Two-parameter estimator for the Tobit regression model

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Abstract
The ridge estimator has been shown to be an effective shrinkage method for reducing the impacts of multicollinearity on a number of occasions. When there is just little information about the dependent variable for some observations, the Tobit regression model is a well-known model. However, it is well known that in the presence of multicollinearity, the variance of the maximum likelihood estimator (MLE) of the Tobit regression model coefficients can be negatively affected. In this research, a new two-parameter estimator is proposed to solve the Tobit regression model's multicollinearity problem. In terms of MSE, our Monte Carlo simulation results show that the proposed estimate outperforms the MLE and ridge estimators.
Keywords: Multicollinearity; ridge estimator; Tobit regression model; Monte Carlo simulation.

1. Introduction

Tobit regression is one of the most extensively used statistical approaches for describing the relationship between a response variable and a collection of variables among academics. It has been widely used in medicine, biology, ecology, economics, and the social sciences since its introduction by [1], it has been routinely applied in medicine, ecology, economics, social sciences, ecology, economics, and the biology. The (Tobit) model is one of the linear models that falls within the scope of the Censored regression models, and it is often called the Limited Regression Model and Tobin in 1958 is defined as a process of marriage between the regression model and the (Probir) model.

Tobit models refer to regression models that are characterized by the fact that the response variable is specific in some way depending on the nature of the phenomenon, as it differs from the truncated regression models, so the model is called a truncated regression model when the observations outside a certain range are missing (for the response variable and explanatory variables), while it is called A specific regressi The (Tobit) model is one of the linear models that falls within the scope of the controlled regression models, and it is often called the Limited Regression Model and Tobin in 1958 is defined as a process of marriage between the regression model and the (Probir) model [2-4].

Due to the nature of these models and the structure of their data, estimating their features is an important rhetorical issue, in addition to the accompanying breaches of some classic hypotheses of standard regression models accordingly, hence the importance of studying methods for estimating the features of these models and analyzing the characteristics of their capabilities. Also, proposing an optimal regression model for the data in question will lead to results that are close to the real reality, as for each type of data there is an optimal model that fits with it, for example if we have quantitative data for the response variable and assumptions for the model are available, they can be dealt with using the traditional regression model, Also, if binary data for the response variable are available, it can be dealt with using the logistic regression model.

In the case of availability of observations that are restricted (specific) and free in the other part (unspecified), where this data is called (censored data), then using the traditional regression model with this type of data will lead to biased estimated parameters on the one hand and on the other hand that is not. One of these alternatives is the controlled regression model proposed by Tobin in 1958 [1] known as the Tobit regression model, and it is designed to estimate the linear relationship between variables when there is either a left-wing control. Or right-wing in the response variable (also known as bottom and top control, respectively), The values that exceed a certain threshold level are called the control from the top, while the values less than the threshold value are called the control from the bottom. On the other hand, it is said that the multiple linear relationship problem exists when there are quasi-linear dependencies between the regression variables, as the Tobit model is also used to accommodate the potential correlation between the explanatory variables [5, 6].

2. Tobit Model

In many areas, such as sociology , medical, and econometrics studies, the Tobit regression model (TRM) is often used to evaluate data having left censored outputs. Consider the following latent linear regression model to introduce the TRM:

$$y_i = \beta' + x_i + \varepsilon_i \qquad \dots (1)$$

where $x_i = (x_{i1}, \cdot\cdot\cdot , x_{ik})$, ε_i's are residuals, β = (β_1, · · · , β_k) which can be estimated by using the maximum likelihood method.

The mathematical definition of the Tobit model, which is sometimes called the mathematically standard Tobit model, is as follows:

$$y_i^* = \beta' x_i + \varepsilon_i \qquad i = 1,2,3, \dots , n \qquad (2)$$

$$y_i = \begin{cases} y_i^* & \text{if} \quad y_i^* > 0 \\ 0 & \text{if} \quad y_i^* \le 0 \end{cases} \qquad \dots \quad (3)$$

In the Tobit model , y_i^* Latent variable is generated by a conventional linear regression model that is not seen at $y_i^* \le 0$.

As for [5], it has defined the model of Tobit , assuming that the observed response variable y_i for the observations $i = 1,2,3, \dots , n$ that achieves the following :

$$y_i = \max(y_i^*, 0) \qquad \dots \quad (4)$$

The constraint a and h can be formulated as follows , $y_i^* > \tau$ and $y_i^* \leq \tau$, respectively, without making a fundamental change in the model, whether y or known unknown . For estimation purposes, we assume that:

$\varepsilon_i = \sim N(0, \sigma^2)$

$y_i^*/x_i \sim N(\beta' X, \sigma_\varepsilon^2)$

$$d_i = \begin{cases} 1 & \text{if} \quad y_i^* > \tau \\ 0 & \text{if} \quad y_i^* \leq \tau \end{cases}$$

the density function of the y_i^* observations is given as follows :

$$f\,(y_i^* \mid \beta, \sigma^2) = \prod_{i=0}^{n} \Pr(y_i^* < \tau)^{(1-d_i)} \prod_{i=1}^{n} [\Pr(y_i^* > \tau)\ f(y_i^*/(y_i^* \geq \tau)]^{d_i} \dots \quad 5)$$

The equation (5) can be simplified as follows:

$$\Pr(y_i^* < \tau) = \Pr(\beta' X_i + \varepsilon_i < \tau) = \Pr\left(\frac{\beta' X_i + \varepsilon_i}{\sigma_\varepsilon^2} < \frac{\tau}{\sigma_\varepsilon^2}\right) = \Pr\left(\frac{\varepsilon_i}{\sigma_\varepsilon^2} < \frac{\tau - \beta' X_i}{\sigma_\varepsilon^2}\right)$$

$$= \emptyset\left(\frac{\tau - \beta' X_i}{\sigma_\varepsilon^2}\right) \quad \dots (6)$$

When $\tau = 0$:

$$\Pr(y_i^* < \tau) = \emptyset\left(\frac{-\beta' X_i}{\sigma_\varepsilon^2}\right) = 1 - \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon^2}\right) \quad \dots (7)$$

$$\Pr(y_i^* \geq \tau) = 1 - \emptyset\left(\frac{\tau - \beta' X_i}{\sigma_\varepsilon^2}\right) \quad \dots (8)$$

When $\tau = 1$

$$\Pr(y_i^* \geq \tau) = \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon^2}\right) \quad \dots (9)$$

$$f = (y_i^*/y_i^* > \tau) = \frac{1/_{\sigma}\, \vartheta[(y_i^* - \beta' X_i)/\sigma_\varepsilon]}{\Pr(y_i^* > \tau)} = \frac{1/_{\sigma}\, \vartheta[(y_i^* - \beta' X_i)/\sigma_\varepsilon]}{1/_{\sigma}\, \emptyset[(y_i^* - \beta' X_i)/\sigma_\varepsilon]}$$

Accordingly:

$$L = \prod_{i=0}^{n} \emptyset\left(\frac{\tau - \beta' X_i}{\sigma_\varepsilon}\right) \prod_{i=0}^{n} \left[1 - \emptyset\left(\frac{\tau - \beta' X_i}{\sigma_\varepsilon}\right)\right] \prod_{i=1}^{n} \frac{1/_{\sigma}\, \vartheta[(y_i^* - \beta' X_i)/\sigma_\varepsilon]}{1/_{\sigma}\, \emptyset[(y_i^* - \beta' X_i)/\sigma_\varepsilon]}$$

$$L = \prod_{i=0}^{n} \emptyset\left(\frac{\tau - \beta' X_i}{\sigma_\varepsilon}\right) \prod_{i=0}^{n} \sigma^{-1} \vartheta\left(\frac{y_i^* - \beta' X_i}{\sigma_\varepsilon}\right) \quad \dots (10)$$

When $\tau = 0$:

$$L = \prod_{i=0}^{n} \left[1 - \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)\right] \prod_{i=0}^{n} \sigma^{-1} \vartheta\left(\frac{y_i^* - \beta' X_i}{\sigma_\varepsilon}\right) \quad \ldots \ (11)$$

$$L = \prod_{i=1}^{n} \frac{1/_\sigma\, \vartheta[(y_i^* - \beta' X_i)/\sigma_\varepsilon]}{1/_\sigma\, \emptyset[(y - \beta' X_i)/\sigma_\varepsilon]}$$

$$L = \prod_{i=0}^{n} \left[1 - \emptyset\left(\frac{y - \beta' X_i}{\sigma_\varepsilon}\right)\right]^{-1} \sigma^{-1} \vartheta\left(\frac{y_i^* - \beta' X_i}{\sigma_\varepsilon}\right) \quad \ldots \ (12)$$

When $\tau = 1$:

$$L = \prod_{i=0}^{n} \left[\emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)\right]^{-1} \sigma^{-1} \vartheta\left(\frac{y_i^* - \beta' X_i}{\sigma_\varepsilon}\right) \quad \ldots \ (13)$$

The tobit maximum likelihood estimator maximizes the following censored log likelihood function:

$$L(\beta, \sigma^2/x) = \prod \left[1 - \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)\right] \prod \sigma^{-1} \vartheta\left(\frac{y_i^* - \beta' X_i}{\sigma_\varepsilon}\right)$$

And the logarithm takes the Maximum Likelihood function :

$$\log L(\beta, \sigma^2/x) = \sum \log\left[1 - \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)\right] - \frac{n_1}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum (y_i - \beta' X_i)^2 \ \ldots \ (14)$$

$$\frac{\partial \log L}{\partial \beta} = -\frac{1}{\sigma} \sum \frac{\vartheta\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)}{1 - \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)} + \frac{1}{\sigma^2} \sum (y_i - \beta' X_i)\, X_i \quad \ldots \ (15)$$

Numerical methods can be used to calculate the parameters of the Tobet model , To estimate the coefcient vector β, the derivative $S(\beta)$ of the tobit log-likelihood function should be equated to zero and solved, i.e.

$$S(\beta) = \frac{\partial \log L}{\partial \beta} = -\frac{1}{\sigma} \sum \frac{\vartheta\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)}{1 - \emptyset\left(\frac{\beta' X_i}{\sigma_\varepsilon}\right)} + \frac{1}{\sigma^2} \sum (y_i - \beta' X_i)\, X_i = 0 \quad \ldots \ (16)$$

The Newton-Raphson method is used to solve Equation (16) such that the iterations of the method is given as :

$$\beta_r = \beta_{r-1} + F^{-1}(\beta_{r-1}) S(\beta_{r-1}) \quad \ldots \ (17)$$

where $F(\beta_{r-1})$ is the Fisher information matrix computed using β_{r-1} at the $(r - 1)$ the step of the algorithm, namely, it is defined by :

$$F(\beta_{r-1}) = E\left(\frac{-\partial^2 L(\beta, \sigma^2/x}{\partial \beta\, \partial \beta'}\right) = X' \widehat{W}(\beta_{r-1}) X \quad \ldots \ (18)$$

where $W(\beta_{r-1}) = \text{diag}[\hat{z}_i(\beta r - 1)]$ such that letting $\omega = \frac{x_i' \beta}{\sigma}$, $\hat{z}_i$ is defined to be :

$$\hat{z}_i = \sigma^2 \emptyset(\omega) \left[1 - \left(\frac{\vartheta(\omega)}{\emptyset(\omega)} \right)^2 \right] - \omega \left(\frac{\vartheta(\omega)}{\emptyset(\omega)} \right) + \left(\omega + \frac{\vartheta(\omega)}{\emptyset(\omega)} \right)^2 \left(1 - \emptyset(\omega)\right) \quad \dots \ (19)$$

The vector of coefficients is updated until some convergence criterion is satisfied. At the final step of the algorithm, the vector of coefficients becomes the (mle) denoted by $\hat{\beta}_{mle}$ [5].

The covariance matrix of the Tobit (mle) estimator is obtained as :

$$\text{Cov}(\hat{\beta}_{mle}) = E\left(\frac{-\partial^2 L(\beta, \sigma^2/x}{\partial\beta\, \partial\beta'} \right)^{-1} = \left(X'\widehat{W}X\right)^{-1} \quad \dots \ (20)$$

and the scalar mean squared error (mle) of (mle) equals :

$$\text{MSE}(\hat{\beta}_{mle}) = \text{tr}\left(X'\widehat{W}X\right)^{-1} = \sum_{i=1}^{p+1} \frac{1}{\lambda_i} \quad \dots \ (21)$$

where $\Lambda = \text{diag}(\lambda_i)$ is the diagonal matrix consisting of the eigenvalues of the matrix $X'\widehat{W}X = Q'\widehat{W}Q$ = such that the matrix Q is composed of the eigenvectors of $(X'\widehat{W}X)$ as its columns [6].

It can be easily observed from Equation (21) that the MSE of (mle) is inflated when there is near-linear dependencies in the design matrix or in the weighted matrix of cross products $(X'\widehat{W}X)$ [7] . As a result of this problem , (mle) becomes unstable and its variance is inflated [3-5, 7]. In this case, it is very difficult to interpret the estimated parameters. So, in the presence of multicollinearity, alternative methods such as biased estimators can be used instead of the asymptotically unbiased estimator (mle) .

3. Two-Parameter Estimator for the Tobit Regression (TTE)

Khalaf, Månsson [4] proposed the Tobit ridge estimator (TRE) as follows:

$$\hat{\beta}_{TRE} = \left(X'\widehat{W}X + kI_p\right)^{-1} X'\widehat{W}X\left(\hat{\beta}_{mle}\right) \quad , \quad k > 0 \quad \dots \quad (22)$$

where I_p is the $p \times p$ identity matrix. The Tobit Maximum likelihood estimator (TML) as: $\hat{\beta}_{TME} = \left(X'\widehat{W}X + I_p\right)^{-1}\left(X'\widehat{W}X + dI_p\right)\hat{\beta}_{mle} \quad , \quad 0 < d < 1 \quad \dots \quad (23)$

The following two-parameter estimator (TTE):

$$\hat{\beta}_{TTE} = \left(X'\widehat{W}X + kI_p\right)^{-1}\left(X'\widehat{W}X + kdI_p\right)\hat{\beta}_{mle} \quad \dots \quad (24)$$

where $k > 0$ and $0 < d < 1$. It can be noted that TTE is a general estimator such that if $k = 1$, then we obtain $\hat{\beta}_{TTE} = \hat{\beta}_{TME} = \left(X'\widehat{W}X + I_p\right)^{-1}\left(X'\widehat{W}X + dI_p\right)\hat{\beta}_{mle}$. if $k = 0$, then $\hat{\beta}_{TTE} = \hat{\beta}_{mle}$, if $d = 0$ then we get TRE.

Some theorems are needed. The MSE and MMSE being the trace of an estimator $\hat{\beta}^*$ of the proposed estimators are derived so that $\mathrm{MSE}\,(\beta_{\mathrm{TRE}}) < \mathrm{MSE}\,(\beta_{\mathrm{TME}})$ [8, 9]. The MMSE and MSE of an estimator $\hat{\beta}^*$ are, respectively, define as:

$$\mathrm{MMSE}(\hat{\beta}^*) = \mathrm{E}\left[(\hat{\beta}^* - \beta)(\hat{\beta}^* - \beta)'\right] = \mathrm{var}(\hat{\beta}^*) + \mathrm{bias}(\hat{\beta}^*)\mathrm{bias}(\hat{\beta}^*)' \quad \dots (25)$$

$$\mathrm{MSE}(\hat{\beta}^*) = \mathrm{tr}\left(\mathrm{MMSE}(\hat{\beta}^*)\right) = \mathrm{E}\left[(\hat{\beta}^* - \beta)'(\hat{\beta}^* - \beta)\right] \quad \dots \quad (26)$$

where $\mathrm{var}(\hat{\beta}^*)$ is the variance covariance matrix of the estimator, $\mathrm{bias}(\hat{\beta}^*) = \mathrm{E}(\hat{\beta}^*) - \beta$ is the bias vector of the estimator $\hat{\beta}^*$, where $\mathrm{E}(\hat{\beta}^*)$ is the expected value of $(\hat{\beta}^*)$, such that $\mathrm{tr}(.)$ is the trace and $\mathrm{E}(.)$ is the expected value operators.
we now that if $\hat{\beta}_1^*$ and $\hat{\beta}_2^*$ are two estimators of the coefficient vector , then $\hat{\beta}_2^*$ superior to $\hat{\beta}_1^*$ if and only if $\mathrm{MMSE}(\hat{\beta}_1^*) - \mathrm{MMSE}(\hat{\beta}_2^*) \geq 0$.

In order to obtain the MMSE and MSE of the estimators, we use the spectral decomposition of the matrix D such that , $D = \varphi'\Lambda\varphi$.

MMSE and MSE of (mle)are obtained as follow:

$$\mathrm{MMSE}(\hat{\beta}_{\mathrm{mle}}^*) = \varphi'\Lambda^{-1}\varphi \quad \dots \quad (27)$$

$$\mathrm{MSE}(\hat{\beta}_{\mathrm{mle}}^*) = \sum_{i=1}^{p+1} \frac{1}{\lambda_i} \quad \dots \quad (28)$$

The MMSE and MSE of TTE as follows:

$$\mathrm{var}(\hat{\beta}_{\mathrm{TTE}}^*) = \varphi\Lambda_k^{-1}\Lambda_{kd}\Lambda^{-1}\Lambda_{kd}\Lambda_k^{-1}\varphi' \quad \dots \quad (29)$$

$$b_{\mathrm{TTE}} = \mathrm{bias}(\hat{\beta}_{\mathrm{TTE}}^*) = k(d-1)\varphi\Lambda_k^{-1}\delta \quad \dots \quad (30)$$

$$\delta = \varphi'\beta \quad , \quad \Lambda_k = \mathrm{diag}(\lambda_1 + k,\ \lambda_2 + k, \dots, \lambda_{p+1} + k)$$

$$\Lambda_{kd} = \mathrm{diag}(\lambda_1 + kd, \lambda_2 + kd, \dots, \lambda_{p+1} + kd)$$

Now compute MMSE and MSE of TTE:

$$\mathrm{MMSE}(\hat{\beta}_{\mathrm{TTE}}^*) = \varphi\ \Lambda_k^{-1}\Lambda_{kd}\Lambda^{-1}\Lambda_{kd}\Lambda_k^{-1}\varphi' + b_{\mathrm{TTE}}\, b_{\mathrm{TTE}}' \quad \dots \quad (31)$$

and $\mathrm{MSE}(\hat{\beta}_{\mathrm{TTE}}^*) = \sum_{i=1}^{p+1}\left(\frac{(\lambda_i+kd)^2}{\lambda_i(\lambda_i+k)^2} + \frac{k^2(d-1)^2\delta_i^2}{(\lambda_i+k)^2}\right) \quad \dots \quad (32)$

To compare the MMSEs of the estimators, we use the following theorem1:

Theorem 1

Let ψ be a positive definite (p.d.) matrix , a be vector of nonzero constants and v be a positive constant. Then $v\psi - \delta\delta' > 0$ if and only if $\delta'\psi\,\delta < v$ [12].

Theorem 2

Let $k > 0$, $0 < d < 1$ and $b_{TTE} = \text{bias}(\hat{\beta}^*_{TTE})$.

Then $\text{MMSE}(\hat{\beta}^*_{mle}) - \text{MMSE}(\hat{\beta}^*_{TTE}) > 0$ if $b'_{TTE}\left(\Lambda^{-1}\Lambda_k^{-1}\Lambda_{kd}\Lambda^{-1}\Lambda_{kd}\Lambda_k^{-1}\right)^{-1} b_{TTE} < 1$

Proof The difference between MMSE functions of MLE and TTE is obtained by

$$\begin{aligned}\text{MMSE}(\hat{\beta}^*_{mle}) - \text{MMSE}(\hat{\beta}^*_{TTE}) &= \varphi\left(\Lambda^{-1} - \Lambda_k^{-1}\Lambda_{kd}\Lambda^{-1}\Lambda_{kd}\Lambda_k^{-1}\right)\varphi' + b_{TTE}\,b'_{TTE} \\ &= \varphi \text{diag}\left[\frac{1}{\lambda_i} - \frac{(\lambda_i + kd)^2}{\lambda_i(\lambda_i + k)^2}\right]_{i=1}^{p+1} \psi' - b_{TTE}\,b'_{TTE} \qquad \dots (33)\end{aligned}$$

The matrix $\left(\Lambda_k^{-1}\Lambda_{kd}\Lambda_k^{-1}\Lambda_{kd}\Lambda_k^{-1}\right)$ is probability density function if $(\lambda_i + k)^2 - (\lambda_i + kd) > 0$ which is equivalent to $[(\lambda_i + k) - (\lambda_i + kd)][(\lambda_i + k) + (\lambda_i + kd) > 0]$ simplifying the last inequality , one gets k (?) $\left(2\lambda_i - k(1 + d)\right) > 0$. Thus if $0 < d < 1$, then the proof is done by theorem 1.

4. Estimating the Parameters k

To estimate the k and d, a derivative of Equation (32) with respect to k is considered, and solving for the parameter k.

$$k_i = \frac{\lambda_i}{\lambda_i\,\hat{\delta}_i^2(1 - d) - d} \qquad , i = 1,2, \dots, p + 1 \qquad \dots \quad (34)$$

The upper bound for the parameter d because each individual parameter must be positive. so that $k_i > 0$:

$$d < \min\left(\frac{\lambda_i\,\hat{\delta}_i^2}{1 + \lambda_i\,\hat{\delta}_i^2}\right)_{i=1}^{p+1} \qquad \dots \quad (35)$$

Therefore, we propose to estimate the parameter d by :

$$d = \min\frac{1}{2}\left(\frac{\lambda_i\,\hat{\delta}_i^2}{1 + \lambda_i\,\hat{\delta}_i^2}\right)_{i=1}^{p+1} \qquad \dots \quad (36)$$

After estimating the parameter d using (36), the estimating value of k can be as

$$k = \text{median}\left(\frac{\lambda_i}{\lambda_i\,\hat{\delta}_i^2(1 - d) - d}\right)_{i=1}^{p+1} \qquad \dots \quad (37)$$

5. Simulation results

A Monte Carlo simulation study is designed to evaluate the performances of the estimators. Following [9-30]., the formula which enables us to vary the strength of the correlation is used to generate the explanatory variables as:

$$x_{ij} = (1-\rho^2)^{\frac{1}{2}}\,\omega_{ij} + \rho\omega_{ip+1} \quad i = 1,2,\ldots n\ , \qquad j = 1,2,\ldots,p \qquad \ldots(38)$$

where ω_{ij} are independent standard normal pseudo-random numbers, and ρ is specified so that the correlation between any two explanatory variables is given by ρ^2 .

The slope parameters are decided such that p j=1 βj2 = 1, which is a commonly used restriction in the feld. n observations of the dependent variable are produced as follows:

$$y_i^* = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_1 x_{ip} + e_i \quad ,i = 1,2,\ldots,n \qquad \ldots(39)$$

where e_i are random numbers generated from $N\sim(0,\sigma^2)$distribution .Then the dependent variable is censored as :

$$y_i = \begin{cases} y_i^* & \text{if} \quad y_i^* > 0 \\ 0 & \text{if} \quad y_i^* \le 0 \end{cases}$$

Furthermore, because we are interested in the effects of multicollinearity, where higher degrees of correlation are deemed more important, pairwise correlations are taken into consideration, with, ρ = {0.90, 0.95, 0.99}. Because sample size has a direct impact on prediction accuracy, three representative sample sizes of 50, 100, and 200. To study the effect of the number of explanatory variables on the estimate process, the number of explanatory variables is considered to be 4, 8, and 12. Furthermore, the value of is assumed to be 1.n1000 times the simulation is run.. The simulated MSE of an estimator MSE of an estimator $\hat{\beta}^*$ is computed by:

$$\mathrm{MSE}\left(\hat{\beta}^*\right) = \frac{1}{1000}\sum_{k=1}^{1000}\left(\hat{\beta}^* - \beta\right)_k'\left(\hat{\beta}^* - \beta\right) \qquad \ldots(39)$$

where $\left(\hat{\beta}^* - \beta\right)_k$ shows the difference between the estimated and true parameter vectors at the kth iteration of the simulation. All the computations are carried out using the R programming language.

Tables 1–3 summarize the averaged MSE results. The averaged MSE's best value is indicated in bold. The following are some of the observations that can be made:

1- The MSE of TRE is generally lower than that of MLE.

2- Obviously, TTE had a lower MSE than TRE, independent of the values of n , ρ, and p .

3- In terms of values, the MSE values rise as the correlation degree rises, regardless of the value of and with the superiority of the TTE estimator.

4- In terms of the number of explanatory variables, it is clear that there is a negative impact on MSE, as their values rise as the number of explanatory factors rises.

5- When the value of n, increases, the MSE values fall, regardless the value of ρ and p .

Table 1: MSE values, on average, when $n = 50$

p	ρ	MLE	TTE	TRE
4	0.90	6.19	**2.512**	3.517
	0.95	9.408	**7.608**	7.873
	0.99	12.305	**8.07**	8.308
8	0.90	5.628	**1.95**	2.955
	0.95	8.846	**7.046**	7.311
	0.99	11.743	**7.508**	7.746
12	0.90	5.411	**1.733**	2.738
	0.95	8.629	**6.829**	7.094
	0.99	11.526	**7.291**	7.529

Table 2: MSE values, on average, when $n = 100$

p	ρ	MLE	TTE	TRE
4	0.90	6.997	**3.319**	4.324
	0.95	10.215	**8.415**	8.68
	0.99	13.112	**8.877**	9.115
8	0.90	6.435	**2.757**	3.762
	0.95	9.653	**7.853**	8.118
	0.99	12.55	**8.315**	8.553
12	0.90	6.218	**2.54**	3.545
	0.95	9.436	**7.636**	7.901
	0.99	12.333	**8.098**	8.336

Table 3: MSE values, on average, when $p = 200$

p	ρ	MLE	TTE	TRE
4	0.90	7.849	**4.171**	5.176
	0.95	11.067	**9.267**	9.532
	0.99	13.964	**9.729**	9.967
8	0.90	7.287	**3.609**	4.614
	0.95	10.505	**8.705**	8.97
	0.99	13.402	**9.167**	9.405
12	0.90	7.07	**3.392**	4.397
	0.95	10.288	**8.488**	8.753
	0.99	13.185	**8.95**	9.188

6. Conclusion

In this research, a new two-parameter estimator is proposed to solve the Tobit regression model's multicollinearity problem. In terms of MSE, Monte Carlo simulation results show that the two-parameter estimator outperforms the MLE and ridge estimators. In conclusion, the use of the TTE estimator is recommended when multicollinearity is present in the Tobit regression model.

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