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Restricted ride estimator in the Poisson regression model

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Abstract
Poisson regression model is the standard statistical method for analyzing count data. Its parameters are usually estimated using maximum likelihood (ML) method. However, the ML method is very sensitive to multicollinearity. Ridge estimator was proposed in Poisson regression model. A restricted ridge estimator is proposed. Simulation and real data example results demonstrate that the proposed estimator is outperformed ML and Poisson ridge estimator.

Keywords: Multicollinearity; ridge regression; restricted estimator; shrinkage; Monte Carlo simulation.

1. Introduction

In regression modeling, data in the form of counts are usually common. Count data regression modeling has received much attention in medicine, behavioral sciences, psychology, and econometrics (Zakariya Y Algamal, 2012; Asar & Genç, 2017a; Coxe, West, & Aiken, 2009). The Poisson regression model is the most basic models under count data regression models (Wang et al., 2014).

In dealing with the Poisson regression model, it is assumed that there is no correlation among the explanatory variables. In practice, however, this assumption often not holds, which leads to the problem of multicollinearity. In the presence of multicollinearity, when estimating the regression coefficients for Poisson regression model using the maximum likelihood (ML) method, the estimated coefficients are usually become unstable with a high variance, and therefore low statistical significance with incorrect signs (ALheety & Kibria, 2014; Batah, Özkale, & Gore, 2009; Jou, Huang, & Cho, 2014).

Numerous remedial methods have been proposed to overcome the problem of multicollinearity. The ridge regression method (Hoerl & Kennard, 1970) has been consistently demonstrated to be an attractive and alternative to the ML estimation method.

Ridge regression is a shrinkage method that shrinks all regression coefficients toward zero to reduce the large variance (Asar & Genç, 2015). The ridge regression performance greatly relies on the choice of shrinkage parameter. Consequently, choosing a suitable value of the shrinkage parameter is an important part of ridge regression model fitting (Sököt Açar & Özkale, 2015). Several methods, which they are based on the original ridge regression of (Hoerl & Kennard, 1970), are available for estimating the ridge shrinkage parameter in the literature (Alkhamisi, Khalaf, & Shukur, 2006; Asar, Karaibrahimoğlu, & Genç, 2014; Hamed, Hefnawy, & Farag, 2013; Hefnawy & Farag, 2014; Khalaf & Shukur, 2005; Kibria, 2003; Muniz & Kibria, 2009).

2. Poisson ridge regression model

Count data often arise in epidemiology, social, and economic studies. This type of data consists of positive integer values. Poisson distribution is a well-known distribution that fit to such type of data. Poisson regression model is used to model the relationship between the counts as response variable and potentially explanatory variables (Zakariya Yahya Algamal & Alanaz, 2018; Zakariya Y. Algamal & Lee, 2015; Alkhamisi et al., 2006; Alkhateeb & Algamal, 2020; Asar, 2015a, 2015b; Hemmerle & Boulanger Carey, 2007; KaÇiranlar & Dawoud, 2017; Kibria, 2012; Kibria, Månsson, & Shukur, 2013; Rashad & Algamal, 2019).

Let y_i be the response variable and follows a Poisson distribution with mean ϕ_i, then the probability density function is defined as

$$f\left(y_i\right)=\frac{e^{-\phi_i}\phi_i^{y_i}}{y_i\,!},\quad y_i=0,1,\ldots;\ i=1,2,\ldots,n. \tag{1}$$

In a Poisson regression model, $\ln(\phi_i)=\mathbf{x}_i^T\boldsymbol{\beta}$ is expressed as a linear combination of explanatory variables $\mathbf{x}_i=(x_{i1},...,x_{ip})^T$. The $\ln(\phi_i)$ is called as canonical link function which making the relationship between explanatory variables and response variable linear. The most common method of estimating the coefficients of Poisson regression model is to use the maximum likelihood method. Given the assumption that the observations are independent, the log-likelihood function is defined as

$$\ell(\boldsymbol{\beta})=\sum_{i=1}^{n}\left\{y_i\mathbf{x}_i^T\boldsymbol{\beta}-\exp(\mathbf{x}_i^T\boldsymbol{\beta})-\ln y_i\,!\right\}. \tag{2}$$

The ML estimator is then obtained by computing the first derivative of the Eq. (3) and setting it equal to zero, as

$$\frac{\partial\ell(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}}=\sum_{i=1}^{n}\left[y_i-\exp(\mathbf{x}_i^T\boldsymbol{\beta})\right]\mathbf{x}_i=0. \tag{3}$$

Because Eq. (4) is nonlinear in $\boldsymbol{\beta}$, the iteratively weighted least squares (IWLS) algorithm can be used to obtain the ML estimators of the Poisson regression parameters (PR) as

$$\hat{\boldsymbol{\beta}}_{ML}=(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})^{-1}\mathbf{X}^T\hat{\mathbf{W}}\hat{\mathbf{v}}, \tag{4}$$

where $\hat{\mathbf{W}} = \mathrm{diag}(\hat{\phi}_i)$ and $\hat{\mathbf{v}}$ is a vector where i$^{\text{th}}$ element equals to $\hat{v}_i = \ln(\hat{\phi}_i) + ((y_i - \hat{\phi}_i)/\hat{\phi}_i)$. The ML estimator is asymptotically normally distributed with a covariance matrix that corresponds to the inverse of the Hessian matrix

$$\mathrm{cov}(\hat{\boldsymbol{\beta}}_{ML}) = \left[-E\left(\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial \beta_i \, \partial \beta_k} \right) \right]^{-1} = (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1}. \tag{5}$$

The mean squared error (MSE) of Eq. (5) can be obtained as

$$\begin{aligned} \mathrm{MSE}(\hat{\boldsymbol{\beta}}_{ML}) &= E(\hat{\boldsymbol{\beta}}_{ML} - \hat{\boldsymbol{\beta}})^T (\hat{\boldsymbol{\beta}}_{ML} - \hat{\boldsymbol{\beta}}) \\ &= tr[(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1}] \\ &= \sum_{j=1}^{p} \frac{1}{\lambda_j}, \end{aligned} \tag{6}$$

where λ_j is the eigenvalue of the $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ matrix.

In the presence of multicollinearity, the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ becomes ill-conditioned leading to high variance and instability of the ML estimator of the Poisson regression parameters. As a remedy, Månsson and Shukur (2011) proposed the Poisson ridge estimator (PRE) as

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{PRE} &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} \hat{\boldsymbol{\beta}}_{ML} \\ &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{v}}, \end{aligned} \tag{7}$$

where $k \geq 0$. The ML estimator can be considered as a special estimator from Eq. (7) with $h = 0$. Regardless of h value, the MSE of the $\hat{\boldsymbol{\beta}}_{PRE}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{ML}$ because the MSE of $\hat{\boldsymbol{\beta}}_{PRE}$ is equal to (Kibria, Månsson, & Shukur, 2015)

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PRE}) = \sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + h)^2} + h^2 \sum_{j=1}^{p} \frac{\alpha_j}{(\lambda_j + h)^2}, \tag{8}$$

where α_j is defined as the j$^{\text{th}}$ element of $\gamma \hat{\boldsymbol{\beta}}_{ML}$ and γ is the eigenvector of the $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ matrix. Comparing with the MSE of Eq. (6), $\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{PRE})$ is always small for $h > 0$.

3. The proposed estimator

In addition to the sample information, there are some exact or restrictions for the unknown parameter of the model exist which may help to reduce the multicollinearity problem. Therefore, suppose that we have some prior information about $\boldsymbol{\beta}$ in the form of independent linear restrictions as:

$$\mathbf{H}\boldsymbol{\beta} = h, \tag{9}$$

where $\mathbf{H}$ denotes a $q \times (p+1)$ $(q \le p+1)$ known matrix and h shows a q x I vector of pre-specified know constants. Considering such a restriction, Duffy and Santner (1989) defined the restricted maximum likelihood estimator (RMLE) with the following from:

$$\hat{\boldsymbol{\beta}}_{RMLE} = \hat{\boldsymbol{\beta}}_{MLE} - (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1} \mathbf{H}^T (\mathbf{H}(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1} \mathbf{H}^T)^{-1} (\mathbf{H}\hat{\boldsymbol{\beta}}_{MLE} - h) \tag{10}$$

Based on the Eq. (10) and Eq. (7), we propose a restricted Poisson ridge estimator (RPRE) which is given as follows:

$$\begin{aligned}\hat{\boldsymbol{\beta}}_{RPRE} &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{v}} - (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{H}^T \left[\mathbf{H}(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{H}^T \right]^{-1} \\ &\quad \left[\mathbf{H}(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{v}} - h \right]\end{aligned} \tag{11}$$

It is easy to see that when the biasing parameter, $k = 0$, Eq. (11) becomes the RMLE in Eq. (10). The restricted ridge estimator was studied by several authors, such as (ALheety & Kibria, 2014; Asar, Arashi, & Wu, 2016; Duffy & Santner, 1989; Kurtoğlu, Özkale, & Computation, 2019; Nagarajah & Wijekoon, 2015; Najarian, Arashi, & Kibria, 2013). The MSE of $\hat{\boldsymbol{\beta}}_{RPRE}$ is defined as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{RPRE}) = \sum_{j=1}^{p} \frac{(\lambda_j (\lambda_j + k - h_{jj})^2}{(\lambda_j + k)^4} + k \left[\sum_{j=1}^{p} \frac{\alpha_j (\lambda_j + k - h_{jj})}{(\lambda_j + 1)^2} \right]^2, \tag{12}$$

4. Simulation results

In this section, a Monte Carlo simulation experiment is used to examine the performance of the new estimator with different degrees of multicollinearity. The response variable of n observations is generated from Poisson regression model by

$$\phi_i = \exp(\mathbf{x}_i^T \boldsymbol{\beta}), \tag{13}$$

where $\boldsymbol{\beta} = (\beta_0, \beta_1, ..., \beta_p)$ with $\sum_{j=1}^{p} \beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$ (Kibria, 2003; Månsson & Shukur, 2011). In addition, because the value of intercept, β_0, has an effect on ϕ_i, three values are chosen $\beta_0 \in \{1, 0, -1\}$, where decreasing the value of β_0 leads to lower average value of ϕ_i, which leads to less variation (Asar & Genç, 2017b; Månsson & Shukur, 2011).

The explanatory variables $\mathbf{x}_i^T = (x_{i1}, x_{i2}, ..., x_{in})$ have been generated from the following formula

$$x_{ij} = (1-\rho^2)^{1/2} w_{ij} + \rho w_{ip}, \; i = 1, 2, ..., n, \quad j = 1, 2, ..., p, \tag{14}$$

where ρ represents the correlation between the explanatory variables and w_{ij}'s are independent standard normal pseudo-random numbers. Because the sample size has direct impact on the prediction accuracy, three representative values of the sample size are considered: 30, 50 and 100. In addition, the number of the explanatory variables is considered as $p = 4$ and $p = 7$ because increasing the number of explanatory variables can lead to increase the MSE. Further, because we are interested in the effect of multicollinearity, in which the degrees of correlation considered more important, three values of the pairwise correlation are considered with $\rho = \{0.90, 0.95, 0.99\}$. According to Asar et al. (2016), two restricted matrices are explained as

$\mathbf{H}_{p=4} = \begin{pmatrix} 1 & 0 & -3 & 2 \\ 1 & -2 & 1 & -1 \end{pmatrix}$ and $\mathbf{H}_{p=7} = \begin{pmatrix} 1 & 0 & -3 & 1 & -1 & 2 & 1 \\ 1 & -2 & 1 & -1 & 0 & 1 & 1 \end{pmatrix}$ with $h = (0,0)$. In addition, the method of determining the value of k is defined as $k = (1/\hat{\alpha}^2_{\max})$. For a combination of these different values of n, p, β_0, and ρ the generated data is repeated 1000 times and the averaged mean squared errors (MSE) is calculated as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}), \qquad (15)$$

where $\hat{\boldsymbol{\beta}}$ is the estimated coefficients for the used estimator.

The estimated MSE of Eq. (15) for ML, PRE, and our proposed estimator, RPRE, for the combination of n, p, β_0, and ρ, are respectively summarized in Tables 1, 2, and 3. Several observations can be made.

First, in terms of ρ values, there is increasing in the MSE values when the correlation degree increases regardless the value of n, p, β_0. However, RPRE performs better than PRE and ML. For instance, in Table 1, when $p = 7$, $n = 100$, and $\rho = 0.99$, the MSE of MCV was about 26.72% and 23.38% lower than that of ML and PRE respectively.

Second, regarding the number of explanatory variables, it is easily seen that there is increasing in the MSE values when the p increasing from four variables to seven variables. Although this increasing can affected the quality of an estimator, RPRE is achieved the lowest MSE comparing with ML and PRE, for different n, ρ, β_0.

Third, with respect to the value of n, The MSE values decreases when n increases, regardless the value of ρ, p, β_0. However, RPRE still consistently outperforms PRE by providing the lowest MSE.

Fourth, in terms of the value of the intercept and for a given values of ρ, p, n, RPRE is always show smaller MSE comparing with the other methods.

To summary, all the considered values of n, ρ, p, β_0, RPRE is superior to PRE, clearly indicating that the new proposed estimator is more efficient.

Table 1: MSE values when $\beta_0 = -1$

			ML	PRE	RPRE
		ρ			
$p = 4$	$n = 30$	0.9	6.2814	6.0344	5.1671
		0.95	6.9094	6.6624	5.7954
		0.99	7.3074	7.0604	6.1933
	$n = 50$	0.9	4.6524	4.4054	3.5384

		0.95	5.7274	5.4804	4.6130
		0.99	5.9194	5.6724	4.8054
	$n=100$	0.9	4.4954	4.2484	3.3811
		0.95	4.7054	4.4582	3.5914
		0.99	5.4604	5.2130	4.3464
$p=7$	$n=30$	0.9	6.3864	6.1394	5.2724
		0.95	7.0054	6.7584	5.8914
		0.99	7.4204	7.1731	6.3064
	$n=50$	0.9	4.9214	4.6744	3.8074
		0.95	6.0644	5.8174	4.9504
		0.99	6.3894	6.1424	5.2754
	$n=100$	0.9	4.8314	4.5844	3.7174
		0.95	5.1064	4.8594	3.9924
		0.99	5.6644	5.4174	4.1504

Table 2: MSE values when $\beta_0 = 0$

		ρ	ML	PRE	RPRE
$p=4$	$n=30$	0.90	6.3051	6.0581	5.1913
		0.95	6.9331	6.6862	5.8191
		0.99	7.3311	7.0841	6.2171
	$n=50$	0.90	4.6761	4.4291	3.5621
		0.95	5.7511	5.5041	4.6371
		0.99	5.9431	5.6962	4.8291
	$n=100$	0.90	4.5191	4.2721	3.4051
		0.95	4.7291	4.4821	3.6151
		0.99	5.4841	5.2371	4.3701
$p=7$	$n=30$	0.90	6.4101	6.1632	5.2961
		0.95	7.0291	6.7821	5.9151
		0.99	7.4441	7.1971	6.3301
	$n=50$	0.90	4.9451	4.6981	3.8311

		0.95	6.0881	5.8411	4.9741
		0.99	6.4131	6.1661	5.2991
	$n=100$	0.90	4.8551	4.6081	3.7411
		0.95	5.1301	4.8831	4.0161
		0.99	5.6881	5.4411	4.5741

Table 3: MSE values when $\beta_0 = 1$

			ML	PRE	RPRE
		ρ			
$p=4$	$n=30$	0.90	6.503	6.256	5.389
		0.95	7.131	6.884	6.017
		0.99	7.529	7.282	6.415
	$n=50$	0.90	4.874	4.627	3.76
		0.95	5.949	5.702	4.835
		0.99	6.141	5.894	5.027
	$n=100$	0.90	4.717	4.47	3.603
		0.95	4.927	4.68	3.813
		0.99	5.682	5.435	4.568
$p=7$	$n=30$	0.90	6.608	6.361	5.494
		0.95	7.227	6.98	6.113
		0.99	7.642	7.395	6.528
	$n=50$	0.90	5.143	4.896	4.029
		0.95	6.286	6.039	5.172
		0.99	6.611	6.364	5.497
	$n=100$	0.90	5.053	4.806	3.939
		0.95	5.328	5.081	4.214
		0.99	5.886	5.639	4.772

5. Real application

To investigate the usefulness of reviewed biased estimators, an application related to the football English league, season 2016-2017 is employed. This data contains twenty teams, where the response variable represents the number of won matches. The six considerable predictors included the number of yellow cards (x_1), the number of red cards (x_2), the total number of substitutions (x_3), the number of matches with 2.5 goals on average (x_4), the number of matches that ended with goals (x_5), and the ratio of the goal scores to the number of matches (x_6).

First, the deviance test (Montgomery, Peck, & Vining, 2015) is used to check whether the Poisson regression model is fit well to this data or not. The result of the residual deviance test is equal to 8.373 with 14 degrees of freedom and the p-value is 0.869. It is indicated form this result that the Poisson regression model fits very well to this data.

Second, to check whether there are relationships between the explanatory variables or not, Figure 1 displays the correlation matrix among the six explanatory variables. It is obviously seen that there are correlations greater than 0.86 between x_1 and x_6, x_1 and x_4, x_2 and x_4, and, x_4 and x_6.

Third, to test the existence of multicollinearity, the eigenvalues of the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ are obtained as 993.758, 142.907, 75.560, 38.999, 21.424, and 1.016. The determined condition number $\text{CN} = \sqrt{\lambda_{\max} / \lambda_{\min}}$ of the data is 31.274 indicating that the multicollinearity issue is exist.

The estimated Poisson regression coefficients, standard errors which are computed by using bootstrap with 500 replications, and MSE values for the ML, PRE and RPRE estimators are listed in Table 4. According to Table 4, it is clearly seen that the RPRE estimator shrinkages the value of the estimated coefficients efficiently. Additionally, in terms of the calculated standard errors, the RPRE and PRE show substantial decreasing comparing with ML.

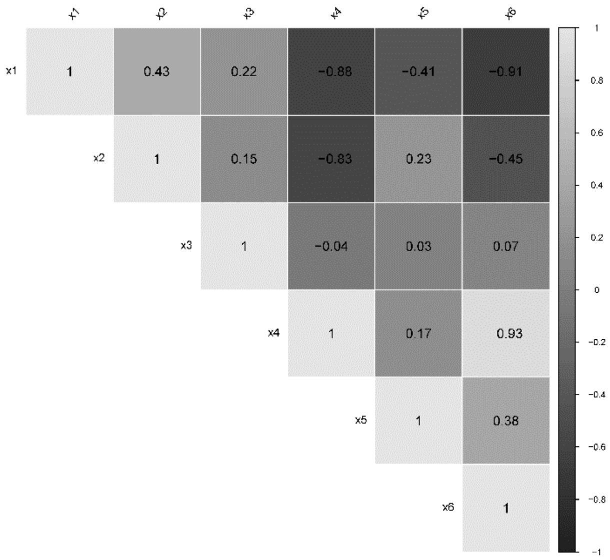

Figure 1: The correlation matrix among the six explanatory variables.

Table 4: The estimated coefficients and MSE values for the used estimators

	ML	PRE	RPRE
$\hat{\beta}_1$	-1.219	-1.016	-0.813
	(0.151)	(0.007)	(0.125)
$\hat{\beta}_2$	0.441	0.510	0.031

	(0.151)	(0.001)	(0.132)
$\hat{\beta}_3$	0.575	0.416	0.224
	(0.175)	(0.008)	(0.103)
$\hat{\beta}_4$	-3.476	-2.034	-0.131
	(0.313)	(0.008)	(0.229)
$\hat{\beta}_5$	-2.432	-2.12	-1.007
	(0.160)	(0.004)	(0.132)
$\hat{\beta}_6$	5.121	0.004	0.173
	(0.387)	(0.003)	(0.225)
MSE	3.681	2.108	1.548

6. Conclusion

In this paper, a restricted ridge estimator of Poisson regression model was proposed. This proposed estimator allows us to handle multicollinearity. According to Monte Carlo simulation studies, the restricted estimator has better performance than maximum likelihood estimator and Poisson ridge estimator, in terms of MSE. Additionally, a real data application is also considered to illustrate benefits of using the new estimator in the context of Poisson regression model. The superiority of the new estimator based on the resulting MSE was observed and it was shown that the results are consistent with Monte Carlo simulation results.

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