The effect dynamic mathematics software GeoGebra, achievement in matrixes and determinants

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Abstract

This paper presents the meaning of using e-learning and its effect on the productivity and positivity of education and its reflection on students Especially in calculating and knowing everything related to matrices in presenting the methodological frameworks, there are several specific examples within this topic, in a creative and innovative way This is because it is important in the future To develop learning in general. This paper discusses some of the mathematical experiments that we can get more accurate and faster using this technology, it is considered the use of the GeoGebra program Necessary To promote mathematics and its impact on teacher's development for easy understanding the concepts of learning and teaching mathematics, This program is an exploratory method with accurate results, by enhancing or supporting it with mathematical operations

Keywords: mathematics, GeoGabra, matrix, determinants

I. Introduction

In this paper the effect of technology is explained to complete mathematical operations in an easy and accurate way, And the use of an advanced program that is easy to use and apply This program is GeoGebra.

GeoGebra is an open source, multi-platform dynamic math program It is used for all educational levels Combines geometry, algebra, spreadsheets, statistics, and calculus, all within one package It can be used easily, and it is considered one of the societies that expanded rapidly Among millions of users Almost all over the world. GeoGebra is the most widely used provider of dynamic mathematics.

We also explained that GeoGebra is involved in many mathematical operations, but here the focus is on matrixes and determinants Through experiments applied to the program.

Matrices are a set of arranged numbers in a constant number Of columns and rows, and it is considered one of the most powerful tools in mathematics, as a result of attempts to obtain compact and simple methods the concept of matrices has greatly evolved for solving a system of linear equations.

II. Example 1: Addition, subtraction, multiplication of matrices

When adding and subtracting the matrices they must be the same size, Meaning, the number of rows must be equal to the number of columns in both matrices.

For example, if the number of rows in a matrix is 4 and the number of columns is 5

Here it is not possible to add or subtract it unless the other matrix contains 4 rows and 5 columns.

And the process of addition or subtraction by addition or subtraction every two elements are identical on site Between the two matrices. as in the example below

A= $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$ b = $\begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}$ find A – B

A – B = $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$ - $\begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$ = $\begin{bmatrix} 1 & 6 \\ -4 & 2 \end{bmatrix}$

Matrix multiplication

In matrix multiplication, there are two types of multiplication ,it will be explained

Scalar multiplication Here the multiplication is performed by multiplying one element in all the elements of the matrix in the sense that this process takes place

when one element outside the matrix is multiplied by every element of the matrix.

. as in the example below.

Find 4 * $\begin{bmatrix} 1 & 5 \\ 8 & 7 \end{bmatrix}$ = $\begin{bmatrix} 4 & 20 \\ 32 & 28 \end{bmatrix}$

Matrix multiplication This is the second type

Multiplication is done by multiplying two matrices (matrix * matrix), That is if the number of columns in the matrix is equal to the number of rows in the second matrix and the result becomes.

The number of rows of the first * the number of columns of the second matrix, you should follow these steps when multiplying this type of matrix.

1- Make sure that the number of columns in the first matrix equals the number of columns in the second array.
2- Multiply each element of each row of the first row of the matrix * each element of the matrix opposite him from each column of the second matrix.
as in the example below.

$$\begin{matrix} 2 & 3 & 1 \\ 2 & -7 & 4 \end{matrix} \quad * \quad \begin{matrix} 3 & 4 & 5 \\ 1 & 1 & 4 \\ 2 & 1 & 4 \end{matrix}$$ find multiplication

= 2*3 + 1*3 + 1*2 = 11
=2*4 + 3*1 + 1*1 = 12
=2*5 + 3*4 + 1*4 = 26
=2*3 + (-7) *1 + 4*2 = 7
=2*4 + (-7) *1 + 4*1 = 5
=2*5 + (-7) * 4 + 4*4 = -2

$$= \begin{matrix} 11 & 12 & 26 \\ 7 & 5 & -2 \end{matrix}$$

Therefore, the result is that this method is very long it consumes a lot of effort and time.
This question can be solved in the GeoGabra , By entering the values of matrices and placing the multiplication sign between them.
As in the figure below.

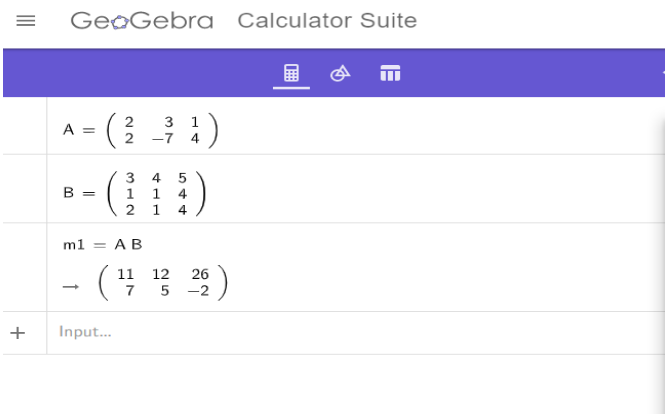

Fig 1: Addition, subtraction, multiplication of matrices

III. Example 2: Find the rank of the matrix

Let it be a matrix of capacity m * n, the rank of the matrix is A is the capacitance of the larger sub-matrix of matrix A. The determinant is not equal to 0.

That is, the rank of matrix A is the largest capacity that can remain after deleting a row or column or both from the original matrix A.

Since matrixes contain two types:

- **Square matrix:** Is a square matrix with a capacity We find a determinant for it if it's not equal to 0. Its capacity is n, but if its determinant is equal to 0, We delete a row and a column, then we extract a sub-matrix from the original matrix A. then we find its determinant If the determinant is not equal to 0. The rank of matrix A is equal to the amplitude of that sub-matrix. If the determinant is 0, then we extract a sub-matrix with a smaller amplitude by deleting another row and column then we find its determinant, through this determinant we find the rank of the matrix and so on.
- **The non-square matrix** :We delete the extra row or column It is preferred that contains more zeros; the rank of that matrix A is equal to the amplitude of that matrix after deleting, If it is equal to 0, we delete another row and column again And we extract a sub-matrix with a smaller amplitude by deleting another row and column Then we find the determinant of it, through this determinant we determine the rank of the matrix and so on.

$$A=\begin{bmatrix} 3 & 1 & 0 \\ 5 & 7 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \\ 1 & 4 & 8 \end{bmatrix}$$

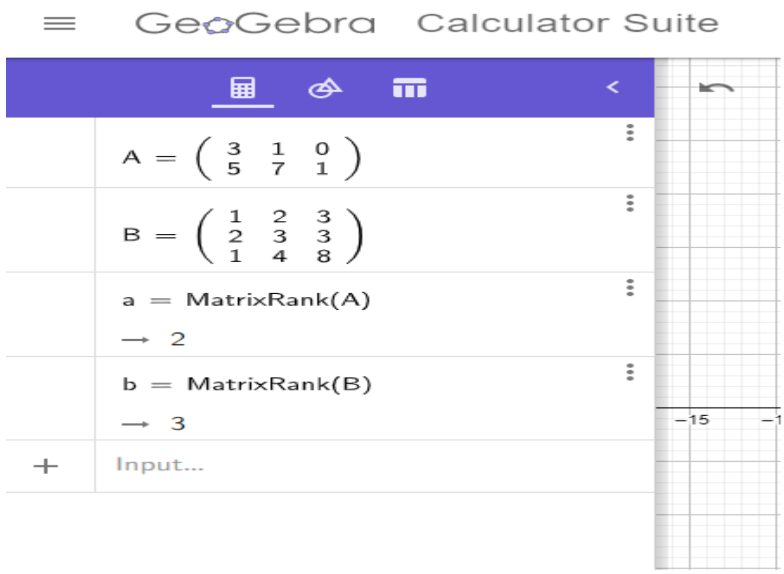

Fig.2: Find the rank of the matrix

IV. Example 3: Find the inverse of a matrix using the determinant

When we want to extract the inverse of a square matrix by using the determinant, using the determinant of the matrix through a theorem that says:

If A is a square matrix and A is not equal to 0 then the determinant of the matrix is not equal to zero, but if it is equal to 0 then the matrix has no inverse

Let A $=\begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$ Find A^{-1} by determinant of?

To solve this matrix, we must find the determinant and then we must find the Adjoint (adj (A)) And then we use the law below to find the invers:

$$A^{-1} = \frac{1}{[A]} * \operatorname{adj}(A)$$

And finding the determinant is by repeating the first and second columns in the order to the right of the determinant as in the figure:

$|A| = (4 + 0 + (-2)) - 0 - 12 - (-3) = 2 - 9 = -7 \neq 0$

Since it is the determinant is -7 and not equal to 0, then the matrix has an inverse.

Now we will move on to the next step which is extracting adj(A)

$\operatorname{Adj}(A) = (\operatorname{Cof}(A))^{T}$

To extract an adj(A), we must first find the Cof(A) using this law:

$\operatorname{Cof}(A) = \operatorname{Cof}(a_{ij}) = A_{ij} = (-1)^{i+j} M_{ij}$

$\operatorname{Cof}(2) = A_{11} = (-1)^{1+1} M_{11} = (+1) \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = (+1) * (2-6) = -4$

$\text{Cof }(3) = A_{1\,2} = (-1)^{1+2}\, M_{1\,2} = (-1)\begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix} = (-1) * (-1-0) = -1$

$\text{Cof }(1) = A_{1\,3} = (-1)^{1+3}\, M_{1\,3} = (+1)\begin{vmatrix} -1 & 2 \\ 0 & 2 \end{vmatrix} = (+1) * (-2-(0)) = -2$

$\text{Cof }(-1) = A_{2\,1} = (-1)^{2+1}\, M_{2\,1} = (-1)\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = (-1) * (3-2) = -1$

$\text{Cof }(2) = A_{2\,2} = (-1)^{2+2}\, M_{2\,2} = (+1)\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = (+1) * (2-0) = 2$

$\text{Cof }(3) = A_{2\,3} = (-1)^{2+3}\, M_{2\,3} = (-1)\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = (-1) * (4-0) = 4$

$\text{Cof }(0) = A_{3\,1} = (-1)^{3+1}\, M_{3\,1} = (+1)\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = (+1) * (9-2) = 7$

$\text{Cof }(2) = A_{3\,2} = (-1)^{3+2}\, M_{3\,2} = (-1)\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} = (-1) * (6-(-1)) = -7$

$\text{Cof }(1) = A_{3\,3} = (-1)^{3+3}\, M_{3\,3} = (+1)\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = (+1) * (4-(-3)) = 7$

$\text{Cof }(A) = \begin{bmatrix} -4 & 1 & -2 \\ -1 & 2 & -4 \\ 7 & -7 & 7 \end{bmatrix}$

Now we take a $\text{Cof}(A)^T$, which means we turn every row into a column

$\text{Adj}(A) = (\text{Cof}(A))^T \begin{bmatrix} -4 & -1 & 7 \\ 1 & 2 & -7 \\ -2 & -4 & 7 \end{bmatrix}$

Then we applied $A^{-1} = \frac{1}{[A]} * \text{adj}(A) = \frac{1}{-7} * \begin{bmatrix} -4 & -1 & 7 \\ 1 & 2 & -7 \\ -2 & -4 & 7 \end{bmatrix} = \begin{bmatrix} 4/7 & 1/7 & -1 \\ -1/7 & -2/7 & 1 \\ 2/7 & 4/7 & -1 \end{bmatrix}$

All of these processes can be abbreviated by GeoGebra by inserting the elements of the matrix into a program and then giving the command inverted, we get the figure below.

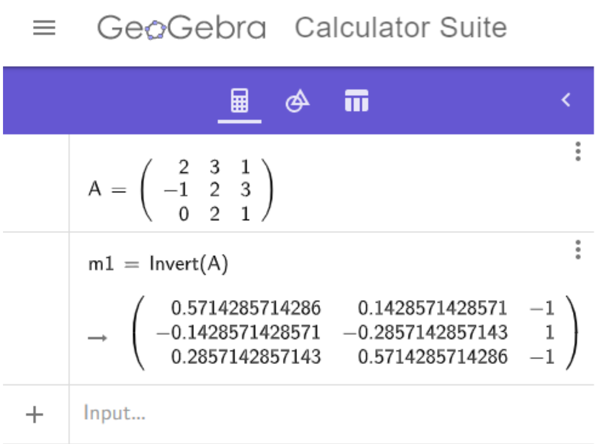

Fig.3: Find the inverse of a matrix using the determinant

V. Conclusion

There is no doubt that GeoGabra is a very excellent program, and it is applicable in learning mathematics especially in matrices and determinants.

The program was very helpful in finding concepts How to calculate, find the rank, and find the inverse of the matrix we have shown it in the experiments, the results are very accurate.

The number of lessons used by GeoGabra within the framework of different mathematical approaches it is evidence of its diversity it also has the advantage of rapid improvements that are made continuously.

Looking forward to more progress:

1- Establishing educational research in the GeoGabra department in order to share ideas and cooperation.
2- Display and editing improvements such as colors.
3- Add questions with multiple options, questions that contain spaces, and even questions that containing truth and false.

VI. References

1. Hohenwarter, M. and Preiner, J., 2007. Creating mathlets with open source tools. The Journal of Online Mathematics and Its Applications, 7, pp.1-29.
2. Hohenwarter, M. and Lavicza, Z., 2007. Mathematics teacher development with ICT: towards an International GeoGebra Institute. Proceedings of the British Society for Research into Learning Mathematics, 27(3), pp.49-54.
3. Preiner, J., 2008. Introducing dynamic mathematics software to mathematics teachers: the case of GeoGebra (Doctoral dissertation in Mathematics Education, University of Salzburg, Salzburg, Austria). Recuperado de http://www. pucrs. br/ciencias/viali/tic_ literatura/teses/Preiner_Judith. pdf.
4. Burke, M. and Kennedy, P., 2011. GeoGebra: From simulation to formalization in teacher preparation and inservice programs. In Model-Centered Learning (pp. 57-72). Brill Sense.
5. Tezer, M. and Kanbul, S., 2009. Opinions of teachers about computer aided mathematics education who work at special education centers. Procedia-Social and Behavioral Sciences, 1(1), pp.390-394.
6. BEAUREGARD, R.A.F., 1973. A FIRST COURSE IN LINEAR ALGEBRA: WITH OPTIONAL INTRO (No. QA251. B42 1973.).
7. Farebrother, R.W., 1997. Ac aitken and the consolidation of matrix theory. Linear algebra and its applications, 264, pp.3-12.
8. Pace, R.K. and LeSage, J.P., 2004. Chebyshev approximation of log-determinants of spatial weight matrices. Computational Statistics & Data Analysis, 45(2), pp.179-196.
9. Deift, P., Its, A. and Krasovsky, I., 2012. Toeplitz matrices and Toeplitz determinants under the impetus of the Ising model. Some history and some recent results. arXiv preprint arXiv:1207.4990.
10. Barouch, E., McCoy, B.M. and Wu, T.T., 1973. Zero-field susceptibility of the two-dimensional Ising model near T c. Physical Review Letters, 31(23), p.1409.

11. Parker, J.S., Mullins, M., Cheang, M.C., Leung, S., Voduc, D., Vickery, T., Davies, S., Fauron, C., He, X., Hu, Z. and Quackenbush, J.F., 2009. Supervised risk predictor of breast cancer based on intrinsic subtypes. Journal of clinical oncology, 27(8), p.1160.
12. Troyanskaya, O., Cantor, M., Sherlock, G., Brown, P., Hastie, T., Tibshirani, R., Botstein, D. and Altman, R.B., 2001. Missing value estimation methods for DNA microarrays. Bioinformatics, 17(6), pp.520-525.
13. Keshavan, R.H., Montanari, A. and Oh, S., 2009, September. Low-rank matrix completion with noisy observations: a quantitative comparison. In 2009 47th Annual Allerton Conference on Communication, Control, and Computing (Allerton) (pp. 1216-1222). IEEE.
14. Drineas, P., Kannan, R. and Mahoney, M.W., 2006. Fast Monte Carlo algorithms for matrices II: Computing a low-rank approximation to a matrix. SIAM Journal on computing, 36(1), pp.158-183.
15. Wright, J., Ganesh, A., Rao, S., Peng, Y. and Ma, Y., 2009. Robust principal component analysis: Exact recovery of corrupted low-rank matrices via convex optimization. In Advances in neural information processing systems (pp. 2080-2088).
16. Fazel, M., Hindi, H. and Boyd, S.P., 2003, June. Log-det heuristic for matrix rank minimization with applications to Hankel and Euclidean distance matrices. In Proceedings of the 2003 American Control Conference, 2003. (Vol. 3, pp. 2156-2162). IEEE.
17. Banerjee, S. and Roy, A., 2014. Linear algebra and matrix analysis for statistics. Crc Press.
18. Zawadzka, K., 1993. The product and the determinant of matrices with entries in a field. Formalized Mathematics, 4(1), pp.1-8.
19. Nirmala, T., Datta, D., Kushwaha, H.S. and Ganesan, K., 2011. Inverse interval matrix: A new approach. Applied Mathematical Sciences, 5(13), pp.607-624.
20. Raz, R., 2002, May. On the complexity of matrix product. In Proceedings of the thiry-fourth annual ACM symposium on Theory of computing (pp. 144-151).
21. Magnus, J.R. and Neudecker, H., 2019. Matrix differential calculus with applications in statistics and econometrics. John Wiley & Sons.
22. Lin, L., Lu, J., Ying, L., Car, R. and Weinan, E., 2009. Fast algorithm for extracting the diagonal of the inverse matrix with application to the electronic structure analysis of metallic systems. Communications in Mathematical Sciences, 7(3), pp.755-777.
23. Masarova, R., Juhas, M., Juhasova, B. and ŠUTOVÁ, Z., 2015. Inverse Matrix in the theory of Dynamic Systems. ICCIT 2015, Copenhagen, Denmark, 11, 12(2015), pp.1878-1881.
24. Pavlikova, S., 2009. Extending the Class of Matrices Invertable using Graphs. In 6th Conference on Mathematics and Physics at Technical Universities 2009 (pp. 203-210).
25. Sankowski, P., 2004, October. Dynamic transitive closure via dynamic matrix inverse. In 45th Annual IEEE Symposium on Foundations of Computer Science (pp. 509-517). IEEE.