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Stochastic Modelling for a System with Two Types of Failure and Possibility of Wrong Diagnosis on Major Failure

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Abstract
The present paper investigates a single-unit system where a failed system is inspected by a first repairman to see if it has a major or minor failure. Minor faults are rectified by the first repairman; however major faults require a second opinion on the type of failure from another repairman, referred to as the second repairman. The second repairman might or might not concur with the first. If the second repairman agrees with the first, the repair is completed by the second; if he does not agree, a third opinion is sought from an expert repairman. Following the third opinion, the system is repaired by an expert repairman, whether the third opinion agreed with either of the previous two opinions or contradicted with both. The Markov and regenerative processes are used to derive reliability, availability, and other system profitability measures. The system's profit equation, sensitivity functions are also determined and the derived measures are demonstrated with numerical examples.

Keywords- Reliability, Major/Minor failure, Wrong diagnosis, Profit equation, Sensitivity functions

1. Introduction

Recent technological advancements have paved the way for a plethora of complex and sophisticated systems. The need of the hour is to reduce failures, increase availability, and improve operational capacity of such systems. A vast number of researchers, including [1-4], have thoroughly analyzed the reliability and profitability of standby systems. Various researchers have also considered the concept of major and minor system failures in their study. Bhatti et al. (2011) discussed the concept of inspection for standby system to detect major or minor faults. Parashar and Bhardwaj (2013) compared the profitability of a hot standby system reliability model. The minor fault was repaired first, followed by the major fault. Kumar and Bhatia (2013) investigated a centrifuge system in which a minor fault causes degradation while a major fault causes the failure. While discussing standby system, Ahmad and Kumar (2015) consider partial system failure on minor faults and complete system failure on major faults. Repair on minor failure and replacement on major failure was addressed by Sharma and Joorelnd (2019) for standby system. The concept of wrong diagnosis was introduced by Bala et. al (2017) wherein they considered the aspect of undertaking the repair by the second repairman if found improper by the first repairmen. Bala et. al (2019, 2022) further extended their work considering the concept of wrong diagnosis along with instruction time, two opinions for diagnosis, post repair intensive inspection, three opinions on failure.

The concept of incorrect diagnosis with two types of faults—major or minor is yet to be addressed in the literature. Keeping that in mind, the present study focuses on a single-unit system in which the failed unit is assessed by the first repairman for minor or major faults. In case of minor fault, there is no need of taking second opinion and hence the repair is done by the first repairman in this case. However, a second opinion is sought from another repairman in the case of major fault. The opinion of the second repairman may or may not be in consonance with the first repairman. If the second repairman agrees with the first, the repair is completed by the second; if he does not agree, a third opinion is sought from an expert repairman. Following the third opinion, the system is repaired by an expert repairman, irrespective of whether the third opinion agreed with either of the previous two opinions or contradicted with both. The stochastic model for the described system is developed using Markov and regenerative processes. Reliability, MTSF, availability, and other parameters that have a substantial influence on system profitability are obtained. The system's profit equation and sensitivity functions are derived. For the obtained results, numerical computations are performed.

2. Notations

The notations for different probabilities/density functions are as follows:

I_0 : Initial state of system

f(t) :failure rate

p_1 / q_1 : probability of minor/major failure after first opinion

p_2 / q_2 : probability that opinion given by second repairman is inconsonance / contradictory with the opinion given by the first one

$h_1(t)/h_2(t)/h_e(t)$: pdf of the inspection time to have first/second/third opinion on type of failure

$g_1(t)$ / $g_2(t)$/ $g_e(t)$:pdf of the repair time by first ordinary/second ordinary / expert repairman.

2. System Description and Assumptions

The following is a description of the system under study and the assumptions used to conduct the analysis:

1) System comprised of single unit
2) When a fault occurs in a system, first repairman inspects it to determine whether the fault is minor or major.
3) Minor faults are repaired by first repairman
4) If a major fault is identified by the first repairman, there is a risk of incorrect diagnosis, so a second opinion is obtained from another repairman known as the second ordinary repairman.
5) The second repairman inspects the failed system again, and he may or may not agree with the first repairman
6) If the second repairman agrees with the first, the second one performs the repair
7) If the second repairman disagrees with the first, the unit is inspected again by an

expert repairman for a third opinion.

8) Expert repairman repairs the failed unit irrespective of the opinions of the first two repairmen.
9) The first and second repairmen are always remains with the system, and an expert repairman is called in when needed.
10) All the time distributions are taken arbitrary
11) Variables involved are independent.

3. Modelling of the System

States of the system are represented by the following symbols

Op : operative unit

$F_{i1}/F_{i2}/F_{ie}$: failed unit under inspection for a first/second/third opinion to diagnose type of failure

$F_{r1}/F_{r2}/F_{re}$: failed unit under repair by first ordinary/second ordinary/expert repairman

Using these symbols, the different states of the system are.

State 0: (Op) State 1: (F_{i1}) State 2: (F_{r1})
State 3: (F_{i2}) State 4: (F_{r2}) State 5: (F_{ie})
State 6: (F_{re})

Here all the states 0, 1, 2, 3, 4, 5 and 6 are regenerative states. State 0 is an operative state whereas all the other states are failed states. The possible transitions between various states are shown in Fig.1.

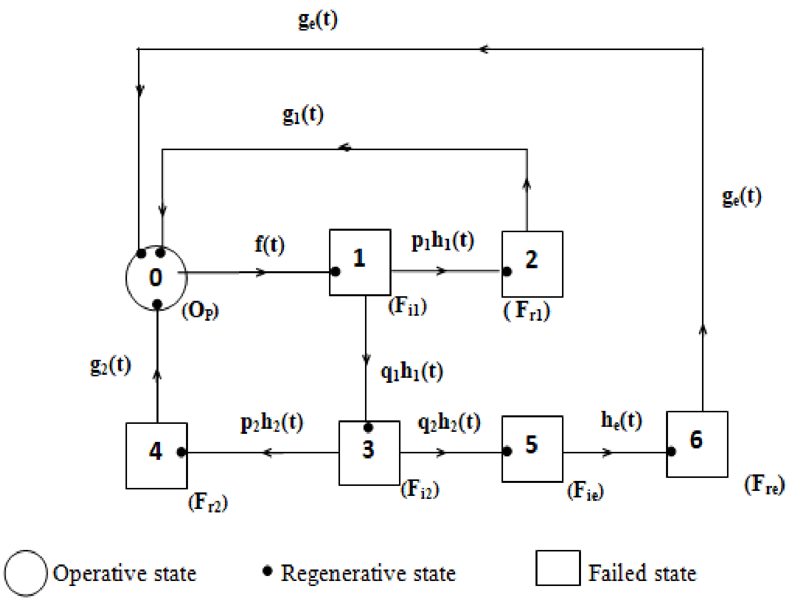

Figure 1: State Transition Diagram

With transition probabilities, $p_{ij} = \int_0^\infty q_{ij}(t)dt$

$p_{01} = \int_0^\infty f(t)\,dt$, $p_{12} = \int_0^\infty p_1h_1\,(t)dt$ $p_{13} = \int_0^\infty q_1h_1\,(t)dt$
$p_{34} = \int_0^\infty p_2h_2\,(t)dt$ $p_{35} = \int_0^\infty q_2h_2\,(t)dt$ $p_{20} = \int_0^\infty g_1\,(t)dt$,
$p_{40} = \int_0^\infty g_2\,(t)dt$, $p_{56} = \int_0^\infty h_e\,(t)dt$, $p_{60} = \int_0^\infty g_e\,(t)dt$, (1-10)

Eventually,

$p_{01} = p_{20} = p_{40} = p_{56} = p_{60} = 1$, $p_{12} + p_{13} = 1$, $p_{34} + p_{35} = 1$, (11-13)

Mean Sojourn time(μ_i) in regenerative state i are

$\mu_0 = \int_0^\infty \overline{F(t)}dt$, $\mu_1 = \int_0^\infty \overline{H_1(t)}dt$ $\mu_2 = \int_0^\infty \overline{G_1(t)}dt$
$\mu_3 = \int_0^\infty \overline{H_2}(t)dt$ $\mu_4 = \int_0^\infty \overline{G_2}(t)dt$ $\mu_5 = \int_0^\infty \overline{H_e}(t)dt$ (14-20)
$\mu_6 = \int_0^\infty \overline{G_e}(t)dt$

Defining $m_{ij} = E(q_{ij})$ and $m_{ij}^{(k)} = E(q_{ij}^{(k)})$, we have

$m_{01} = \mu_0$, $m_{12} + m_{13} = \mu_1$, $m_{20} = \mu_2$,
$m_{34} + m_{35} = \mu_3$, $m_{40} = \mu_4$, $m_{56} = \mu_5$, $m_{60} = \mu_6$
(21-29)

4. System Reliability and MTSF

Let $\pi_0(t)$ be c.d.f. of the first passage time from regenerative state 0 to a failed state, then from transition diagram, we have

$\pi_0(t) = Q_{01}(t)$ (30)

Taking Laplace-Stieljes transformation of the above equation, we obtain

$\pi_0^{**}(s) = Q_{01}^{**}(s)$ (31)

The reliability of the system at time t is given by,

$R(t) = L^{-1}[\{1 - \pi_0^{**}(s)\}/s]$
$= \overline{F(t)}$ (32-33)

Now the mean time to system failure (MTSF)

$MTSF = \int_0^\infty R(t)dt = \mu_0$ (34)

5. System Availability

If $AV_i(t)$ =P[system is operative at instant t | I_0= i], then from transition diagram, $AV_i(t)$ seen to satisfy the following recursive relations

$AV_0(t) = W_0(t) + q_{01}(t) © AV_1(t)$
$AV_1(t) = q_{12}(t) © AV_2(t) + q_{13}(t) © AV_3(t)$
$AV_2(t) = q_{20}(t) © AV_0(t)$
$AV_3(t) = q_{34}(t) © AV_4(t) + q_{35}(t) © AV_5(t)$
$AV_4(t) = q_{40}(t) © AV_0(t)$
$AV_5(t) = q_{56}(t) © AV_6(t)$
$AV_6(t) = q_{60}(t) © AV_0(t)$
Where, $W_0(t) = \overline{F}(t)$ (35- 42)

Taking Laplace transform eqns (35-41) and solving them for $AV_0^*(s)$, we obtain

$AV_0^*(s) = L_1(s)/M_1(s)$

Where, $L_1(s) = W_0^*(s)$
and $M_1(s) = 1- q_{01}^*(s)\,(q_{12}^*(s)\,q_{20}^*(s)+q_{13}^*(s)\,q_{34}^*(s)\,q_{40}^*(s)+q_{13}^*(s)\,q_{35}^*(s)\,q_{56}^*(s)\,q_{60}^*(s))$
In steady-state, the availability of system, is given by
$AV_0 = \lim_{t\to\infty} AV_0(t) = \lim_{s\to 0} sAV_0^*(s) = L_1/M_1$
Where, $L_1 = \mu_0$
and $M_1 = \mu_0 + \mu_1 + p_{12}\mu_2 + p_{13}\mu_3 + p_{13}p_{34}\mu_4 + p_{13}p_{35}\mu_5 + p_{13}p_{35}\mu_6$ (43-48)

6. Expected Time for Repairing the Minor Failure by First Repairman (B_0)
Defining, $B_0'(t)$= P [first repairman is busy in minor repair at time t| I_0= i], we obtain
$B_0'(t) = q_{01}(t) © B_1'(t)$
$B_1'(t) = q_{12}(t) © B_2'(t) + q_{13}(t) © B_3'(t)$
$B_2'(t) = W_2(t) + q_{20}(t) © B_0'(t)$
$B_3'(t) = q_{34}(t) © B_4'(t) + q_{35}(t) © B_5'(t)$
$B_4'(t) = q_{40}(t) © B_0'(t)$
$B_5'(t) = q_{56}(t) © B_6'(t)$
$B_6'(t) = q_{60}(t) © B_0'(t)$
where, $W_2(t) = \overline{G_1}(t)$ (49-56)
Taking LT of eqns (49)-(55) and solving them for $B_0'^*(s)$ we get
$B_0'^*(s) = L_2(s)/M_1(s)$
Where,
$L_2(s) = q_{01}^*(s)\,q_{12}^*(s)\,W_2^*(s)$
In steady-state, expected time for which system is in repair for minor failure is
$B_0' = \lim_{t\to\infty} B_0'(t) = \lim_{s\to 0} sB_0'^*(s) = L_2/M_1$
Where,
$L_2 = p_{12}\mu_2$ (57-60)

Similarly, the parameters that may affect the system profitability are:
Expected time for repairing the major failure by second repairman (B_0'') = L_3/M_1
Expected time for repairing the major failure by expert repairman (B_0^e) = L_4/M_1
Expected inspection time for first opinion by first repairman(I_0') $= L_5/M_1$
Expected inspection time for second opinion by second repairman(I_0'') $= L_6/M_1$
Expected inspection time by expert repairman(I_0^e) $= L_7/M_1$
Expected number of visits by both the repairmen (V_0) $= L_8/M_1$
Expected number of visits by expert repairman (V_0^e) $= L_9/M_1$
where,
$L_3 = p_{13}p_{34}\,\mu_4$; $L_4 = p_{13}p_{35}\,\mu_6$; $L_5 = \mu_1$; $L_6 = p_{13}\,\mu_6$;
$L_7 = p_{13}p_{35}\,\mu_5$; $L_8 = 1 + p_{13}$; $L_9 = p_{13}p_{35}$ (61-67)
and M_1 is specified in eqn (48)

7. Profit Analysis

The expected profit per unit time incurred to the system in steady-state is given by

Profit (P) = $R_0AV_0 - E_1B_0{}' - E_2B_0'' - E_3B_0^e - E_4I_0' - E_5I_0'' - E_6\, I_0^e - E_7\, V_0 - E_8\, V_0^e$

(68)

R_0 = System revenue

E_1 / E_2/ E_3= expense of engaging a first/second/expert repairman to repair the failed system

E_4 / E_5/ E_6= expense of engaging a first/second/expert repairman for inspecting the failed system

E_7 / E_8= charges for first, second and expert repairmen's visits

All the costs considered above are per unit time.

8. Sensitivity and Relative Sensitivity Analysis

Sensitivity analysis is conducted to identify how variations in the independent variable affect the dependent variable under certain conditions. Because there is a significant difference in the values of independent variables, the relative sensitivity function is employed to compare their impacts on dependent variables. The relative sensitivity function is defined as the percentage change caused by a percentage change in one of the variables. For availability (AV_0) and profit (P), the sensitivity and relative sensitivity functions are as follows:

$$\Phi_k = \frac{\partial AV_0}{\partial k} \quad ; \quad \delta_k = \Phi_k \left(\frac{k}{AV_0}\right)$$

$$\rho_k = \frac{\partial P}{\partial k} \quad ; \qquad \tau_k = \rho_k \left(\frac{k}{P}\right)$$

(69-72)

where k= λ, β_1 , β_2, β_e, α_1, α_2, α_e, R_0, E_1, E_2, , E_4, E_5, E_6, E_7, E_8

9. Results & Discussion

For numerical calculations, all time distributions are assumed to be exponential i.e.

$f(t) = \lambda\, e^{-\lambda t}, h_1(t) = \beta_1 e^{-\beta_1 t}, h_2(t) = \beta_2 e^{-\beta_2 t}\ , h_e(t) = \beta_e e^{-\beta_e t}$

$g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t},\ g_e(t) = \alpha_e e^{-\alpha_e t}$

Furthermore, the values of the parameters/probabilities/costs involved are assumed to be

λ= 0.001, β_1=0.5, β_2= 0.8, β_e= 1, α_1= 0.2, α_2= 0.4, α_e= 0.8, p_1= 0.4, q_1=0.6, p_2=0.5, q_2=0.5, R_0= 10, E_1=1000, E_2= 1000, E_3= 1100, E_4= 1400, E_5= 1600, E_6= 1800, E_7= 1000, E_8=1100

9.1 Reliability & MTSF

The reliability and MTSF of the defined system are expressed as

R(t)= $\overline{F(t)}$ = $1 - e^{-\lambda t}$

MTSF = $\int_0^\infty R(t)dt = \mu_0$ = $1/\lambda$

MTSF clearly declines as λ rises.

9.2 Trend of Availability (AV_0) for varied Rate (λ, α_e)

Keeping the other variables constant as defined above, the trend of availability for varied λ

and α_e is observed and results are compiled in Table 1. It is notice that
(i) As λ increases, AV_0 decreases
(ii) AV_0 increases as the repair rate α_e rises

Table1: AV_0 for varied λ and α_e

λ	Availability (AV_0)		
	α_e =0.05	α_e =0.2	α_e = 0.8
0.0001	0.9988	0.9993	0.9994
0.0006	0.9930	0.9957	0.9963
0.0011	0.9873	0.9921	0.9933
0.0016	0.9816	0.9886	0.9903
0.0021	0.9760	0.9850	0.9873
0.0026	0.9705	0.9815	0.9943

9.3 Trend of Profit (P) for varied Rate and Costs

The trend of profit (P) for varied (λ, β_2), (R_0, E_2) and (E_4, E_8) is shown in Table 2, Fig.2 and Fig.3 respectively. The values of other variables are taken constant as assumed. It is observed that,

(i) P declines as λ rises while it increases as β_2.
(ii)

Table 2: P for varied λ and β_2

λ	Profit (P)		
	β_2 =0.6	β_2=0.7	β_2 = 0.8
0.0001	9.0309	9.0311	9.0312
0.0002	8.0631	8.0634	8.0635
0.0003	6.1312	7.0969	7.0971
0.0004	5.1672	6.1316	6.1319
0.0005	4.2043	5.1675	5.1678
0.0006	3.2427	4.2047	4.2049

(ii) P rises as R_0 increases, but drops as E_2 rises. Further,

a. For E_2= 1000, P>0 if R_0 >9.63
b. For E_2= 2000, P>0 if R_0 >10.38
c. For E_2= 3000, P>0 if R_0 >11.13

(iii) P declines as E_4 and E_8 increases. Moreover,

a. For E_8= 1000, P>0 if E_4 < 1598.75
b. For E_8= 2000, P>0 if E_4 < 1448.73
c. For E_8= 3000, P>0 if E_4 < 1298.75

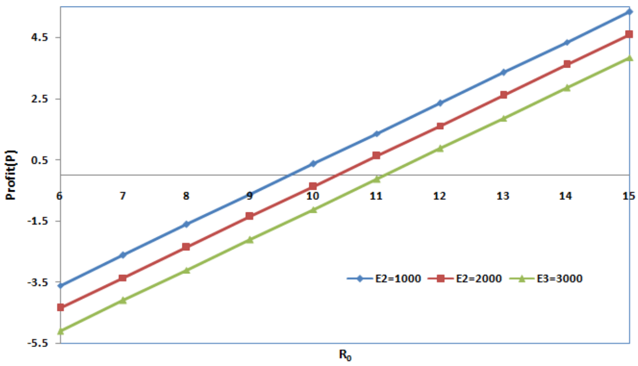

Figure 2: P for varied R_0 and E_2

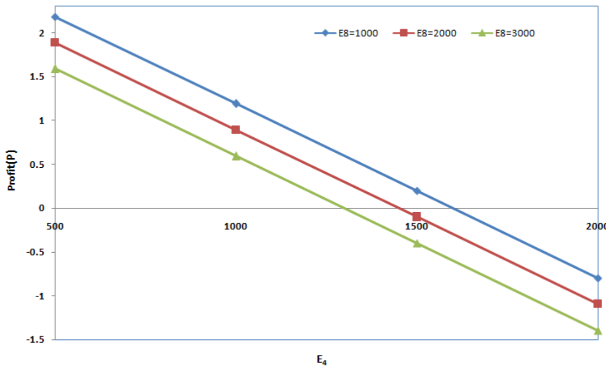

Figure 3: P for varied E_4 and E_8

9.4 Calculations for sensitivity analysis

Using the values of parameters assumed above, Tables 3 and 4 illustrate the sensitivity and relative sensitivity functions for AV_0 and P, respectively. Considering the absolute values of defined functions, Table 3 and Table 4 shows that the AV_0 and P both are sensitive to failure rate λ . However, the sequence in which the parameters influence the AV_0 and P , is:

AV_0 : λ, β_1 , α_1, α_2, β_2, β_e, α_e,

P : R_0 , λ, β_1 , E_4, E_1, α_1, α_e, E_7, E_5, α_2, E_2 , β_e, E_6, E_4 , E_8, β_2

Table 3: Sensitivity and Relative sensitivity of Availability (AV_0) for different parameters

Variable(k)	Availability(AV_0)	
	$\Phi_k = \frac{\partial AV_0}{\partial k}$	$\delta_k = \Phi_k \left(\frac{k}{AV_0}\right)$
λ	-6.0994	-0.0061
α_1	0.0099	0.0020
α_2	0.0019	0.0015
α_e	$4.63X10^{-4}$	$1.86X10^{-4}$
β_1	0.0040	0.0040
β_2	$9.28X10^{-4}$	$7.45X10^{-4}$
β_e	$2.96X10^{-4}$	$2.96X10^{-4}$

Table 4: Sensitivity and Relative sensitivity of Profit (P) for different parameters

Variable(k)	Profit(P_0) $\rho_k = \frac{\partial P}{\partial k}$	Profit(P_0) $\tau_k = \rho_k \left(\frac{k}{P}\right)$
λ	$-9.57X10^{3}$	-26.170
α_1	9.9423	5.442
α_2	1.8642	2.0416
α_e	2.0034	4.3881
β_1	5.5671	7.6211
β_2	$3.40X10^{-4}$	$7.45X10^{-4}$
β_e	0.5368	1.469
R_0	0.9939	27.219
E_1	-0.0020	-5.475
E_2	$-7.454X10^{-4}$	-2.0408
E_3	$-3.727X10^{-4}$	-1.1225
E_4	-0.0020	-7.661
E_5	$-7.454X10^{-4}$	-3.2653
E_6	$-2.981X10^{-4}$	-1.4694
E_7	-0.0016	-4.3806
E_8	$-2.981X10^{-4}$	-0.0458

10. Conclusion

In this article, a stochastic model for a single unit system with incorrect diagnosis of major faults is developed. System performance measures and factors influencing system profitability are derived. Profit equation is formulated based on the obtained measures. To illustrate the developed model, numerical computations are performed for the exponential case, and various conclusions regarding system profitability are drawn. Sensitivity analysis is also performed. The model is cost-effective for making accurate and validated failure decisions and diagnosing problems.

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