On Estimating the Reliability Function of an Exponential-Pareto II Distribution Using Weighted Least Squares Method

Rasha Tariq Habeeb, Jubran A Labban

Department of Mathematics,

College of Science, University of Al-Qadisiyah, Al-Diwaniyah, Iraq.

Corresponding author: Jubran.Labban@qu.edu.iq

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Abstract

The reliability function of the two-parameter Pareto distribution fits income and property value distributions pretty well and explains much of what we see in the real world. For controlled data, the reliability function is usually estimated by the least square method if there is no contamination; if there is contamination, the computation process gets complicated, especially if the model is not linear.

This study suggests the weighted least square estimation to figure out the reliability function's parameters. This method is short and easy to implement. It can be used with the nonlinear model with simulated cases; this study will show that the weighted least square estimator performs better than the ordinary least unbiased estimator.

Keywords: The Reliability function, Pareto II distribution, Weighted Least Squares (WLS) ,Ordinary least square (OLS).

Introduction:

There are some natural phenomena such as medical, engineering, financial, geophysical and natural phenomena (rain, hurricanes, earthquakes), etc., which cannot be represented by a single distribution, but need to merge two distributions such as Pareto II distribution and ace distribution to obtain a more flexible distribution to describe complex phenomena and heterogeneous societies, one of the most commonly used methods of estimation is the method of Ordinary least Square when data are distributed naturally and the Ordinary least Square way has been one of the methods of estimating parameters for an extended period of time because of the good advantages of their capabilities, but in many cases and in practice researchers face the problem of data deviation from distribution by containing abnormal values Outliers arising from errors in data description or recording false observations, Many of the issues that accompany the occupation and dominance of the great powers over developing countries are causing human, moral and material losses and delays in economic and healthy growth, because such data have some measurements or recordings of their variables exceeding their natural ratios, these measurements are then abnormal observations, so the classic method of estimating parameters is a failure here, so the fortified methods become necessary in such a case because these methods do not reduce the importance of abnormal viewing within the data set, the main objective of this research after estimating the parameters is to conduct a test on the parameters of Pareto II distribution is contaminated by the exponential distribution [1].

Several of these methods have been selected, including the usual Ordinary least Square method for nonlinear models that give optimal results in the case of Pareto II distribution, as well as the method of estimating the Weighted Least Squares. At the same time, the most influential researchers mentioned these studies, Grubbs defined "abnormal value as that value that seems to deviate from the rest of the other sample values" [2]. The anomalous view was described as "viewing that follows a probability distribution different from the distribution of the rest of the sample views" [3]. Explained the impact of outliers values on the capabilities of least Squares in his famous sentence, "Having one anomalous value destroys the good advantages of Least Squares, and it pulls the line of reconciliation for least squares." There is no doubt that the process of obtaining estimates that are very close to its parameters is the goal of any study or effort to represent the community or study accordingly better; most studies and research in the field of immunity and within multivariable have been interested in searching for alternative methods to the method of least squares, which are called Robust methods as Rousseeuw and Leroy [4]. Explained by examples of the effect of abnormal values on the capabilities of the small squares and explained by drawing how one eccentric view impacts the direction of the reconciliation line for the smaller squares.

The Reliability Function

Reliability is abbreviated from "reliable," which means trusting and depending on something. This term was used first after the First World War by 1920 in the productivity improvement process through using the statistical control operations; at the end of 1950 and the beginning of 1960, the USA worked to release intercontinental ballistic missiles and competition with the Russians to be the first who will be on the moon. The significant role of reliability appears in study and application, and from then, reliability has become very popular in the applied aspects [5].

During the last year, many studies estimated the reliability function of many distributions using different counting methods. Also, there are other techniques in assessing this function through metrics depending on the time average fail and another [6].

Definition 1

Let T be a random variable and $P(T \leq t)$ be the probability failure function in the period (t, t+Δt), and F(t) be the cumulative distribution function (cdf) at the time of failure t, The **Reliability Function,** denoted by $R(t)$, can be defined as

$$R(t) = P(T > t) = 1 - P(T \leq t) = 1 - F(t) \quad (1)$$

Statistical Distribution

Pareto II Distribution

The Pareto distribution is attributed to the Italian economist, Vilfredo Pareto, who used this distribution for the first time and extensively in the subject of economics as a model for studying the income distribution in the applications of dependency theory because it is one of the failure distributions for models of stress, durability, and mechanical engineering [7-8]. The average time taken for devices and equipment until failure is essential in determining the reliability of devices and equipment; the p.d.f of Pareto II becomes [9].

$$f(x) = \begin{cases} \dfrac{\alpha\delta}{(1+\delta t)^{(\alpha+1)}} & x > 0 \\ 0 & o.w \end{cases} \qquad (2)$$

Exponential Distribution

The exponential distribution is a continuous probability distribution named after the exponential function. The time intervals between occurrences are estimated using this distribution. The exponential distribution is frequently utilized in time measurement concerns. This includes the post office service, the duration of a telephone call, the period of unloading the freighter, the period of repairing a machine, and the period of waiting for a customer before obtaining the service. In the exact sciences, the exponential distribution represents the lifespan of radioactive atoms before they decay.

Definition 2

We say that the random variable T has the exponential distribution if the PDF given by

$$f(t, \delta) = \delta e^{-\delta t} \quad , \qquad \delta > 0$$

and CDF, Reliability

$$F(t) = 1 - e^{-\delta t} \quad , \qquad R(t) = e^{-\delta t}$$

Nonlinear regression analysis

Nonlinear regression models are a topic of high importance despite the scarcity of studies related to them compared to linear regression models. Still, they have wide applications in applied and natural studies. It is known that the relationships are either linear in terms of parameters in terms of variables, which are (linear regression relationships), and the general relationship that includes (k) of the explanatory variables takes the following formula:

$$Y = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + U \qquad (3)$$

This relationship is linear in the parameters' allowance, as well as in the variables.

Or the regression relationships should be nonlinear and are also either linear in the parameters and nonlinear allowances of variables, such as:

$$Y = \ln(\beta_0 + \beta_1 X) + u$$

That the regression relationship is not linear in the parameters' allowance but linear in the allocation of variables such as

$$Y = \beta_0 + \sqrt{\beta_1} X + u$$

$$Y = \beta_0 + {\beta_1}^2 X + u \quad [10].$$

Shifting nonlinear regression to linear regression

There are several ways to convert nonlinear models into linear models where some models have a linear regression somewhat similar to linear regression, as linear regression can be converted into a linear relationship when the linear slope is very similar to linear regression can be altered by making a simple conversion of data or using logarithmic transformation. Some models are difficult to convert according to the following definition:

Definition 3

Let X matrix variables and $\Theta = (\theta_1, \theta_2, \ldots. \theta_i); i \in N$ is the vector of parameters $f(\underline{X}, \Theta)$ is the nonlinear model's function, and Y is the viewing values of response variables. The nonlinear regression equation is

$$Y = f(\underline{X}, \Theta) + u \tag{4}$$

As Greene 2003 [11] suggested converting nonlinear models into linear models and their formulas (4) and using the method of Ordinary least Square nonlinear can minimize the amount that follows

$$S(\Theta) = \grave{u}u = [Y - f(\underline{X}, \Theta)]\grave{}[Y - f(\underline{X}, \Theta)] \tag{5}$$

by deriving the formula (5) for Θ

$$\frac{\delta S}{\delta \Theta} = -2\frac{\delta f(\underline{X}, \Theta)}{\delta \Theta}[Y - f(\underline{X}, \Theta)] = 0 \tag{6}$$

Where $\Theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_i \end{bmatrix}; \forall i, \in N$

$$\frac{\delta f(\underline{X}, \Theta)}{\delta \Theta} = Z(\Theta) = \begin{bmatrix} \frac{\delta f(x_1, \Theta)}{\delta \theta_0} & \cdots & \frac{\delta f(x_1, \Theta)}{\delta \theta_i} \\ \vdots & \ddots & \vdots \\ \frac{\delta f(x_n, \Theta)}{\delta \theta_0} & \cdots & \frac{\delta f(x_n, \Theta)}{\delta \theta_i} \end{bmatrix} \quad \forall i, n \in N \tag{7}$$

Let $Z(\Theta_{(1)})$ be the matrix calculated for particular values Θ and using the Tylor Neighborhood Selector $\Theta_{(1)}$ the view can be rounded

$$f(\underline{X}, \Theta) \approx f(\underline{X}, \Theta_{(1)}) + Z(\Theta_{(1)})(\Theta - \Theta_{(1)}) \tag{8}$$

Then

$$Y \approx f(\underline{X}, \Theta_{(1)}) + Z(\Theta_{(1)})\Theta - Z(\Theta_{(1)})\Theta_{(1)} + u \tag{9}$$

Let

$$Y^*(\Theta_{(1)}) = Y - f(\underline{X}, \Theta_{(1)}) + Z(\Theta_{(1)})\Theta_{(1)} \tag{10}$$

We can bring the nonlinear model closer to a linear form through the last version.

$$\widetilde{Y}(\Theta_{(1)}) = Z(\Theta_{(1)})\,\Theta + u \tag{11}$$

Now we can estimate the features of the linear model and use the Newton-Raphson method and the following form:

$$\Theta_{(2)} = \Theta_{(1)} + \left[Z(\grave{\Theta}_{(1)})\, Z(\Theta_{(1)})\right]^{-1}\; Z(\grave{\Theta}_{(1)})(Y - f(\underline{X}, \Theta_{(1)})) \quad (12)$$

Which also gives new values to its $\Theta = \Theta_{(2)}$ vector and can continue the process of searching for optimal values for the product of landmarks until p, we note the relative stillness of the estimated parameters:

$$\widehat{\Theta} = \Theta_{(p)} = \Theta_{(p-1)} \quad (13)$$

Estimation reliability Function

The primary purpose of the estimation methods is to obtain the capabilities of the parameters of the model to be studied, and it must have good specifications this time to make a model able to benefit from it, and be the estimation methods differ according to different ideas and techniques, to achieve two purposes important [7].

The first is that the capabilities contain good specifications, the purpose the other is the method appears in an easy-to-implement way; in recent years, intelligent techniques have been used to estimate the function of reliability, and there are several ways to estimate the function of the reliability of the total data, including classical and non-classical methods [8].

Ordinary least square (OLS)

The method of least squares depends on finding the estimators that are characterized as providing the smallest sum of squares error resulting from the difference between the estimated value and the actual value. The least-squares method can be used for the reliability function of Pareto II distribution. Therefore, the estimations of least squares for the parameters can be found through the following equation

$$1 - F(t) = (1 + \delta t)^{-\alpha}$$

Using method (Greene, 2000) and through formulas (12) and by imposing the following

$$f(\underline{X}, \Theta_{(1)}) = (1 + \delta t)^{-\alpha}$$

$$Y = 1 - F(t)$$

$$\Theta = \begin{bmatrix} \delta \\ \alpha \end{bmatrix}$$

$$Z(\Theta_{(1)}) = \begin{bmatrix} \frac{\partial f(t_1, \Theta)}{\partial \delta} & \frac{\delta f(t_1, \Theta)}{\partial \alpha} \\ \vdots & \vdots \\ \frac{\partial f(t_n, \Theta)}{\partial \sigma} & \frac{\partial f(t_n, \Theta)}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} -\alpha t_1 (1 + \delta t_1)^{-(\alpha+1)} & -(1 + \delta t_1)^{-\alpha} \ln(1 + \delta t_1) \\ \vdots & \vdots \\ -\alpha t_n (1 + \delta t_n)^{-(\alpha+1)} & -(1 + \delta t_1)^{-\alpha} \ln(1 + \delta t_n) \end{bmatrix}$$

THEN

$$\left[Z\left(\grave{\Theta}_{(1)}\right) Z\left(\Theta_{(1)}\right)\right]^{-1} = \begin{bmatrix} \sum_{i=1}^{n} \alpha^2 {t_1}^2 (1+\delta t_i)^{-2(\alpha+1)} & \sum_{i=1}^{n} \alpha t_i (1+\delta t_i)^{-(2\alpha+1)} \ln(1+\delta t_n) \\ \sum_{i=1}^{n} \alpha t_i (1+\delta t_i)^{-(2\alpha+1)} \ln(1+\delta t_n) & \sum_{i=1}^{n} (1+\delta t_i)^{-2\alpha} (\ln(1+\delta t_i))^2 \end{bmatrix}^{-1}$$

$$Z\left(\grave{\Theta}_{(1)}\right)\left(Y - f\left(\underline{X}, \Theta_{(1)}\right)\right) = \begin{bmatrix} \sum_{i=1}^{n} \alpha t_i (1+\delta t_i)^{-(2\alpha+1)} - \sum_{i=1}^{n} \alpha t_i (1+\delta t_i)^{-(\alpha+1)} y_i \\ \sum_{i=1}^{n} (1+\delta t_i)^{-2\alpha} \ln(1+\delta t_i) - \sum_{i=1}^{n} (1+\delta t_i)^{-\alpha} \ln(1+\delta t_i) y_i \end{bmatrix}$$

Then

$$\Theta_{(2)} = \Theta_{(1)} + \left[Z\left(\grave{\Theta}_{(1)}\right) Z\left(\Theta_{(1)}\right)\right]^{-1} Z\left(\grave{\Theta}_{(1)}\right)\left(Y - f\left(\underline{X}, \Theta_{(1)}\right)\right)$$

Substituting the value of the estimators, we get an estimate of the reliability function

$$\hat{R}_{OLS}(t) = (1 + \hat{\delta}_{OLS} t)^{-\hat{\alpha}_{OLS}} \tag{14}$$

Weighted Least Squares (WLS)

Several estimation methods will adopt the height of the capabilities adopted by the regression subject, which studies have suggested estimating the beta parameter and can be used to estimate other distributions [12].

Definition 4 [13].

Let the $(y_1, y_2, \dots, y_n)$ be a random sample that follows the distribution of what G(.), and that $y_i, i = 1,2,\dots,n$ represents the statistics ranked for the sample, then the prediction and variation of the distribution $G(y_i)$ is

$$E\left(G(y_i)\right) = \frac{i}{n+1}$$

$$V\left(G(Y_i)\right) = \frac{i(n+i+1)}{(n+1)^2(n+2)}$$

We use definition 4 to get a WLS estimator by minimizing the total of the following weighted squares.

$$T_w = \sum_{i=1}^{n} w_i \left(G(Y_i) - \frac{i}{n+1}\right)^2 \tag{15}$$

$$w_i = \frac{1}{V(G(Y_i))} = \frac{(n+1)^2(n+2)}{i(n+i+1)} \tag{16}$$

As for the Pareto II distribution contaminated with an exponential distribution, the estimate of parameters $\hat{\delta}_1, \hat{\delta}_2, \hat{\alpha}_1$ to find the estimate of the function of the reliability and by imposing the density function for distributions

$$G(Y_i) = (1-\tau)f_1(t,\delta_1,\alpha_1) + \tau f_2(t,\delta_2) = (1-\tau)\frac{\alpha_1\delta_1}{(1+\delta_1 t)^{(\alpha_1+1)}} + \tau\ \delta_2 e^{-\delta_2 t}$$

Then $f_1(t)$ is the primary distribution (distribution of Pareto II), $f_2(t)$ contaminated distribution (exponential distribution), and τ is the amount of contaminated that can be reduced formula (15)

$$T_w = \sum_{i=1}^{n} w_i \left((1-\tau)\frac{\alpha_1\delta_1}{\left(1+\delta_1 t_{(i)}\right)^{(\alpha_1+1)}} + \tau\ \delta_2 e^{-\delta_2 t_{(i)}} - \frac{i}{n+1} \right)^2 \qquad (17)$$

Taking the partial derivative for his teacher $\underline{\theta}$ to equalize (17), we get

$$\frac{\partial}{\partial \underline{\theta}} T_w = \begin{bmatrix} \frac{\partial}{\partial \alpha} T_w \\ \frac{\partial}{\partial \delta_1} T_w \\ \frac{\partial}{\partial \delta_2} T_w \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = G \qquad \text{such that} \qquad \underline{\theta} = \begin{bmatrix} \alpha \\ \delta_1 \\ \delta_2 \end{bmatrix} \qquad (18)$$

$$\frac{\partial T_w}{\partial \alpha} = 2\sum_{i=1}^{n} w_i \left((1-\tau)\frac{\alpha\delta_1}{(1+\delta_1 t_i)^{(\alpha+1)}} + \tau\ \delta_2 e^{-\delta_2 t_i} - \frac{i}{n+1} \right)(1 - \tau)\frac{\delta_1 - \alpha\delta_1 \ln(1+\delta_1 t_i)}{(1+\delta_1 t_i)^{(\alpha+1)}}$$

$$\frac{\partial T_w}{\partial \alpha} = \sum_{i=1}^{n} w_i \left((1-\tau)^2 \frac{\alpha{\delta_1}^2 - \alpha^2{\delta_1}^2 \ln(1+\delta_1 t_i)}{(1+\delta_1 t_i)^{2(\alpha+1)}} + \tau(1-\tau)\frac{\delta_2 e^{-\delta_2 t_i}(\delta_1 - \alpha\delta_1 \ln(1+\delta_1 t_i))}{(1+\delta_1 t_i)^{(\alpha+1)}} - \frac{i}{n+1}\ (1-\tau)\frac{\delta_1 - \alpha\delta_1 \ln(1+\delta_1 t_i)}{(1+\delta_1 t_i)^{(\alpha+1)}} \right) \qquad (19)$$

$$\frac{\partial T_w}{\partial \delta_1} = 2\sum_{i=1}^{n} w_i \left((1-\tau)\frac{\alpha\delta_1}{(1+\delta_1 t_i)^{(\alpha+1)}} + \tau\ \delta_2 e^{-\delta_2 t_i} - \frac{i}{n+1} \right)(1 - \tau)\frac{\alpha(1+\delta_1 t_i) - \alpha\delta_1(\alpha+1)t_i}{(1+\delta_1 t_i)^{(\alpha+2)}} \qquad (20)$$

$$\frac{\partial T_w}{\partial \delta_1}=\sum_{i=1}^{n} w_i\Bigg((1-\tau)^2\frac{\alpha^2\delta_1(1+\delta_1 t_i)-\alpha^2{\delta_1}^2(\alpha+1)t_i}{(1+\delta_1 t_i)^{(2\alpha+3)}} + \tau\,(1-\tau)\frac{\delta_2 e^{-\delta_2 t_i}(\alpha(1+\delta_1 t_i)-\alpha\delta_1(\alpha+1)t_i)}{(1+\delta_1 t_i)^{(\alpha+2)}} - \frac{i}{n+1}(1-\tau)\frac{\alpha(1+\delta_1 t_i)-\alpha\delta_1(\alpha+1)t_i}{(1+\delta_1 t_i)^{(\alpha+2)}}\Bigg) \qquad (21)$$

$$\frac{\partial T_w}{\partial \delta_2}=2\sum_{i=1}^{n} w_i\left((1-\tau)\frac{\alpha\delta_1}{(1+\delta_1 t_i)^{(\alpha+1)}}+\tau\ \delta_2 e^{-\delta_2 t_i}\ -\frac{i}{n+1}\right)\tau\, e^{-\delta_2 t_i}(1-\ \delta_2)$$

$$\frac{\partial T_w}{\partial \delta_2}=\sum_{i=1}^{n} w_i\left(\tau(1-\tau)\frac{\alpha\delta_1\ e^{-\delta_2 t_i}(1-\ \delta_2)}{(1+\delta_1 t_i)^{(\alpha+1)}}+\tau^2\ \delta_2 e^{-2\delta_2 t_i}(1-\ \delta_2)\ -\frac{i\tau\ e^{-\delta_2 t_i}(1-\ \delta_2)}{n+1}\right) \qquad (22)$$

The second derivative of the equation (17) is as follows:

$$\frac{\partial}{\partial\underline{\theta}\,\partial\underline{\theta}}T_w=\begin{bmatrix}\frac{\partial T_w}{\partial\alpha^2} & \frac{\partial T_w}{\partial\alpha\,\partial\delta_1} & \frac{\partial T_w}{\partial\alpha\,\partial\delta_2}\\ \frac{\partial T_w}{\partial\alpha\,\partial\delta_1} & \frac{\partial T_w}{\partial\delta_1^2} & \frac{\partial T_w}{\partial\delta_1\,\partial\delta_2}\\ \frac{\partial T_w}{\partial\delta_2\,\partial\alpha} & \frac{\partial T_w}{\partial\delta_2\,\partial\delta_1} & \frac{\partial T_w}{\partial\delta_2^2}\end{bmatrix}$$

$$\frac{\partial}{\partial\underline{\theta}\,\partial\underline{\theta}}T_w=\begin{bmatrix}h_{11} & h_{12} & h_{13}\\ h_{21} & h_{22} & h_{23}\\ h_{31} & h_{32} & h_{33}\end{bmatrix}=H \qquad (23)$$

To find the estimate of parameters (α, δ_1, δ_2) in formula (6), Newton- Raphson numerical method of solving this equation is adopted and based on formulas (9), (8) as follows:

$$\underline{\theta}_{n+1}=\theta_n-H^{-1}G \qquad (24)$$

Formula (10) is the only robust maximum likelihood estimator method parameter of landmarks (α,δ_1, δ_2) as follows:

$$\hat{\underline{\theta}}_{RML}=\underline{\theta}_{n+1} \qquad (25)$$

Using stability, the reliability function is estimated to be as follows:

$$R(t)=(1-\tau)R_1(t)+\tau R_2(t)$$

$$=(1-\tau)(1+\delta_1 t)^{-\alpha_1}+\tau(e^{-\delta_2 t})$$

$$\boxed{\hat{R}_{WLS}(t)=(1-\hat{\tau})(1+\hat{\delta}_{1_{WLS}}t)^{-\hat{\alpha}_{WLS}}+\hat{\tau}(e^{-\hat{\delta}_{2WLS}t})} \qquad (26)$$

Simulation

In our study, the Monte Carlo simulation method was used to test the applicability of the estimation methods presented in the theoretical side of the research and then compared those methods using the MSE and IMSE criteria.

In this part, we'll describe the stages of simulation experiments.

Stage 1: This is one of the most critical stages on which the rest of the subsequent stages depend, where the default values are selected as follows:

Choosing arbitrary values for α, δ_1, δ_2 with $\tau = 10\%, 20\%, 30\%$

Table 1: The default value of the scale parameter

Model	1	2	3
α	1.2	2.4	3.1
δ_1	1.3	1.4	2.3
δ_2	2.1	3.5	1.6

For simulation n = (50, 150, 250) virtual samples size

The repetition of each experiment is r =1000

Stage 2: Pareto II distribution and exponential distribution data are generated using the CDF

$$F_1(t) = 1 - (1 + \delta_1 t)^{-\alpha}$$

$$F_2(t) = 1 - e^{-\delta_2 t}$$

Suppose the U represents the CDF of the Pareto II distribution

$U = 1 - (1 + \delta_1 t)^{-\alpha}$

$$\Rightarrow t = \delta_1^{-1}\left((1 - U)^{-1/\alpha} - 1\right) \qquad (27)$$

As for generating exponential distribution and letting U ~ uniform (0,1) of the distribution

$$U = 1 - e^{-\delta_2 t}$$

Then

$$t = -\delta_2^{-1}\ln(1 - u) \qquad (28)$$

The distribution of the contaminated Pareto II under study in this research is obtained according to the following formula and based on the equation (27), (28) as follows:

$$t = \begin{cases} \delta_1^{-1}\left((1-U)^{-1/\alpha} - 1\right) \\ -\delta_2^{-1}\ln(1-u) \end{cases}$$

Stage 3: At this stage, the function of the reliability of the Pareto II model was estimated according to the OLS method of formula (14) and WLS method of formula (26)

Stage 4: This is the last stage where the two methods of estimation of the reliability function are compared on the adoption of MSE and IMSE according to the following formulas:

$$\text{MSE}(\hat{R}(t)) = \frac{1}{r}\sum_{i=1}^{r}\left[\hat{R}_i(t_j) - R(t_j)\right]^2$$

$$\text{IMSE}(\hat{R}(t)) = \frac{1}{n_t}\sum_{j=1}^{n_t}\left\{\frac{1}{r}\sum_{i=1}^{r}\left[R_i(t_j) - R(t_j)\right]^2\right\} = \frac{1}{n_t}\sum_{j=1}^{n_t}\left(\text{MSE}(\hat{R}(t_j))\right)$$

The following tables show reliability function estimate, MSE, and IMSE on all sample sizes.

Table 2: Times of estimating the reliability function

t_i	Model 1	Model 2	Model 3
t_1	0.1	0.1	0.1
t_2	0.18	0.15	0.2
t_3	0.26	0.2	0.3
t_4	0.34	0.25	0.4
t_5	0.42	0.3	0.5
t_6	0.5	0.35	0.6
t_7	0.58	0.4	0.7
t_8	0.66	0.45	0.8
t_9	0.74	0.5	0.9

a- Generating random numbers for the Pareto II distribution with two parameters (α,δ_1), an exponential distribution with a single-parameter (δ_2). According to the available generation function in (VB.NET) language:

t ~ Pareto(α,δ) or t ~ exp(δ_2)

b. Find estimators:

At this stage, the estimation of the reliability function of the Pareto II distribution and the contaminated Pareto II distribution through the exponential distribution is dealt with through estimation methods in the theoretical part of this thesis, according to the methods: MLE and RMLE.

c. Comparison:

After finding estimators of reliability function, two criteria were used to evaluate the accuracy of estimation methods: Mean squares error (MSE) and Integral Mean square Error (IMSE). The way that yields the smallest value of MSE and IMSE is best fitted.

Results

Table 3: Estimators of the parameters, the reliability function of the estimation methods when contamination ratio (10%) and for all selected sample sizes and all model

n	Model 1				Model 2				Model 3			
	t_i	R_{real}	R_{OLS}	R_{WLS}	t_i	R_{real}	R_{OLS}	R_{WLS}	t_i	R_{real}	R_{OLS}	R_{WLS}
50	0.1	0.944115	0.819626	0.863755	0.1	0.654865	0.919553	0.654786	0.1	0.503056	0.593453	0.603995
	0.18	0.844606	0.671657	0.766234	0.15	0.565865	0.69233	0.566635	0.2	0.386173	0.291348	0.302421
	0.26	0.761771	0.866176	0.692234	0.2	0.602058	0.781603	0.494326	0.3	0.262686	0.230314	0.243327
	0.34	0.566139	0.806931	0.516506	0.25	0.431688	0.592726	0.530853	0.4	0.15467	0.157508	0.137949
	0.42	0.517469	0.642632	0.579664	0.3	0.465213	0.551115	0.469398	0.5	0.115936	0.092014	0.099223
	0.5	0.475619	0.702793	0.535312	0.35	0.412492	0.628119	0.417415	0.6	0.089554	0.067992	0.073406
	0.58	0.536969	0.656985	0.496812	0.4	0.300797	0.480513	0.305249	0.7	0.070755	0.051666	0.055512
	0.66	0.498247	0.503037	0.378898	0.45	0.269377	0.550487	0.274082	0.8	0.05688	0.040183	0.052228
	0.74	0.379809	0.471245	0.433352	0.5	0.296165	0.517179	0.24712	0.9	0.056647	0.038954	0.033384

	0.82	0.355184	0.441909	0.332942	0.55	0.218884	0.398398	0.273353	1	0.046657	0.025706	0.032284
150	0.1	0.772457	0.881795	0.8841	0.1	0.654865	0.724915	0.802186	0.1	0.503056	0.562609	0.57135
	0.18	0.691042	0.752059	0.770593	0.15	0.565865	0.802003	0.694627	0.2	0.315959	0.379976	0.333145
	0.26	0.623267	0.649788	0.560921	0.2	0.492593	0.596916	0.60626	0.3	0.214925	0.331539	0.210305
	0.34	0.691947	0.464436	0.618344	0.25	0.527618	0.545438	0.532895	0.4	0.189042	0.246758	0.115091
	0.42	0.632462	0.409586	0.460766	0.3	0.380629	0.500433	0.3857	0.5	0.1417	0.189724	0.080458
	0.5	0.475619	0.445133	0.422743	0.35	0.412492	0.460855	0.343184	0.6	0.109454	0.122502	0.058253
	0.58	0.536969	0.326188	0.39015	0.4	0.367641	0.520491	0.37519	0.7	0.070755	0.098761	0.043402
	0.66	0.407657	0.294015	0.442261	0.45	0.329239	0.482478	0.275918	0.8	0.06952	0.081052	0.033124
	0.74	0.464211	0.325753	0.33703	0.5	0.296165	0.44854	0.24911	0.9	0.056647	0.06753	0.025805
	0.82	0.434114	0.242841	0.38511	0.55	0.267524	0.342092	0.225833	1	0.038174	0.069664	0.025012
250	0.1	0.944115	0.764515	0.760814	0.1	0.654865	0.590084	0.656649	0.1	0.503056	0.639445	0.613853
	0.18	0.844606	0.661948	0.824402	0.15	0.691613	0.594344	0.695971	0.2	0.315959	0.409186	0.312965
	0.26	0.761771	0.601629	0.604411	0.2	0.602058	0.494758	0.608585	0.3	0.262686	0.2291	0.210096
	0.34	0.691947	0.664766	0.546644	0.25	0.431688	0.340052	0.53608	0.4	0.15467	0.201346	0.148601
	0.42	0.632462	0.616207	0.609159	0.3	0.380629	0.288018	0.38886	0.5	0.115936	0.123055	0.133409

	0.5	0.475619	0.554342	0.559318	0.35	0.412492	0.24578	0.423789	0.6	0.089554	0.094736	0.100791
	0.58	0.536969	0.417649	0.422766	0.4	0.300797	0.258104	0.310766	0.7	0.070755	0.091355	0.063659
	0.66	0.498247	0.472263	0.47992	0.45	0.329239	0.22316	0.341992	0.8	0.05688	0.073557	0.049982
	0.74	0.464211	0.44535	0.447847	0.5	0.242317	0.158781	0.30921	0.9	0.056647	0.049294	0.048624
	0.82	0.355184	0.342851	0.419652	0.55	0.267524	0.169667	0.229603	1	0.038174	0.050073	0.039146

Table 4: Estimators of the parameters, the reliability function of the estimation methods when contamination ratio (20%) and for all selected sample sizes and all model

n	Model 1				Model 2				Model 3			
	t_i	R_{real}	R_{OLS}	R_{WLS}	t_i	R_{real}	R_{OLS}	R_{WLS}	t_i	R_{real}	R_{OLS}	R_{WLS}
50	0.1	0.767687	0.774304	0.673779	0.1	0.652571	0.781308	0.599531	0.1	0.532375	0.72719	0.645607
	0.18	0.682782	0.571599	0.705412	0.15	0.687069	0.731258	0.500419	0.2	0.432016	0.603112	0.423138
	0.26	0.611942	0.959895	0.512037	0.2	0.595856	0.686251	0.516976	0.3	0.309128	0.510521	0.297328
	0.34	0.552203	0.644221	0.463125	0.25	0.425408	0.789077	0.441839	0.4	0.232484	0.439288	0.178936
	0.42	0.612783	0.730424	0.423753	0.3	0.373331	0.744044	0.381321	0.5	0.147987	0.383116	0.135784
	0.5	0.559492	0.555093	0.390747	0.35	0.32937	0.57524	0.271574	0.6	0.117892	0.412993	0.128742
	0.58	0.513461	0.630941	0.442941	0.4	0.356931	0.544637	0.238186	0.7	0.116749	0.300882	0.101495
	0.66	0.387369	0.587459	0.41275	0.45	0.317957	0.516612	0.210296	0.8	0.078364	0.27012	0.066274

	0.74	0.358748	0.547602	0.386157	0.5	0.284496	0.490873	0.186764	0.9	0.079311	0.298506	0.053404
	0.82	0.407722	0.511016	0.296631	0.55	0.255628	0.570985	0.166723	1	0.066149	0.271585	0.053012
	0.1	0.767687	0.971081	0.931316	0.1	0.797587	0.677489	0.565928	0.1	0.532375	0.70426	0.647908
	0.18	0.834512	0.726149	0.674451	0.15	0.687069	0.452033	0.561905	0.2	0.432016	0.397276	0.428014
	0.26	0.747929	0.668429	0.601993	0.2	0.595856	0.371083	0.378797	0.3	0.252923	0.352974	0.304494
	0.34	0.552203	0.619066	0.541283	0.25	0.519944	0.315275	0.316168	0.4	0.232484	0.266989	0.18617
	0.42	0.501368	0.576375	0.598759	0.3	0.456293	0.255262	0.326387	0.5	0.147987	0.208291	0.143857
	0.5	0.457766	0.658897	0.545128	0.35	0.402564	0.220041	0.278684	0.6	0.14409	0.136283	0.139081
	0.58	0.420105	0.50627	0.408259	0.4	0.292035	0.184781	0.196629	0.7	0.116749	0.13593	0.111878
	0.66	0.387369	0.583175	0.459008	0.45	0.260147	0.209095	0.171104	0.8	0.078364	0.112816	0.074548
	0.74	0.43847	0.45113	0.424164	0.5	0.23277	0.149026	0.150118	0.9	0.079311	0.094982	0.061284
150	0.82	0.407722	0.522817	0.322054	0.55	0.20915	0.156948	0.162176	1	0.066149	0.080952	0.062022
	0.1	0.767687	0.794521	0.931316	0.1	0.652571	0.563309	0.565928	0.1	0.532375	0.549167	0.650196
	0.18	0.682782	0.887515	0.674451	0.15	0.562147	0.552485	0.561905	0.2	0.353468	0.446003	0.42815
	0.26	0.747929	0.816969	0.735769	0.2	0.595856	0.371083	0.462974	0.3	0.309128	0.314721	0.247042
250	0.34	0.552203	0.619066	0.541283	0.25	0.425408	0.315275	0.316168	0.4	0.232484	0.190005	0.182284

	0.42	0.612783	0.704459	0.598759	0.3	0.456293	0.22685	0.267044	0.5	0.180873	0.145126	0.169561
	0.5	0.457766	0.658897	0.545128	0.35	0.402564	0.220041	0.278684	0.6	0.14409	0.139253	0.107925
	0.58	0.420105	0.50627	0.498983	0.4	0.356931	0.184781	0.240325	0.7	0.095522	0.111799	0.085321
	0.66	0.473451	0.477143	0.375552	0.45	0.317957	0.234065	0.242127	0.8	0.095778	0.091441	0.083453
	0.74	0.43847	0.45113	0.347044	0.5	0.284496	0.171142	0.183478	0.9	0.079311	0.075973	0.055168
	0.82	0.407722	0.427759	0.393622	0.55	0.255628	0.145948	0.162176	1	0.054122	0.063974	0.054907

Table 5: Estimators of the parameters, the reliability function of the estimation methods when contamination ratio (30%) and for all selected sample sizes and all model

n	Model 1				Model 2				Model 3			
	t_i	R_{real}	R_{OLS}	R_{WLS}	t_i	R_{real}	R_{OLS}	R_{WLS}	t_i	R_{real}	R_{OLS}	R_{WLS}
50	0.1	0.932455	0.856914	0.867912	0.1	0.650278	1.039136	0.691689	0.1	0.686516	0.75051	0.669313
	0.18	0.674523	0.671933	0.601696	0.15	0.558428	1.011805	0.56816	0.2	0.390976	0.63926	0.454351
	0.26	0.600616	0.594957	0.635143	0.2	0.589655	0.986255	0.475823	0.3	0.35557	0.553707	0.329264
	0.34	0.538267	0.448279	0.557868	0.25	0.51227	0.787338	0.331488	0.4	0.275926	0.594201	0.248509
	0.42	0.593104	0.397482	0.497178	0.3	0.366033	0.768921	0.349862	0.5	0.220046	0.527617	0.157544
	0.5	0.439913	0.513727	0.367038	0.35	0.392635	0.918592	0.25013	0.6	0.146231	0.386953	0.12425
	0.58	0.400872	0.577422	0.334637	0.4	0.346222	0.898582	0.269803	0.7	0.120288	0.427364	0.121263

	0.66	0.448655	0.531793	0.376143	0.45	0.306677	0.879658	0.196438	0.8	0.122036	0.388872	0.097706
	0.74	0.412729	0.401292	0.285081	0.5	0.223223	0.861727	0.215133	0.9	0.101974	0.291271	0.079265
	0.82	0.311996	0.452977	0.324716	0.55	0.243733	0.84471	0.193883	1	0.085641	0.327643	0.052895
	0.1	0.932455	0.731263	0.733919	0.1	0.650278	0.309965	0.699899	0.1	0.686516	0.794275	0.677332
	0.18	0.674523	0.630383	0.633293	0.15	0.558428	0.189192	0.583264	0.2	0.390976	0.601085	0.465444
	0.26	0.734086	0.544392	0.553623	0.2	0.482445	0.118154	0.406523	0.3	0.290921	0.385474	0.3411
	0.34	0.538267	0.596087	0.48982	0.25	0.51227	0.092091	0.430603	0.4	0.275926	0.379475	0.26008
	0.42	0.485267	0.421798	0.438127	0.3	0.366033	0.05986	0.309426	0.5	0.180038	0.255576	0.203438
	0.5	0.439913	0.380824	0.395754	0.35	0.321247	0.032399	0.335602	0.6	0.178727	0.261744	0.161899
	0.58	0.400872	0.433182	0.360612	0.4	0.283272	0.026622	0.245646	0.7	0.120288	0.18212	0.106701
	0.66	0.367081	0.404558	0.331127	0.45	0.306677	0.014865	0.270335	0.8	0.099848	0.191681	0.105987
	0.74	0.337687	0.377436	0.30611	0.5	0.272829	0.012572	0.20021	0.9	0.083434	0.136506	0.070953
150	0.82	0.381328	0.431973	0.347908	0.55	0.199418	0.008813	0.222478	1	0.07007	0.119923	0.071339
	0.1	0.762917	0.640282	0.901456	0.1	0.650278	1.097281	0.578098	0.1	0.686516	0.546719	0.685901
	0.18	0.674523	0.501427	0.780094	0.15	0.558428	1.469456	0.590632	0.2	0.390976	0.358861	0.385336
250	0.26	0.734086	0.400757	0.68334	0.2	0.482445	1.311458	0.412606	0.3	0.290921	0.30466	0.342936

	0.34	0.538267	0.398273	0.495311	0.25	0.41913	1.74138	0.358205	0.4	0.275926	0.18078	0.259199
	0.42	0.485267	0.328655	0.541884	0.3	0.366033	1.542162	0.385026	0.5	0.220046	0.165791	0.164294
	0.5	0.439913	0.274698	0.489601	0.35	0.321247	2.033295	0.342025	0.6	0.178727	0.104619	0.129465
	0.58	0.489954	0.189968	0.364973	0.4	0.283272	2.186626	0.25055	0.7	0.120288	0.082539	0.103271
	0.66	0.448655	0.162162	0.409455	0.45	0.250917	2.344815	0.275904	0.8	0.099848	0.081121	0.08313
	0.74	0.337687	0.17068	0.309526	0.5	0.272829	2.507817	0.249862	0.9	0.101974	0.054243	0.067392
	0.82	0.311996	0.121207	0.287634	0.55	0.199418	2.189123	0.227249	1	0.07007	0.054944	0.054946

To reach the best estimator through preference between different estimated methods, this study has generally depended on the following statistical measures for Comparison: Mean Square Error (MSE) and Integral Mean Square Error (IMSE). From tables (6), (7), and (8), we noticed the following MSE and IMSE:

Table 6: MSE and IMSE of different reliability functions of the estimation methods when contamination ratio (10%) and for all selected sample sizes and all model

n	**Model 1**			**Model 2**			**Model 3**		
	t_i	**R_{OLS}**	**R_{MLS}**	**t_i**	**R_{OLS}**	**R_{MLS}**	**t_i**	**R_{OLS}**	**R_{MLS}**
	0.1	0.002475	0.000437	0.1	0.00077	3.48E-05	0.1	0.001322	4.79E-05
	0.18	0.005931	0.000485	0.15	0.001236	7.34E-05	0.2	0.002104	7.36E-05
	0.26	0.007622	0.000477	0.2	0.001343	0.000102	0.3	0.002393	0.000102
	0.34	0.012178	0.000546	0.25	0.001624	0.000102	0.4	0.002067	0.000105
	0.42	0.014396	0.0005	0.3	0.001861	0.000142	0.5	0.002623	0.000103
50	0.5	0.013088	0.000372	0.35	0.002062	0.000126	0.6	0.002712	8.01E-05

	0.58	0.013955	0.000338	0.4	0.002729	0.000159	0.7	0.0028	7.46E-05
	0.66	0.014459	0.000375	0.45	0.002907	0.00016	0.8	0.002887	6.84E-05
	0.74	0.014674	0.000342	0.5	0.003058	0.000129	0.9	0.002969	6.2E-05
	0.82	0.014665	0.000256	0.55	0.002608	0.000124	1	0.002492	6.82E-05
	0.1	0.000328	4.98E-05	0.1	0.000849	4.23E-05	0.1	0.001178	3.37E-05
	0.18	0.000637	7.76E-05	0.15	0.001164	8.25E-05	0.2	0.002425	8.9E-06
	0.26	0.001015	7.8E-05	0.2	0.001583	0.000171	0.3	0.002367	2.76E-06
	0.34	0.001673	8.71E-05	0.25	0.001939	0.000246	0.4	0.002596	3.8E-06
	0.42	0.001686	9.25E-05	0.3	0.002233	0.000263	0.5	0.002772	5.75E-06
	0.5	0.001963	9.53E-05	0.35	0.003024	0.000323	0.6	0.002926	9.12E-06
	0.58	0.002204	9.62E-05	0.4	0.002671	0.000381	0.7	0.003068	1.07E-05
	0.66	0.002945	9.56E-05	0.45	0.002833	0.000437	0.8	0.00391	9.64E-06
	0.74	0.00316	0.000115	0.5	0.003625	0.000491	0.9	0.004056	1.25E-05
150	0.82	0.003341	9.17E-05	0.55	0.003759	0.000542	1	0.003427	1.06E-05
	0.1	0.000189	0.000164	0.1	0.000632	1.26E-06	0.1	0.001201	4.05E-05
	0.18	0.000388	0.000176	0.15	0.000891	3.53E-06	0.2	0.002581	8.16E-05
	0.26	0.000653	0.000204	0.2	0.001524	8.5E-06	0.3	0.002577	7.96E-05
	0.34	0.001128	0.000187	0.25	0.001569	1.57E-05	0.4	0.003495	0.000106
	0.42	0.001451	0.000171	0.3	0.001855	2.38E-05	0.5	0.00375	0.00011
	0.5	0.001755	0.000157	0.35	0.002106	3.86E-05	0.6	0.003245	8.92E-05
	0.58	0.002038	0.000144	0.4	0.002842	3.83E-05	0.7	0.004158	8.61E-05
	0.66	0.002296	0.000108	0.45	0.003075	5.29E-05	0.8	0.004332	9.93E-05
	0.74	0.002069	9.93E-05	0.5	0.003279	5.68E-05	0.9	0.004488	7.54E-05
250	0.82	0.00224	9.11E-05	0.55	0.003456	4.79E-05	1	0.004629	8.44E-05

Table 7: MSE and IMSE of different reliability functions of the estimation methods when contamination ratio (20%) and for all selected sample sizes and all model

n	Model 1			Model 2			Model 3		
	t_i	R_{OLS}	R_{MLS}	t_i	R_{OLS}	R_{MLS}	t_i	R_{OLS}	R_{MLS}
50	0.1	0.002627	0.001575	0.1	0.002348	0.001254	0.1	0.002391	0.000136
	0.18	0.006498	0.002036	0.15	0.003054	0.001436	0.2	0.004475	0.00023
	0.26	0.01286	0.001533	0.2	0.003135	0.002007	0.3	0.005699	0.000387
	0.34	0.017307	0.001645	0.25	0.004957	0.002081	0.4	0.007978	0.000375
	0.42	0.021043	0.001162	0.3	0.005961	0.002043	0.5	0.008775	0.0005
	0.5	0.024026	0.000999	0.35	0.005665	0.00159	0.6	0.007738	0.000519
	0.58	0.021523	0.001052	0.4	0.006414	0.001482	0.7	0.010065	0.000521
	0.66	0.022884	0.000912	0.45	0.008656	0.001364	0.8	0.00868	0.000511
	0.74	0.023821	0.000796	0.5	0.007668	0.00152	0.9	0.011094	0.000404
	0.82	0.029834	0.000701	0.55	0.010016	0.001378	1	0.00943	0.000472
50	0.1	0.000582	3.41E-05	0.1	0.001588	0.000201	0.1	0.004065	5.7E-06
	0.18	0.001281	7.31E-05	0.15	0.002419	0.000235	0.2	0.006273	9.44E-06
	0.26	0.002761	0.000107	0.2	0.003612	0.000217	0.3	0.008051	1.12E-05
	0.34	0.004036	0.000134	0.25	0.004794	0.000196	0.4	0.009281	9.67E-06
	0.42	0.00531	0.000155	0.3	0.007233	0.000175	0.5	0.012543	1.16E-05
	0.5	0.006532	0.000139	0.35	0.006959	0.000156	0.6	0.013579	8.77E-06
	0.58	0.007671	0.000182	0.4	0.007906	0.000139	0.7	0.014501	7.82E-06
	0.66	0.008713	0.000191	0.45	0.008756	0.000125	0.8	0.015328	8.29E-06
	0.74	0.009652	0.000161	0.5	0.009513	0.000113	0.9	0.016067	7.04E-06
	0.82	0.008582	0.000201	0.55	0.010182	0.000103	1	0.013682	4.81E-06
250	0.1	3.56E-04	0.000618	0.1	0.001484	0.001412	0.1	0.004245	0.000167

	0.18	7.64E-04	0.00068	0.15	0.003361	0.001025	0.2	0.008016	0.000305
	0.26	1.10E-03	0.000813	0.2	0.004078	0.000826	0.3	0.010303	0.000379
	0.34	0.000741	0.000583	0.25	0.005373	0.000819	0.4	0.011896	0.000341
	0.42	0.000681	0.0006	0.3	0.006586	0.00057	0.5	0.010777	0.000352
	0.5	0.000888	0.000501	0.35	0.007694	0.000514	0.6	0.01168	0.000349
	0.58	0.00092	0.000345	0.4	0.01062	0.000591	0.7	0.012485	0.000336
	0.66	0.000934	0.00036	0.45	0.009572	0.000571	0.8	0.016142	0.000387
	0.74	0.000936	0.000256	0.5	0.01035	0.000558	0.9	0.016932	0.000294
	0.82	0.000761	0.000278	0.55	0.011032	0.000546	1	0.017633	0.000269

Table 8: MSE and IMSE of different reliability functions of the estimation methods when contamination ratio (30%) and for all selected sample sizes and all model

n	Model 1			Model 2			Model 3		
	t_i	**R_{OLS}**	**R_{MLS}**	**t_i**	**R_{OLS}**	**R_{MLS}**	**t_i**	**R_{OLS}**	**R_{MLS}**
50	0.1	0.003869	0.002592	0.1	0.002406	0.003348	0.1	0.004564	0.000175
	0.18	0.009802	0.003011	0.15	0.004596	0.003557	0.2	0.009225	0.000367
	0.26	0.016231	0.002347	0.2	0.007001	0.003838	0.3	0.012466	0.000554
	0.34	0.018256	0.002546	0.25	0.007721	0.004658	0.4	0.014926	0.000705
	0.42	0.02265	0.001792	0.3	0.009643	0.004447	0.5	0.016965	0.000989
	0.5	0.026366	0.001522	0.35	0.013982	0.003407	0.6	0.015332	0.00087
	0.58	0.035947	0.001571	0.4	0.015992	0.003159	0.7	0.016617	0.001096
	0.66	0.038917	0.001328	0.45	0.017802	0.002916	0.8	0.021705	0.000897
	0.74	0.041228	0.001125	0.5	0.019414	0.003279	0.9	0.01877	0.000879
	0.82	0.042975	0.000784	0.55	0.017048	0.00301	1	0.01966	0.001038
150	0.1	0.000784	0.001189	0.1	0.002825	0.003217	0.1	0.005323	0.000195
	0.18	0.002275	0.001443	0.15	0.006554	0.002353	0.2	0.012943	0.000471

	0.26	0.004237	0.001798	0.2	0.008107	0.001809	0.3	0.01416	0.000575
	0.34	0.007887	0.001297	0.25	0.013258	0.001332	0.4	0.020583	0.000706
	0.42	0.010709	0.001308	0.3	0.016451	0.00122	0.5	0.019063	0.000781
	0.5	0.013504	0.000857	0.35	0.019409	0.000791	0.6	0.021002	0.000991
	0.58	0.016183	0.000682	0.4	0.022093	0.000677	0.7	0.022729	0.000986
	0.66	0.015293	0.000668	0.45	0.020041	0.000759	0.8	0.029664	0.000779
	0.74	0.020999	0.000446	0.5	0.021782	0.000599	0.9	0.025642	0.000736
	0.82	0.018898	0.000456	0.55	0.023314	0.000592	1	0.032822	0.000685
250	0.1	0.000971	0.000839	0.1	0.002761	0.002349	0.1	0.007319	0.000314
	0.18	0.002303	0.001083	0.15	0.005291	0.002182	0.2	0.014358	0.000494
	0.26	0.004284	0.001148	0.2	0.008065	0.002077	0.3	0.019005	0.000641
	0.34	0.006517	0.001266	0.25	0.013281	0.001518	0.4	0.022445	0.000887
	0.42	0.010803	0.001058	0.3	0.013565	0.000912	0.5	0.020687	0.000932
	0.5	0.011135	0.000854	0.35	0.016088	0.000863	0.6	0.022719	0.000931
	0.58	0.016296	0.000558	0.4	0.022486	0.000594	0.7	0.029984	0.000735
	0.66	0.015388	0.00055	0.45	0.025033	0.000542	0.8	0.026158	0.000692
	0.74	0.021117	0.000453	0.5	0.022342	0.000526	0.9	0.033745	0.000782
	0.82	0.018994	0.000315	0.55	0.029322	0.000526	1	0.035317	0.000584

Conclusions

During conducting the simulation experiments and according to the analyses of the results from the practical part, the following conclusions have been drawn:

1. It was shown that the actual value of the reliability function and estimated reliability function decrease with the increase of time t ; and it is always between (0-1), which coincides with the theoretical aspect of characteristics of the reliability function.
2. The values of the two statistical measures (MSE) in estimating the reliability function were decreased by increasing the sample sizes and all estimation methods, which aligns with statistical theory.

3- The results of (IMSE) in estimating the reliability function in the (WLS) methods were better than the (OLS) method in the presence or absence of contamination.

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