).
3. ***Neutrosophic-beta-continuous***($N\beta_{cont}$) if $f^{-1}(G_N)$ is $N\beta$Cset in (Δ_N, τ_N) for eachNC-set G_N in (Γ_N, σ_N)
4. ***Neutrosophic-alpha-generalized-continuous***($N\alpha G_{cont}$) if $f^{-1}(G_N)$ is $N\alpha GC$set in (Δ_N, τ_N) for eachNC-set G_N in (Γ_N, σ_N).

III. APPLICATIONS OF NEUTROSOPHIC BETA OMEGA CLOSED SETS

Definition 3.1.A neutrosophic topological space(Δ_N,τ_N) is called a

1. $T_{N\beta\omega}$-space($T_{N\beta\omega-sp}$) if all$N\beta\omega C$set is NC

2. Pre-$T_{N\beta\omega}$-space($PT_{N\beta\omega-sp}$) if all$N\beta\omega C$set is NPC.
3. Beta-$T_{N\beta\omega}$-space($\beta T_{N\beta\omega-sp}$) if all$N\beta\omega C$set is $N\beta C$.
4. Semi-generalized-$T_{N\beta\omega}$-space($SGT_{N\beta\omega-sp}$) if all$N\beta\omega C$set is $NSGC$.
5. Generalized-semi-$T_{N\beta\omega}$-space($GST_{N\beta\omega-sp}$) if all$N\beta\omega C$set is $NGSC$.

Example 3.1. Let Δ_N = [0, 1], $\tau_N = \{0_N, \ G_N{}^c(\lambda), H_N{}^c(\lambda), 1_N\}$,$(\Delta_N, \tau_N) = \{0_N,$ $G_N(\lambda), H_N(\lambda), 1_N\}$. where

$$G_N(\lambda) = \begin{cases} <\lambda,\ \lambda,\ 1-\lambda> & where\ 0 \le \lambda\ \le \frac{1}{2} \\ <1-\lambda, 1-\ \lambda,\ \lambda> & \frac{1}{2} \le \lambda\ \le 1 \end{cases} \text{and}$$

$$H_N(\lambda) = \begin{cases} <\lambda, \lambda, 1-\ \lambda> & where\ 0 \le \lambda\ \le \frac{1}{2} \\ <\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2}> & \frac{1}{2} \le \lambda\ \le 1 \end{cases}.$$

Then τ_N is a NT. Here, (Δ_N, τ_N) is $T_{N\beta\omega-sp}$.

Example 3.2. Let $\Delta_N = \{\lambda_1, \lambda_2,\ \lambda_3\}$, $\tau_N = \{0_N,\ G_N, 1_N\}$ where $G_N = <\zeta,\ \left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.5}\right),$ $\left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.7}\right),\ \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.5}, \frac{\lambda_3}{0.4}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N,\ \tau_N)$ = $(\Delta_N,\ \tau_N)$-{ P_N / $G_N \subseteq P_N \subset 1_N$} $= NPC(\Delta_N,\ \tau_N)$. Therefore,every $N\beta\omega C$set isNPC. Hence(Δ_N, τ_N) is $PT_{N\beta\omega-sp}$.

Example 3.3. Let $\Delta_N = \{\lambda_1, \lambda_2,\ \lambda_3\}$, $\tau_N = \{0_N,\ G_N, 1_N\}$ where $G_N = <\zeta,\ \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.6}\right),$ $\left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.7}, \frac{\lambda_3}{0.7}\right),\ \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.4}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N,\ \tau_N)$ = $(\Delta_N,\ \tau_N)$-{ P_N / $G_N \subseteq P_N \subset 1_N$} $= N\beta C(\Delta_N,\ \tau_N)$. Therefore,every $N\beta\omega C$set is NβC. Hence$(\Delta_N,\ \tau_N)$ is $\beta T_{N\beta\omega-sp}$.

Example 3.4. Let $\Delta_N = \{\lambda_1,\ \lambda_2,\ \lambda_3\}$, $\tau_N = \{0_N,\ G_N, 1_N\}$ where $G_N = <\zeta,\ \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.4}, \frac{\lambda_3}{0.4}\right),$ $\left(\frac{\lambda_1}{0.2}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.1}\right),\ \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.9}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N, \tau_N) = (\Delta_N, \tau_N) = NGSC(\Delta_N,$ $\tau_N)$. Therefore every $N\beta\omega C$set is $NGSC$. Hence(Δ_N, τ_N) is $GST_{N\beta\omega-sp}$.

Example 3.5. Let $\Delta_N = \{\lambda_1,\ \lambda_2,\ \lambda_3\}$,$\tau_N = \{0_N,\ G_N, 1_N\}$ where $G_N = <\zeta,\ \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.3}\right),$ $\left(\frac{\lambda_1}{0.1}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.1}\right),\ \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.8}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N, \tau_N) = (\Delta_N, \tau_N) = NSGC(\Delta_N,$ $\tau_N)$. Therefore every $N\beta\omega C$set is $NSGC$. Hence$(\Delta_N, \tau_N)$is $SGT_{N\beta\omega-sp}$.

Theorem3.1.

(a). Every $PT_{N\beta\omega-sp}$ is $\beta T_{N\beta\omega-sp}$.
(b). Every $T_{N\beta\omega-sp}$ is $PT_{N\beta\omega-sp}$.
(c). Every $T_{N\beta\omega-sp}$ is $\beta T_{N\beta\omega-sp}$.
(d). Every $T_{N\beta\omega-sp}$ is $GST_{N\beta\omega-sp}$.
(e). Every $T_{N\beta\omega-sp}$ is $SGT_{N\beta\omega-sp}$.

Proof :

(a). Let(Δ_N, τ_N) be$PT_{N\beta\omega-sp}$. We know that, every NPC set is $N\beta C$ set and since every $N\beta\omega C$set is NPC, we get $N\beta\omega C$set is $N\beta C$ set.Hence(Δ_N, τ_N) is a $\beta T_{N\beta\omega-sp}$.

(b), (c), (d) follow from the facts that every NC set is NPC set, $N\beta C$ set, NGSC set. $NSGC$ set respectively.

Remark 3.1. The examples below show that the converse of the preceding theorems does not have to be true.

Example 3.6. Take$\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.4}\right), \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.3}\right), \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.6}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N, \tau_N)$ = (Δ_N, τ_N) = $N\beta C(\Delta_N, \tau_N)$. But $NPC(\Delta_N, \tau_N)$ = (Δ_N, τ_N)-$\{P_N \ / \ G_N \subset P_N \subset {G_N}^C\}$-$\{Q_N \ / \ G_N \subset Q_N \nsubseteq {G_N}^C\}$ .Hence$(\Delta_N, \tau_N)$is $\beta T_{N\beta\omega-sp}$but not $PT_{N\beta\omega-sp}$.

Example 3.7. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.5}\right), \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.7}\right), \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.5}, \frac{\lambda_3}{0.4}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N, \tau_N)$ = (Δ_N, τ_N)-$\{P_N \ / \ G_N \subseteq P_N \subset 1_N\}$ $= NPC(\Delta_N, \tau_N)$and $NC(\Delta_N, \tau_N)$=$\{0_N, {G_N}^C, 1_N\}$. Hence$(\Delta_N, \tau_N)$is $PT_{N\beta\omega-sp}$but not $T_{N\beta\omega-sp}$.

Example 3.8. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.6}\right), \left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.7}, \frac{\lambda_3}{0.7}\right), \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.4}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N, \tau_N)$= (Δ_N, τ_N)-$\{P_N \ / \ G_N \subseteq P_N \subset 1_N\}$ $= N\beta C(\Delta_N, \tau_N)$ and $NC(\Delta_N, \tau_N)$ =$\{0_N, {G_N}^C, 1_N\}$. Hence$(\Delta_N, \tau_N)$is $\beta T_{N\beta\omega-sp}$but not $T_{N\beta\omega-sp}$.

Example 3.9. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.4}, \frac{\lambda_3}{0.4}\right), \left(\frac{\lambda_1}{0.2}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.1}\right), \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.9}\right)>$.Then τ_N is a NT. Then $N\beta\omega C(\Delta_N, \tau_N)$ = (Δ_N, τ_N)= $NGSC(\Delta_N, \tau_N)$ and $NC(\Delta_N, \tau_N)$ =$\{0_N, {G_N}^C, 1_N\}$. Hence$(\Delta_N, \tau_N)$is $GST_{N\beta\omega-sp}$but not $T_{N\beta\omega-sp}$.

Example 3.10. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.3}\right), \left(\frac{\lambda_1}{0.1}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.1}\right), \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.8}\right)>$.Then $\tau_N$is a NT. Then $N\beta\omega C(\Delta_N, \tau_N)$ = (Δ_N, τ_N)= $NSGC(\Delta_N, \tau_N)$ and $NC(\Delta_N, \tau_N)$ =$\{0_N, {G_N}^C, 1_N\}$. Therefore every $N\beta\omega C$ set is $NSGC$. Hence(Δ_N, τ_N) is $SGT_{N\beta\omega-sp}$but not $T_{N\beta\omega-sp}$.

IV. NEUTROSOPHIC BETA OMEGACONTINUOUS MAPPING

Definition4.1. A mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$is called***neutrosophic betaomegacontinuous***$(N\beta\omega_{cont})$ if $f^{-1}(G_N)$ is $N\beta\omega C$set in $(\Delta_N, \tau_N)$for allNC-set G_N in (Γ_N, σ_N).

Example 4.1. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$,$\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, W_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.7}, \frac{\lambda_3}{0.6}\right), \left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.7}\right), \left(\frac{\lambda_1}{0.4}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.4}\right)>$, $H_N = <$

$\zeta, \left(\frac{\delta_1}{0.7}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.8}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.3}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.2}\right) >$, W_N = $< \zeta, \left(\frac{\delta_1}{0.8}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.8}\right), \left(\frac{\delta_1}{0.9}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.2}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.2}\right) >$.Then τ_N and σ_N are NTs. Define a mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then $f^{-1}(V_N)$ is NβωC in (Δ_N, τ_N) for every NCset $V_N$in (Γ_N, σ_N). Hence$f$ is $N\beta\omega_{cont}$.

Theorem 4.1.

1. Every N_{cont} mapping is $N\beta\omega_{cont}$ mapping.
2. Every $N\beta_{cont}$ mapping is $N\beta\omega_{cont}$ mapping.
3. Every NG^*_{cont} mappingis $N\beta\omega_{cont}$ mapping.
4. Every $N\psi_{cont}$ mapping is $N\beta\omega_{cont}$ mapping.
5. Every $NW\mathrm{G}^*_{cont}$ mapping is $N\beta\omega_{cont}$ mapping.
6. Every NP_{cont} mapping is $N\beta\omega_{cont}$ mapping.
7.

Proof:

1. Taking$f$as aN_{cont} mappingandV_N be any NC-setin (Γ_N, σ_N). Since,f is N_{cont} mapping, we get $f^{-1}(V_N)$ is NC-set in (Δ_N, τ_N) implies $f^{-1}(V_N)$ is $N\beta\omega C$set. Hence f is $N\beta\omega_{cont}$ mapping in (Δ_N, τ_N).
2. Let f be any $N\beta_{cont}$ mapping and V_N be any NC-set in (Γ_N, σ_N). Since f is $N\beta_{cont}$ mapping, we get $f^{-1}(V_N)$ is $N\beta C$ in(Δ_N, τ_N).Since every$N\beta C$set is$N\beta\omega C$,$f^{-1}(V_N)$ is $N\beta\omega C$. Hence f is $N\beta\omega_{cont}$ mapping in (Δ_N, τ_N).
3. Let f beNG^*_{cont} mapping and V_N be any NC-set in (Γ_N, σ_N). Since f is NG^*_{cont} mapping, we get $f^{-1}(V_N)$ is NG^*C in (Δ_N, τ_N). Therefore, $f^{-1}(V_N)$ is $N\beta\omega C$, because every NG^*C set is $N\beta\omega C$ set. Hence f is $N\beta\omega_{cont}$ mapping in (Δ_N, τ_N)
4. Let f be $N\psi_{cont}$ mapping and V_N be any NC-set in (Γ_N, σ_N). Since f is $N\psi_{cont}$ mapping, we get $f^{-1}(V_N)$ is $N\psi C$in (Δ_N, τ_N). Since all$N\psi C$ set is $N\beta\omega C$,$f^{-1}(V_N)$ is NβωC. Hence f is $N\beta\omega_{cont}$ mapping in (Δ_N, τ_N).
5. Let f be $NW\mathrm{G}^*_{cont}$ mapping and V_N be any NC-set in (Γ_N, σ_N). Since f is $NW\mathrm{G}^*_{cont}$ mapping, we get $f^{-1}(V_N)$ is NWG^*C in (Δ_N, τ_N).We know that every NWG^*C set is $N\beta\omega$ set,$f^{-1}(V_N)$ is $N\beta\omega C$. Hence f is $N\beta\omega_{cont}$ mapping in (Δ_N, τ_N).
6. Let f be NP_{cont} mapping in (Δ_N, τ_N). Let V_N be any NC-set in (Γ_N, σ_N)). Since f is N_{cont} mapping, we get $f^{-1}(V_N)$ is NPCin (Δ_N, τ_N). Every NPC is $N\beta\omega C$ that implies$f^{-1}(V_N)$ is$N\beta\omega C$. Hence f is $N\beta\omega_{cont}$ mapping in (Δ_N, τ_N).

Example 4.2. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$,$\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = < \zeta, \left(\frac{\lambda_1}{0.2}, \frac{\lambda_2}{0.2}, \frac{\lambda_3}{0.1}\right), \left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.7}, \frac{\lambda_3}{0.7}\right), \left(\frac{\lambda_1}{0.7}, \frac{\lambda_2}{0.7}, \frac{\lambda_3}{0.7}\right) >$, $H_N = < \zeta, \left(\frac{\delta_1}{0.2}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.1}\right), \left(\frac{\delta_1}{0.2}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.2}\right), \left(\frac{\delta_1}{0.7}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.8}\right) >$.Then τ_N and σ_N are NTs. Define a mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is $N\beta\omega_{cont}$ mapping but not $N\beta_{cont}$ mapping.

Example 4.3. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$,$\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, W_N\ 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.5}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.5}\right), \left(\frac{\lambda_1}{0.4}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.4}\right), \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.7}\right)>$, $H_N = <\zeta, \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.8}, \frac{\delta_2}{0.8}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.3}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.3}\right)>$, $W_N = <\zeta, \left(\frac{\delta_1}{0.7}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.8}\right), \left(\frac{\delta_1}{0.8}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.2}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.2}\right)>$.Then τ_N and σ_N are NTs. Define a mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is $N\beta\omega_{cont}$ mapping but not $NG^*{}_{cont}$ mapping.

Example 4.4. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$,$\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\zeta, \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.1}\right), \left(\frac{\lambda_1}{0.2}, \frac{\lambda_2}{0.2}, \frac{\lambda_3}{0.2}\right), \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.7}\right)>$, $H_N = <\zeta, \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.6}\right), \left(\frac{\delta_1}{0.5}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.6}\right), \left(\frac{\delta_1}{0.4}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.4}\right)>$.Then $\tau_N$and σ_N are NT. Define a mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is $N\beta\omega_{cont}$ mapping but not $NWG^*{}_{cont}$ mapping.

Remark 4.1. The converse of the preceding theorem does not have to be true.

1. The example 4.1. shows that the converse of theorem 4.1(1) is not true.
2. Theexample 4.3. shows that the converse of the 4.1(4) does not exist.
3. The example 4.2 shows that $N\beta\omega_{cont}$ need not beNP_{cont} .

Remark 4.2. The following examples show that $N\beta\omega_{cont}$ mapping set and NGS_{cont} mapping are independent in (Δ_N, τ_N).

Example 4.5. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, W_N\ 1_N\}$, where $G_N = <\zeta, \left(\frac{\lambda_1}{0.9}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.9}\right), \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.7}\right), \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.2}, \frac{\lambda_3}{0.2}\right)>$, $H_N = <\zeta, \left(\frac{\delta_1}{0.7}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.8}\right), \left(\frac{\delta_1}{0.7}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.6}\right), \left(\frac{\delta_1}{0.4}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.7}\right)>$, $W_N = <\zeta, \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.8}\right), \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.6}\right), \left(\frac{\delta_1}{0.5}, \frac{\delta_2}{0.4}, \frac{\delta_3}{0.7}\right)>$.Then τ_N and σ_N are NT. Define a mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is $N\beta\omega_{cont}$ mapping notNGS_{cont} mapping.

Example 4.6. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$,$\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$, where $G_N = <\zeta, \left(\frac{\lambda_1}{0.9}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.9}\right), \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.7}, \frac{\lambda_3}{0.7}\right), \left(\frac{\lambda_1}{0.2}, \frac{\lambda_2}{0.2}, \frac{\lambda_3}{0.2}\right)>$, $H_N = <\zeta, \left(\frac{\delta_1}{0.1}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.1}\right), \left(\frac{\delta_1}{0.1}, \frac{\delta_2}{0.1}, \frac{\delta_3}{0.1}\right), \left(\frac{\delta_1}{0.9}, \frac{\delta_2}{0.9}, \frac{\delta_3}{0.9}\right)>$.Then τ_N and σ_N are NT. Define a mapping $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$.Then f isNGS_{cont} mapping but not $N\beta\omega_{cont}$ mapping.

Remark 4.3. Consider the examples4.5. and 4.6., these examples supports the fact thatNG_{cont} and $N\alpha G_{cont}$ mappings are independent with $N\beta\omega_{cont}$ in (Δ_N, τ_N).

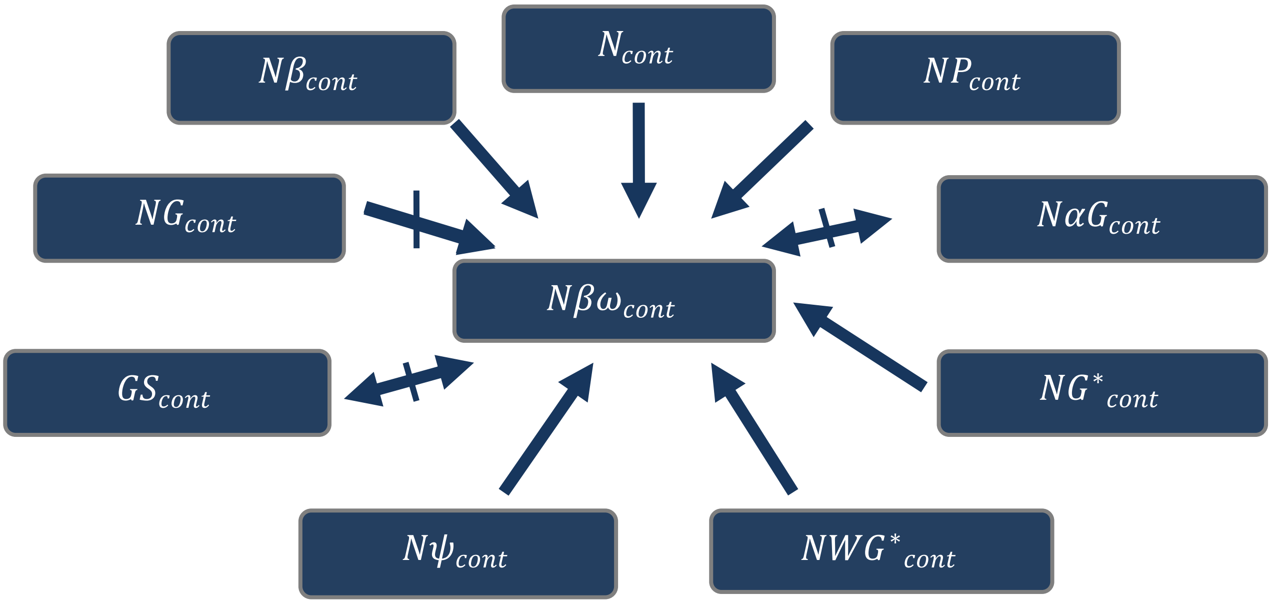

Figure.1(The implications of $N\beta\omega_{cont}$ mapping)

V. SOME THEOREMS AND PROPERTIES OF Nβω-CONTINUOUS MAPPING

Theorem5.1. If $f:(\Delta_N,\tau_N)\to(\Gamma_N,\sigma_N)$is $N\beta\omega_{cont}$ and$g:(\Gamma_N,\sigma_N)\to(\Omega_N,\varphi_N)$ is N_{cont} then their composition gof: $(\Delta_N,\tau_N)\to(\Omega_N,\varphi_N)$ is $N\beta\omega_{cont}$.

Proof : Let G_N be any N-set of (Ω_N,φ_N). Since g is N_{cont} , $g^{-1}(G_N)$is NC-set in (Γ_N,σ_N). Since f is $N\beta\omega_{cont}$, $f^{-1}(g^{-1}(G_N))=(gof)^{-1}(G_N)isN\beta\omega C$in (Δ_N,τ_N). Therefore gof is $N\beta\omega_{cont}$.

Remark 5.1. If two mappings are$N\beta\omega_{cont}$, Then theircompositionneed not be $N\beta\omega_{cont}$.

Example 5.1. Let $\Delta_N=\{\lambda_1,\lambda_2,\lambda_3\}$, $\Gamma_N=\{\delta_1,\delta_2,\delta_3\}$, $\Omega_N=\{\gamma_1,\gamma_2,\gamma_3\}$, $\tau_N=\{0_N,G_N,1_N\}$ and $\sigma_N=\{0_N,\ H_N,\ 1_N\}$, $\sigma_N=\{0_N,\ I_N,\ 1_N\}$, where $G_N=<\zeta,\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.8},\frac{\lambda_3}{0.7}\right),\left(\frac{\lambda_1}{0.8},\frac{\lambda_2}{0.7},\frac{\lambda_3}{0.8}\right),\left(\frac{\lambda_1}{0.3},\frac{\lambda_2}{0.2},\frac{\lambda_3}{0.3}\right)>$, $H_N=<\zeta,\left(\frac{\delta_1}{0.9},\frac{\delta_2}{0.9},\frac{\delta_3}{0.9}\right),\left(\frac{\delta_1}{0.9},\frac{\delta_2}{0.9},\frac{\delta_3}{0.9}\right),\left(\frac{\delta_1}{0.1},\frac{\delta_2}{0.1},\frac{\delta_3}{0.1}\right)>$, $I_N=<\zeta,\left(\frac{\delta_1}{0.2},\frac{\delta_2}{0.1},\frac{\delta_3}{0.2}\right),\left(\frac{\delta_1}{0.1},\frac{\delta_2}{0.1},\frac{\delta_3}{0.1}\right),\left(\frac{\delta_1}{0.8},\frac{\delta_2}{0.8},\frac{\delta_3}{0.8}\right)>$.Then τ_N and σ_N are NTs. Define the mappings$f:(\Delta_N,\tau_N)\to(\Gamma_N,\sigma_N)$ and$g:(\Gamma_N,\sigma_N)\to(\Omega_N,\varphi_N)$by $f(\lambda_1)=\delta_1$, $f(\lambda_2)=\delta_2$,$f(\lambda_3)=\delta_3$, $g(\delta_1)=\gamma_1$, $g(\delta_2)=\gamma_2$ and $g(\delta_3)=\gamma_3$. Then $f$and g are$N\beta\omega_{cont}$ mapping but $f\circ g$ is not$N\beta\omega_{cont}$ mapping.

Theorem 5.2. A map $f:(\Delta_N,\tau_N)\to(\Gamma_N,\sigma_N)$is $N\beta\omega_{cont}$ if and only if $f^{-1}(G_N)$ is $N\beta\omega O$ in (Δ_N,τ_N), for every NO-set G_N in (Γ_N,σ_N).

Proof : Let $f:(\Delta_N,\tau_N)\to(\Gamma_N,\sigma_N)beN\beta\omega_{cont}$ and G_N be aNO-set in (Γ_N,σ_N). Then $f^{-1}({G_N}^C)$ is $N\beta\omega C$in (Δ_N,τ_N). But $f^{-1}({G_N}^C)$=$(f^{-1}(G_N))^C$ and $f^{-1}(G_N)$is NβωO in (Δ_N,τ_N). Converse is similar.

Theorem 3.3.3. Let $f:(\Delta_N,\tau_N)\to(\Gamma_N,\sigma_N)$be a map. Assume that NβωO(Δ_N,τ_N) is NC-set under any union. Then the following are equivalent :

(i) The map f is $N\beta\omega_{cont}$;
(ii) $\beta\omega cl_N(G_N)) \subseteq cl_N\big(f(G_N)\big)$where $G_N \in (\Delta_N, \tau_N)$;
(iii) $\beta\omega cl_N\big(f^{-1}(H_N)\big) \subseteq f^{-1}\big(cl_N(H_N)\big)$ where $H_N \in (\Gamma_N, \sigma_N)$;
(iv) $f^{-1}\big(\beta\omega int_N(H_N)\big) \subseteq \beta\omega int_N\big(f^{-1}(H_N)\big) where\ H_N \in (\Gamma_N, \sigma_N)$.

Proof : (i) ⇔ (ii) : Assume that f is $N\beta\omega_{cont}$ and $G_N \in (\Delta_N, \tau_N)$. Since $G_N \subseteq f^{-1}(f(G_N))$. We have $G_N \subseteq f^{-1}(cl_N(f(G_N)))$. Since $cl_N(f(G_N))$ is a NC-set in (Γ_N, σ_N), by assumption $f^{-1}(cl_N(f(G_N)))$ is aNβωC containing G_N. Consequently,$cl_N(G_N) \subseteq f^{-1}(cl_N(f(G_N)))$. Thus $f(\beta\omega cl_N(G_N)) \subseteq f(f^{-1}(cl_N(f(G_N))) \subseteq cl_N(f(G_N))$. Conversely, suppose that (ii) holds for any subset G_N of (Δ_N, τ_N). Let W_N be a NC-set of (Γ_N, σ_N). Then by assumption $f(\beta\omega cl_N(f^{-1}(W_N))) \subseteq cl_N(f(f^{-1}(W_N))) \subseteq cl_N(W_N) = W_N$. Thus $\beta\omega cl_N(f^{-1}(W_N)) \subseteq f^{-1}(W_N)$ and so $f^{-1}(W_N)$ is *NβωC*.

(ii) ⇔ (iii) : Assume that $\beta\omega cl_N(G_N)) \subseteq cl_N\big(f(G_N)\big)$where $G_N \in (\Delta_N, \tau_N)$. Then putting$G_N$=$f^{-1}(H_N)$ in (ii) we get $f(\beta\omega cl_N(f^{-1}(H_N))) \subseteq cl_N(f(f^{-1}(H_N)) \subseteq cl_N(H_N)$. Thus $\beta\omega cl_N(f^{-1}(H_N)) \subseteq f^{-1}(cl_N(H_N))$. Conversely, Assume that (iii) holds. Let $H_N = f(G_N)$ where G_N is a subset of (Δ_N, τ_N). Then we have $\beta\omega cl_N(G_N) \subseteq \beta\omega cl_N(f^{-1}(H_N)) \subseteq f^{-1}(cl_N(f(G_N)))$ so $f(\beta\omega cl_N(G_N)) \subseteq cl_N(f(G_N))$.

(iii) ⇔ (iv) : Let H_N be any subset of (Γ_N, σ_N). Then by (iii) we have $\beta\omega cl_N(f^{-1}({H_N}^C)) \subseteq f^{-1}(cl_N({H_N}^C))$.

Hence $(\beta\omega int_N(f^{-1}(H_N))^C \subseteq (f^{-1}(int_N(H_N))^C$. Therefore, we obtain $f^{-1}(int_N(H_N)) \subseteq \beta\omega int_N(f^{-1}(H_N))$.

(iv) ⇔ (i) : Assume that $f^{-1}\big(\beta\omega int_N(H_N)\big) \subseteq \beta\omega int_N\big(f^{-1}(H_N)\big)\ and\ H_N \in (\Gamma_N, \sigma_N)$. Let W_N be any NC-set of (Γ_N, σ_N). Wehave $f^{-1}\big({W_N}^C\big)$ =$(int_N\big({W_N}^C\big) \subseteq \beta\omega int_N\left(f^{-1}\big({W_N}^C\big)\right) = (\beta\omega cl_N(f^{-1}(W_N)))^C$ and hence $\beta\omega cl_N(f^{-1}(W_N)) \subseteq f^{-1}(W_N)$ implies that $f^{-1}(W_N)$ is $N\beta\omega C$in (Δ_N, τ_N). Hence f is $N\beta\omega_{cont}$.

Proposition 3.2.3 Let $f: (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$be a $N\beta\omega_{cont}$.
(a). If (X, τ_N) is a $T_{N\beta\omega-sp}$, then f is N_{cont}.
(b). If (X, τ_N) is a $PT_{N\beta\omega-sp}$, then f is NP_{cont}.
(c). If (X, τ_N) is a $\beta T_{N\beta\omega-sp}$, then f is $N\beta_{cont}$.
(d). If (X, τ_N) is a $\alpha GT_{N\beta\omega-sp}$, then f is $N\alpha G_{cont}$.

Proof : The proof follows from the definitions.

CONCLUSION

In this article, we defined some new spaces of neutrosophic beta omega closed set. we also introduced and studied some characteristics of neutrosophic beta omega continuous mapping. obtained some of their basic properties. We have analyzed the relationship between the $N\beta\omega_{cont}$-Continuous mapping and some other Continuous mapping.

Acknowledgement

We are grateful to all of those with whom working this article and we would like to show our gratitude to the reviewers for their insight.

REFERENCE

1. R. Arokiarani, S. Dhavaseelan, Jafari and M. Parimala, “On Some New Notions and Functions in Neutrosophic Topological Spaces” , in *Neutrosophic Sets and Systems*, Vol.16, pp.16-19(2017).
2. K. Atanassov, “Intuitionistic fuzzy sets” , in VII ITKR’s Session;Publishing House; Sofia, Bulgaria,(1983). 7
3. S. Blessie Rebecca andFrancina Shalini, “Neutrosophic Generalized Regular Continuous Function in Neutrosophic Toplogical Spaces”, in *IJRAR*, February 2019, Vol.6, issue 1(2019).
4. C. L. Chang , “ Fuzzy Topological Spaces”, *J.Math. Anal ,Appl*, pp.182-190(1968).
5. D. Coker, “An introduction to intuitionistic fuzzy topological spaces”, in *Fuzzy sets and systems*, pp.81–89(1997).
6. R. Dhavaseelan and S. Jafari, “Neutrosophic generalized α-Contra-continuity”, in *CREAT. MATH. INFORM* ,No. 2, pp.133 - 139(2018).
7. Florentin Smarandache,“Single Valued Neutrosophic Sets”, in *Technical Sciences and Applied Mathematics*, pp. 10-14(2009).
8. S. Pious Missier and A. Anusuya, “Neutrosophic Beta Omega Closed Sets in Neutrosophic Fuzzy Topological Spaces”, in *the proceedings of 24th FAI-ICDBSMD 2021* Vol. 6(i), pp.42(2021).
9. S. A. Salama, Alblowi,“Neutrosophic set and Neutrosophic topological spaces”, in *IOSR Jour. of Mathematics*, pp. 31-35(2012).
10. Salama, Florentine Smarandache and Valeri Kromov, “Neutrosophic Closed set and Neutrosophic Continuous functions”, in *Neutrosophic sets and system*, Vol-4, pp.4-8(2014).
11. L. Zadeh, “Fuzzy sets”, in *Information and Control*, Vol.8, pp. 338–353(1965).