Restricted ridge estimator in the negative binomial regression model

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Abstract:
The negative binomial regression model is a well-known model in application when the response variable is non-negative integers or counts. However, it is known that multicollinearity negatively affects the variance of maximum likelihood estimator of the negative binomial coefficients. To overcome this problem, a restricted ridge estimator is proposed and derived. Our Monte Carlo simulation results suggest that the proposed estimator can bring significant improvement relative to other existing estimators. In addition, the real application results demonstrate that the proposed estimator outperforms other estimators in terms of predictive performance.

Keywords: Multicollinearity; ridge estimator; negative binomial regression model; shrinkage; Monte Carlo simulation.

1- Introduction

Negative binomial regression model (NBRM) is widely applied for studying several real data problems, "such as in mortality studies where the aim is to investigate the number of deaths and in health insurance where the target is to explain the number of claims made by the individual [De Jong and Heller (2008), Algamal (2012), Cameron and Trivedi (2013), Kandemir Çetinkaya and Kaçıranlar (2019)].

In dealing with the NBRM, it is assumed that there is no correlation among the explanatory variables. In practice, however, this assumption often not holds, which leads to the problem of multicollinearity. In the presence of multicollinearity, when estimating the regression coefficients for NBRM using the maximum likelihood (ML) method, the estimated coefficients are usually become unstable with a high variance, and therefore low statistical significance [Månsson (2012, (2013), Kibria et al. (2015), Türkan and Özel (2017)]. Numerous remedial methods have been proposed to overcome the problem of multicollinearity [Algamal (2018, (2018, (2018), Algamal and Alanaz (2018), Algamal and Asar (2018), Algamal (2018), Yahya Algamal (2018), Rashad and Algamal (2019), Shamany et al. (2019), Al-Taweel et al. (2020), Algamal (2020), Alkhateeb and Algamal (2020), Abdulazeez and Algamal (2021), Algamal and Abonazel (2021), Alobaidi et al. (2021), Lukman, Algamal, et al. (2021), Lukman, Dawoud, et al. (2021), Mohammed and Algamal (2021), Noeel and Algamal (2021), Rashad et al. (2021)]. The ridge regression method [Hoerl and Kennard (1970)] has been consistently demonstrated to be an attractive and alternative to the ML estimation method.

Liu (1993) proposed the Liu estimator having the advantages of being a linear function of the shrinkage parameter as well. This estimator has the advantages of ridge estimator and Stein estimator. Liu (2003) showed the superiority of the Liu-type estimator, which is a two-parameter estimator, over ridge regression. Actually, this estimator uses advantages of both ridge estimator and Liu estimator. In Liu-type estimator, one can use a large shrinkage value, because there is another parameter to make the estimator give a good fit.

In this paper, a restricted ridge estimator is proposed and derived for the negative binomial regression model. The idea behind our proposed estimator is to decrease the shrinkage parameter and, therefore, the resultant estimator can be better with small amount of bias.

2- Negative binomial regression model

Most popular distribution when analyzing count data is Negative Binomial regression, where this type of data used in health, social and physical science , when the dependent variable comes in the form of non-negative integers, the conditional distribution is $y_i|x_i\ , h_i \sim \text{Poisson}\,(h_i, \mu_i), i = 1,2,3,\dots,n$ where $h_i$is a random variable which is $\Gamma(\theta+\theta)$ distributed , x_i is a p×1 vector of covariates , β is a p×1 vector of parameters and $\mu_i = \exp(\acute{x}_i\,\beta)$.

The marginal density function of y_i is

$$\text{pr}(y = y_i|x_i)) = \frac{\Gamma(\theta+y_i)}{\Gamma(\theta)\Gamma(1+y_i)}\left(\frac{\theta}{\theta+\mu_i}\right)^{\theta}\left(\frac{\mu_i}{\theta+\mu_i}\right)^{y_i} \tag{1}$$

The conditional mean and variance of the distribution are given respectively as :

$$E(y_i|x_i) = \mu_i \tag{2}$$

$$V(y_i|x_i) = \mu_i\left(1 + \frac{1}{\theta}\mu_i\right) \tag{3}$$

This model is usually estimated by the maximum likelihood (ML) estimator which is found by maximizing the log-likelihood function

$$l(\theta,\beta) = \sum_{i=1}^{n}\left\{\left(\sum_{t=0}^{y_i=1}\log(t+\theta)\right) - \log y_i! - (y_i+\theta)\log\left(1+\frac{1}{\theta}\exp(\acute{x}_i\beta)\right) + y_i\log\left(\frac{1}{\theta}\right) + y_i\acute{x}_i\beta\right\}$$
(4)

Them the ML can be obtained by solving the following equation :

$$s(\beta) = \frac{\partial l(\theta,\beta)}{\partial \beta} = \sum_{i=1}^{n} \frac{y_i+\mu_i}{1+\left(\frac{1}{\theta}\right)\mu_i} x_i = 0 \qquad (5)$$

Since equation (5) is non-linear in β , then by using weighted least square algorithm, we have:

$$\hat{\beta}_{NBR} = (X\acute{}\hat{W}X)^{-1}X\acute{}\hat{W}\hat{Z} \qquad (6)$$

Where $\hat{Z}$ is a vector with the ith element equaling log $(\hat{u}_i) + (y_i - \hat{u}_i)/\hat{u}_i$,

and $\hat{W} = \text{diag}(\hat{u}_i/(1 + \hat{u}_i)/\hat{\theta}_i$.

the ML estimator of β is normally distributed with asymptotic mean vector $E(\hat{\beta}_{ML})$=0 and asymptotic covariance matrix

$$\text{Cov}(\hat{\beta}_{ML}) = (X\acute{}\hat{W}X)^{-1} \qquad (7)$$

the asymptotic mean –square error (MSE) based on the asymptotic covariance matrix equal

$$\text{MSE}(\hat{\beta}_{ML}) = \text{tr}(X\acute{}\hat{W}X)^{-1} = \sum_{j=1}^{p} \frac{1}{\lambda_j} \qquad (8)$$

Where λ_j is the eigenvalue of $X\acute{}\hat{W}X$ matrix . When there is a multicollinearity problem , the explanatory variables are highly intercorrelated . In that situation the weighted matrix of cross products, $X\acute{}\widehat{W_1}X$,is often ill-conditioned which leads to some eigenvalues being small . in that situation , it is very hard to interpret the estimated parameters since the vector of estimated coefficients become too long.

To avoid this problem the Negative Binomial ridge regression proposed by Mansson and Shukur (2011). By minimizes the weighted sum of squares error (WSSE). Hence $\hat{\beta}_{ML}$ is given by:

$$\hat{\beta}_{NBRR} = (X\acute{}\hat{W}X + KI)^{-1}X\acute{}\hat{W}X\,\hat{\beta}_{NBML} \qquad 0<K<1$$

$$= (\acute{X}\hat{W}X + KI)^{-1}X\acute{}\hat{W}\hat{S} \qquad (8)$$

In mansson and shukur (2011) it is shown that the MSE of this estimator equals :

$$\text{MSE}(\hat{\beta}_{NBRR}) = \sum_{j=1}^{J} \frac{\lambda_j}{(\lambda_j+k)^2} + k^2 \sum_{j=1}^{J} \frac{{\alpha_j}^2}{(\lambda_j+k)^2} = \gamma_1(k) + \gamma_2(k) \qquad (9)$$

$$= \text{Var}(\hat{\beta}_{NBRR}) + \text{Bias}(\hat{\beta}_{NBRR})$$

Where $\gamma_1(k)$ is the variance and $\gamma_2(k)$ is the bias part of $\hat{\beta}_{NBRR}$

The MSE of $\hat{\beta}_{NBRR}$ is lower than $\hat{\beta}_{ML}$ estimate such that when we found k (where k may take on value between zero and infinity) such that the reduction in the variance part is greater than the increase of the squared part, for this reason NBRR estimation is better than ML, furthermore NBRR is simple method since it dose not require any changes of the negative binomial regression.

3. The proposed estimator

In addition to the sample information, there are some exact or restrictions for the unknown parameter of the model exist which may help to reduce the multicollinearity problem. Therefore, suppose that we have some prior information about $\boldsymbol{\beta}$ in the form of independent linear restrictions as:

$$\mathbf{H}\boldsymbol{\beta} = h, \tag{10}$$

where $\mathbf{H}$ denotes a $q \times (p+1)$ $(q \leq p+1)$ known matrix and h shows a q x I vector of pre-specified know constants. Considering such a restriction, Duffy and Santner (1989) defined the restricted maximum likelihood estimator (RMLE) with the following from:

$$\hat{\boldsymbol{\beta}}_{RMLE} = \hat{\boldsymbol{\beta}}_{MLE} - (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})^{-1}\mathbf{H}^T(\mathbf{H}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})^{-1}\mathbf{H}^T)^{-1}(\mathbf{H}\hat{\boldsymbol{\beta}}_{MLE} - h) \tag{11}$$

Based on the Eq. (11), we propose a restricted negative binomial ridge estimator (RNBRE) which is given as follows:

$$\begin{aligned}\hat{\boldsymbol{\beta}}_{RNBRE} &= (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\mathbf{I})^{-1}\mathbf{X}^T\hat{\mathbf{W}}\hat{\mathbf{v}} - (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\mathbf{I})^{-1}\mathbf{H}^T\left[\mathbf{H}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\mathbf{I})^{-1}\mathbf{H}^T\right]^{-1} \\ &\quad \left[\mathbf{H}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\mathbf{I})^{-1}\mathbf{X}^T\hat{\mathbf{W}}\hat{\mathbf{v}} - h\right]\end{aligned} \tag{12}$$

It is easy to see that when the biasing parameter, $k = 0$, Eq. (12) becomes the RMLE in Eq. (10). The restricted ridge estimator was studied by several authors, such as [Duffy and Santner (1989), Najarian et al. (2013), ALheety and Kibria (2014), Nagarajah and Wijekoon (2015), Asar et al. (2016), Kurtoğlu et al. (2019)]. The MSE of $\hat{\boldsymbol{\beta}}_{RNBRE}$ is defined as

$$\text{MSE}(\hat{\boldsymbol{\beta}}_{RNBRE}) = \sum_{j=1}^{p}\frac{(\lambda_j(\lambda_j + k - h_{jj})^2}{(\lambda_j + k)^4} + k\left[\sum_{j=1}^{p}\frac{\alpha_j(\lambda_j + k - h_{jj})}{(\lambda_j + 1)^2}\right]^2, \tag{13}$$

4. Simulation results

In this section, a Monte Carlo simulation experiment is used to examine the performance of the new estimator with different degrees of multicollinearity. The response variable of n observations is generated from negative binomial regression as $NB(\mu_i, \mu_i + \theta\mu_i^2)$ with $\mu_i = \exp(x_i^T\beta)$. Here, $\beta = (\beta_0, \beta_1, ..., \beta_p)$ with $\sum_{j=1}^{p}\beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$ [Kibria (2003), Månsson and Shukur (2011)].

The explanatory variables $\mathbf{x}_i^T = (x_{i1}, x_{i2}, ..., x_{in})$ have been generated from the following formula

$$x_{ij} = (1-\rho^2)^{1/2}w_{ij} + \rho w_{ip}, \; i = 1,2,...,n, \quad j = 1,2,...,p, \tag{14}$$

where ρ represents the correlation between the explanatory variables and w_{ij}'s are independent standard normal pseudo-random numbers. Because the sample size has direct impact on the prediction accuracy, three representative values of the sample size are considered: 50, 100 and 200. In addition, the number of the explanatory variables is considered as $p = 4$ and $p = 7$ because increasing the number of explanatory variables can lead to increase the MSE. Further, because we are interested in the effect of multicollinearity, in which the degrees of correlation considered more important, three values of the pairwise correlation are considered with $\rho = \{0.90, 0.95, 0.99\}$. According to Asar, Arashi, and Wu (2016), two restricted matrices are explained as

$\mathbf{H}_{p=4} = \begin{pmatrix} 1 & 0 & -3 & 2 \\ 1 & -2 & 1 & -1 \end{pmatrix}$ and $\mathbf{H}_{p=7} = \begin{pmatrix} 1 & 0 & -3 & 1 & -1 & 2 & 1 \\ 1 & -2 & 1 & -1 & 0 & 1 & 1 \end{pmatrix}$ with $h = (0,0)$. In addition, the method of determining the value of k is defined as $k = (1/\hat{\alpha}^2_{\max})$. For a combination of these different values of n, p, β_0, and ρ the generated data is repeated 1000 times and the averaged mean squared errors (MSE) is calculated as

$$\text{MSE}(\hat{\boldsymbol{\beta}}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}), \tag{1}$$

where $\hat{\boldsymbol{\beta}}$ is the estimated coefficients for the used estimator.

The estimated MSE of Eq. (15) for ML, PRE, and our proposed estimator, RPRE, for the combination of n, p, β_0, and ρ, are respectively summarized in Tables 1, 2, and 3. Several observations can be made.
First, in terms of ρ values, there is increasing in the MSE values when the correlation degree increases regardless the value of n, p, β_0. However, RNPRE performs better than ridge and ML estimators. For instance, in Table 1, when $p = 7$, $n = 100$, and $\rho = 0.99$, the MSE of RNPRE was about 27.83% and 24.36% lower than that of ML and ridge estimators, respectively.
Second, regarding the number of explanatory variables, it is easily seen that there is increasing in the MSE values when the p increasing from four variables to seven variables. Although this increasing can affected the quality of an estimator, RNBRE is achieved the lowest MSE comparing with ML and ridge estimator, for different n, ρ, β_0.
Third, with respect to the value of n, The MSE values decreases when n increases, regardless the value of ρ, p, β_0. However, RNBRE still consistently outperforms ridge estimator by providing the lowest MSE.
Fourth, in terms of the value of the intercept and for a given values of ρ, p, n, RNBRE is always show smaller MSE comparing with the other methods".
To summary, all the considered values of n, ρ, p, β_0, RNBRE is superior to ridge estimator, clearly indicating that the new proposed estimator is more efficient.

Table 1: MSE values when $\beta_0 = -1$

		ρ	ML	Ridge	RNBRE
$p = 4$	$n = 50$	0.9	6.8024	6.5554	5.6881
		0.95	7.4304	7.1834	6.3164
		0.99	7.8284	7.5814	6.7143
	$n = 100$	0.9	5.1734	4.9264	4.0594
		0.95	6.2484	6.0014	5.134
		0.99	6.4404	6.1934	5.3264
	$n = 200$	0.9	5.0164	4.7694	3.9021
		0.95	5.2264	4.9792	4.1124
		0.99	5.9814	5.734	4.8674
$p = 7$	$n = 50$	0.9	6.9074	6.6604	5.7934
		0.95	7.5264	7.2794	6.4124
		0.99	7.9414	7.6941	6.8274
	$n = 100$	0.9	5.4424	5.1954	4.3284
		0.95	6.5854	6.3384	5.4714
		0.99	6.9104	6.6634	5.7964
	$n = 200$	0.9	5.3524	5.1054	4.2384
		0.95	5.6274	5.3804	4.5134
		0.99	6.1854	5.9384	4.6714

Table 2: MSE values when $\beta_0 = 0$

		ρ	ML	Ridge	RNBRE
$p = 4$	$n = 50$	0.90	6.8261	6.5791	5.7123
		0.95	7.4541	7.2072	6.3401
		0.99	7.8521	7.6051	6.7381
	$n = 100$	0.90	5.1971	4.9501	4.0831
		0.95	6.2721	6.0251	5.1581
		0.99	6.4641	6.2172	5.3501
	$n = 200$	0.90	5.0401	4.7931	3.9261
		0.95	5.2501	5.0031	4.1361
		0.99	6.0051	5.7581	4.8911
$p = 7$	$n = 50$	0.90	6.9311	6.6842	5.8171
		0.95	7.5501	7.3031	6.4361
		0.99	7.9651	7.7181	6.8511
	$n = 100$	0.90	5.4661	5.2191	4.3521
		0.95	6.6091	6.3621	5.4951
		0.99	6.9341	6.6871	5.8201
	$n = 200$	0.90	5.3761	5.1291	4.2621
		0.95	5.6511	5.4041	4.5371

		0.99	6.2091	5.9621	5.0951

Table 3: MSE values when $\beta_0 = 1$

			ML	Ridge	RNBRE
		ρ			
$p = 4$	$n = 50$	0.90	7.024	6.777	5.91
		0.95	7.652	7.405	6.538
		0.99	8.05	7.803	6.936
	$n = 100$	0.90	5.395	5.148	4.281
		0.95	6.47	6.223	5.356
		0.99	6.662	6.415	5.548
	$n = 200$	0.90	5.238	4.991	4.124
		0.95	5.448	5.201	4.334
		0.99	6.203	5.956	5.089
$p = 7$	$n = 50$	0.90	7.129	6.882	6.015
		0.95	7.748	7.501	6.634
		0.99	8.163	7.916	7.049
	$n = 100$	0.90	5.664	5.417	4.55
		0.95	6.807	6.56	5.693
		0.99	7.132	6.885	6.018
	$n = 200$	0.90	5.574	5.327	4.46
		0.95	5.849	5.602	4.735
		0.99	6.407	6.16	5.293

5. Conclusions

In this paper, a restricted ridge estimator of negative binomial regression model was proposed. This proposed estimator allows us to handle multicollinearity. According to Monte Carlo simulation studies, the restricted estimator has better performance than maximum likelihood estimator and ridge estimator, in terms of MSE. The superiority of the new estimator based on the resulting MSE was observed and it was shown that the results are consistent with Monte Carlo simulation results.

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