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On the Detour Based Indices

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Abstract

Topological index is a real value associated with a graph that should be structurally invariant. Many topological indices were defined, and many among them have been used to model chemical and pharmacological properties, as well as many other features of molecules. We study two topological indices based on detour distance. Detour index for any graph that is connected is defined as maximum distances or detour distances between every unordered paired vertices of G. Detour Harary index for any graph that is connected is defined as the sum of reciprocal of the maximum distances or detour distances between every unordered paired vertices of G. Here, in this paper we establish formulae to calculate the Detour index and detour Harary index of a few graph structures and also corona product of two particular graphs. Also we compare the results for both these indices .

1 Introduction

Let G be a simple connected graph with $V(G)$ as the vertex set and $E(G)$ as the edge set. If any two vertices $a, b \in V(G)$ are considered, the distance between them, denoted by d(a,b), is defined as the length of the shortest path between a and b in G, whereas the detour distance, denoted by D(a,b), is defined as the length of the longest path between a and b in G.

Topological index is a real value associated with a graph that should be structurally invariant. Many topological indices were defined, and many among them have been used to model chemical and pharmacological properties, as well as many other features of molecules. [1].

Wiener index [2] for any graph that is connected is defined to be the sum of shortest distances or simple distances between every unordered paired vertices of G.

$$W(G) = \sum_{1\le i\le j\le n} d(vi,vj)$$

Wiener index is known as the first mathematical invariant which reflects a molecular graph's topological structure. It is one of the most widely used topological indices which have high interdependence with numerous physical and chemical properties of molecular compounds (for more on Wiener index refer [3] and [4]). This index was introduced by Harold Wiener and he developed this to find out the physical properties of paraffin in 1947.

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If we are using the distance matrix (an $n \times n$ matrix consisting of all distances between vertices v_i and v_j with vertex set $\{v_1,v_2,...,v_n\}$) then, the Wiener index is half the sum of the distance matrix's off–diagonal elements.

Detour index [5] for any graph that is connected is defined to be the sum of maximum distances or detour distances between every unordered paired vertices of G.

$\omega(G) = \sum_{1\le i\le j\le n} D(vi,vj)$.

The graphs with minimal and maximum detour indices for the n-vertex unicyclic graphs with cycle length r (where $3 \le r \le n-2$ for $n \ge 5$) were found in [6], as were the bounds for the detour index.

If we are using the detour matrix (an $n \times n$ matrix consisting of all detour distances between vertices v_i and v_j with vertex set $\{v_1,v_2,...,v_n\}$) then, the detour index is half the sum of the detour matrix's off–diagonal entries. Frank Harary [7] proposed the detour index into mathematical literature in 1969 and it was discussed by Buckley and Harary [8] in 1990. It was instigated into chemistry under the title "the maximum path matrix of a molecular graph" [9, 10, 11, 12, 13] in 1994 and theoretical graph theory contribution to find some interest in chemical literature [14, 15, 16, 17, 18, 19, 20, 21, 22].

Another important index which we are discussing here is the Harary index, H(G), which was established in 1993 by Plavi et al. [23] and Ivanciuc et al. [24] independently. On the occasion of Professor Frank Harary's 70th birthday, the Harary index was named after him. The Harary index is defined as, sum of the reciprocal of distances between unordered pair of vertices of G.

$H(G) = \sum_{1\le i<j\le n} \frac{1}{d(v_i,v_j)}$.

The Harary index is equal to half the sum of the reciprocal of the off–diagonal components of the distance matrix while using the distance matrix.

Detour Harary index [25], ωH(G) for any graph that is connected is defined to be sum of the reciprocal of maximum distances or detour distances between every unordered paired vertices of G.

$\omega H(G) = \sum_{1\le i<j\le n} \frac{1}{D(v_i,v_j)}$.

The detour Harary index is half the sum of the reciprocal of the detour matrix's off–diagonal components while using the detour matrix.

In [25], it is mentioned that the Detour–Harary index and Harary index of a tree graph are the same and they studied the Detour–Harary index for some topological structures containing cycles. In this paper we are establishing the formulae to calculate Detour index for complete graph, cycle, wheel, friendship graph, lollipop graph, starred complete graph, n-partite graph and a graph obtained by performing the graph operation corona and found the formula to calculate the Detour Harary index for complete graph, wheel, friendship graph, starred complete graph, n-partite graph and a graph obtained by performing the graph operation corona.

2 Detour Index

A complete graph is a simple undirected graph with a unique edge connecting every distinct pair of vertices. K_n denotes the complete graph with n vertices. [26]

Theorem 2.1 If $G \cong K_n$, with $n \geq 3$ then,

$\omega(G) = \frac{n(n-1)^2}{2}$

Proof:

Consider K_n with vertex set, $V(G) = \{x_1, x_2, \dots, x_n\}$.

Distance between any pair of vertices (x_i, x_j) where $(i, j) \in [1, n]$ with $j > i$

i.e, $d(x_i, x_j) = 1$.

Detour distance between any pair of vertices x_i, x_j is $n - 1$.

$$D(x_i, x_j) = n - 1$$

Consider a vertex x_i ,

Distance from x_i to all other vertices [i.e, $(n - 1)$] vertices is then $(n - 1)^2$.

Total number of vertices in $K_n = n$.

Sum of detour distances of every pair of vertices,

$$\sum_{i,j=1}^{n} D(x_i, x_j) = n(n-1)^2$$

therefore Detour index,

$\omega(G) = \frac{n(n-1)^2}{2}$

A cycle graph C_n, sometimes simply known as an n-cycle, is a graph with n vertices and a single cycle that passes through all nodes. [27]

Theorem 2.2 If $G \cong C_n$, with $n \geq 3$, then,

when n is odd, $\omega(G) = \frac{n}{8}(n-1)(3n-1)$

when n is even, $\omega(G) = \frac{n^2}{8}(3n-4)$

Proof:

Consider C_n with vertex set, $V(G) = \{x_1, x_2, \dots, x_n\}$.

Case 1: When n is odd

$$D(x_i, x_{i+1}) = (n-1)n$$

$$D(x_i, x_{i+2}) = (n-2)n$$

. . .

$D(x_i, x_{i+(\frac{n-1}{2})}) = (n-(n-\frac{1}{2}))n$

In general,

$D(x_i, x_j) = n[(n-1)+(n-2)+...+(n-(\frac{n-1}{2}))].$

Therefore,

$\sum_{i<j} D(x_i, x_j) = n[\sum_{k=1}^{\frac{n-1}{2}} (n-k)].$

So, the detour index ω(G) is defined as,

$$\begin{aligned} \omega(G) &= \sum_{i<j} D(x_i, x_j) \\ &= n[\sum_{k=1}^{\frac{n-1}{2}} (n-k)] \\ &= \frac{n}{8}(n-1)(3n-1) \end{aligned}$$

which is the detour index of cycle with odd number of vertices.

Case 2: When n is even

$$D(x_i, x_{i+1}) = (n-1)n$$

$$D(x_i, x_{i+2}) = (n-2)n$$

. . .

$D(x_i, x_{i+(\frac{n-1}{2})}) = (n-(n-\frac{1}{2}))n$

$D(x_i, x_{i+\frac{n}{2}}) = (\frac{n}{2})n$

In general,

$D(x_i, x_j) = n((n-1)+(n-2)+...+(n-(\frac{n-1}{2}))+\frac{n}{2}).$

Therefore,

$\sum_{i<j} D(x_i, x_j) = n[\sum_{k=1}^{\frac{n-2}{2}} (n-k)+\frac{n}{2}].$

So, the detour index ω(G) is defined as,

$\omega(G) = \sum_{i<j} D(x_i, x_j) = n[\sum_{k=1}^{\frac{n-2}{2}} (n-k)+\frac{n}{2}] = \frac{n^2}{8}(3n-4)$

which is the cycle's detour index when the number of vertices are even.

A wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. While some authors [28] use W_n to denote a wheel graph with n vertices ($n \geq 4$) some others [29] use W_n to denote a wheel graph with $n + 1$ vertices ($n \geq 3$), which is formed by connecting a single vertex to n vertices of a cycle. In the rest of this article, we use the latter notation.

Theorem 2.3 If $G \cong W_n$, with $n \geq 3$, then,

$\omega(G) = \frac{1}{2}n^2(n+1)$

Proof:

Consider W_n, with vertex set $V(G) = \{x_1, x_2, \ldots, x_n\}$.

The detour distance between any two pair of vertices $(x_i, x_j) = n$

Total number of such pairs $= \sum_{i=1}^{n} i$

$$\omega(G) = \sum_{i<j} D(x_i, x_j)$$
$$= n\sum_{i=1}^{n} i = n \times \frac{1}{2}(n(n+1)) = \frac{1}{2}n^2(n+1)$$

The friendship graph (or Dutch windmill graph)F_n [31] is a planar undirected graph with $2n + 1$ vertices and $3n$ edges [30] which can be formed by joining n clones of the cycle graph C_3 at a common vertex.

Theorem 2.4 If $G \cong F_n$ with $n \geq 1$, then,

$$\omega(G) = 2n(4n - 1)$$

Proof:

Consider F_n having vertex set $V(G) = \{x_1, x_2, \ldots, x_{2n+1}\}$

The detour distance between any two pair of vertices (x_i, x_j) is denoted by

$D(x_i, x_j)$ where $(i, j) \in [1, n]$ with $i < j$

case 1 : Detour distance from the central vertex to all another vertex = 2

There are 2n such vertices.

Therefore, the sum of detour distance from central vertex to all other $= 2 \times 2n = 4n$

case 2: Detour distance from non-central vertex to other non-central vertices of

a) Same cycle = 2

There are n such vertices

Therefore, the sum of detour distances of such vertices $= 2 \times n = 2n$

b) Different cycle = 4

Number of such vertices

$$= \sum_{i=1}^{n} 4(i - 1)$$

$$= 2n(n - 1)$$

Therefore,

the sum of detour distance of such vertices $= 4 \times 2n(n-1) = 8n(n-1)$

$\omega(G) = 4n + 2n + 8n(2) - 8n = 2n(4n - 1)$.

An (n, m)lollipop graph [32] is a special type of graph consisting of a complete graph on n vertices and a path graph with m vertices, connected with a bridge.

Theorem 2.5 If $G \cong K_n P_m$, with $n \geq 3, m \geq 2$, then,

$$\omega(G) = \frac{n}{2}(n-1)^2 + \frac{m}{6}(m+1)(m+2) + \frac{m}{2}(n-1)(2n+m-1)$$

Proof:

Consider K_n with vertex set $\{y, x_1, x_2, \dots, x_{n-1}\}$ and P_m with vertex set $\{y, y_1, y_2, \dots, y_m\}$

Let y be the vertex common to K_n and P_m

From Theorem 2.1, Detour index of complete graph

$K_n = \frac{n}{2}(n-1)^2$

From Result , Detour index of path

$P_m = \frac{m}{6}(m^2 - 1)$

Here, by definition of lollipop graph, there is an extra edge(bridge) from the vertex u.

So, Considering the path $y - y_1 - y_2 - \dots - y_m$,

Detour index of path

$P_{m+1} = \frac{(m+1)}{6}((m+1)^2 - 1) = \frac{m}{6}(m+1)(m+2)$

Consider vertex x_1

Detour distance from x_1 to $y = n - 1$

So, Detour distance from x_1 to $y_i = (n-1) + i$ [where,$1 \leq i \leq m$]

Sum of detour distances between pair of vertices

$$(x_1, y_i) = \sum_{i=1}^{m}(n - 1 + i)$$

Since there are $n - 1$ such $x_i's$, total sum of detour distances between pair of vertices (x_i, y_j)

$= (n-1) \times \sum_{i=1}^{m}(n+i-1)$
$= (n-1)[mn - m + \frac{m^2+m)}{2}]$
$= \frac{m}{2}(n-1)(2n+m-1)$

Detour Index of K_nP_m ,

$\omega(G) = \frac{n^3-2^n+n}{2} + \frac{m(m^2+3m+2)}{6} + \frac{m}{2}(n-1)(2n+m-1)$

Let K_n be a complete graph on n vertices. The graph obtained by identifying one vertex of K_n with central vertex of star $K_{1,m}$ is called starred complete graph [33]. It is represented as K_nS_m .

Theorem 2.6 If, $G \cong K_nS_m$ with, $n \geq 3, m \geq 1$, then,

$\omega(G) = \frac{n(n-1)^2}{2} + m^2 + n(n-1)m$

Proof:

Consider K_n with vertex set $\{u, u_1, u_2, \ldots, u_{n-1}\}$ and S_m be a star with vertex set $\{u, v_1, v_2, \ldots, v_n\}$

Let u be the identified vertex which is common to K_n and S_m.

Detour index of complete graph

$K_n = \frac{n(n-1)^2}{2}$

Detour index of star $S_m = (m - 1)^2$, where m is the number of vertices of star.

But in the graph K_nS_m we are considering m as the number of leaf vertices of the star.

So here, m = m + 1.

Therefore, Detour index of $S_m = m^2$

Now, Detour distance from u_i's to v_i's and vice versa is, $(n - 1) + 1 = n$

Since there are $(n - 1)m$ such pairs,

Sum of detour distances between all such pairs of vertices= $n(n - 1)m$.

Therefore, Detour index of starred complete graph

$\omega(G) = \frac{n(n-1)^2}{2} + m^2 + n(n-1)m$

An n-partite graph [34] is a graph whose graph vertices can be partitioned into n disjoint sets so that no two vertices within the same set are adjacent.

Theorem 2.7 If, $G \cong K_{1,2,\ldots,n}$ with $n > 3$, then,

$\omega(G) = (S_n - 1)S_nC_2$

where S_n is the sum of elements in each set of $K_{1,2,\ldots,n}$.

Proof:

Consider $K_{1,2,\ldots,n}$ be an n-partite graph having vertex set

$\{u_{11}, u_{12}, \ldots, u_{1k_1}\} \cup \{u_{21}, u_{22}, \ldots, u_{2k_2}\} \cup \ldots \cup \{u_{n1}, u_{n2}, \ldots, u_{nk_m}\}$ where, $n > 3$.

Detour distance from every u_{nk_i} to every $u_{nk_{i+1}} = S_n - 1$

And there are S_nC_2 such possibilities

Therefore, Detour index of n-partite graph,

$\omega(G) = (S_n - 1)S_nC_2$

The corona product [35] of two graphs G and H is defined as the graph obtained by taking one copy of G and |V (G)| copies of H and joining the i^{th} vertex of G to every vertex in the i^{th} copy of H.

Theorem 2.8 If $G \cong K_{n \odot P_1}$, where K_n is a complete graph with $n \geq 3$ and P_1 is a path with 1 vertex, then,

$\omega(G) = n[\frac{(n-1)^2}{2} + n(n-1) + \frac{n^2-1}{2} + 1]$

Proof:

Consider K_n with vertex set $\{u_1, u_2, \ldots, u_n\}$ and let $\{v_1, v_2, \ldots, v_n\}$ represent the outer vertices P_1.

From theorem(2.1), detour index for complete graph is

$\frac{n}{2}(n-1)^2$

Consider vertex u_1

Detour distance from u_1 to $v_i = (n-1) + 1 = n$

Since there are $(n - 1)$ such u_i's

Therefore, Total of detour distances between pair of vertices

$(u_i, v_j) = n(n-1) \times n$

$= n^2(n-1)$

Consider v_1

Detour distance from v_1 to v_i = n + 1

Therefore, Total detour distance between pair of vertices

$(v_i, v_j) = (n + 1)(n - 1) \times n = n(n - 1)(n + 1)$

Detour distance between pair of adjacent vertices u_i and v_i = 1

There are n such pairs

Therefore, Sum of detour distances between $(u_i, v_i) = 1 \times n = n$ Therefore, Detour Index of $K_n \odot P_1$ is,

$$\omega(G) = \frac{n}{2}(n-1)^2 + n^2(n-1) + n(n-1)(n+1) + n$$
$$= n[\frac{(n-1)^2}{2} + n(n-1) + \frac{n^2-1}{2} + 1]$$

3 Detour Harary Index

Theorem 3.1 Let $G \cong K_n$, where K_n represents a complete graph having $n \geq 3$. Then,

$$\omega H(G) = \frac{n}{2}$$

Proof:

Consider K_n to be a complete graph having its vertex set as $\{v_1, v_2, \dots, v_n\}$

Reciprocal of detour distance between vertices v_i and v_j, $1 \leq i, j \leq n$ in a complete graph is,

$$\frac{1}{D(v_i,v_j)} = \frac{1}{n-1}$$

Consider vertex v_1

Since there are $n - 1$ pair of vertices with v_1, sum of reciprocals of the detour distance between vertex v_1 and all other vertices will be

$$\frac{1}{n-1} \times n - 1 = 1$$

Total number of vertices in K_n = n

Sum of reciprocals of detour distance between all the paired vertices is,

$$\sum_{i,j=1}^{n} \frac{1}{D(v_i,v_j)} = n \times 1 = n$$

Detour Harary index of K_n,

$$\omega H(G) = \sum_{1 \leq i < j \leq n} \frac{1}{D(v_i,v_j)} = \frac{n}{2}$$

Theorem 3.2 Let $G \cong W_n$, where W_n represents a wheel graph with $n \geq 3$. Then,

$\omega H(G) = \frac{n+1}{2}$

Proof:

Consider W_n to be a wheel graph with its vertex set $\{v_1, v_2, \ldots, v_n\}$.

The reciprocal of detour distances between the vertices v_i, v_j, $1 \leq i, j \leq n$ is,

$\frac{1}{D(v_i,v_j)} = \frac{1}{n}$

Total number of such pairs are

$\sum_{k=1}^{n} k$

Detour Harary index of W_n,

$\omega H(G) = \sum_{i<j} \frac{1}{D(v_i,v_j)} = \frac{1}{n} \times \sum_{k=1}^{n} k = \frac{1}{n} \times \frac{n(n+1)}{2} = \frac{n+1}{2}$

Theorem 3.3 If $G \cong F_n$, where F_n is a friendship graph, with $n \geq 1$ then,

$\omega H(G) = \frac{n(n+2)}{2}$

Proof:

Consider F_n to be a friendship graph with the vertex set $\{v_1, v_2, \ldots, v_{2n+1}\}$.

Case 1: Reciprocal of detour distance from the central vertex to all another vertex is $\frac{1}{2}$There are 2n such vertices.

Sum of reciprocal of detour distances from central vertex to all other vertices= n

Case 2: Reciprocal of detour distance form non-central vertex to other non-central vertices of

a) Same cycle =$\frac{1}{2}$

There are n such vertices.

Sum of reciprocal of detour distances between such pair of vertices= $\frac{n}{2}$

b) Different cycle =$\frac{1}{4}$

Number of such vertices,

$\sum_{i=1}^{n} 4(i-1)$

$= 2n(n-1)$

Sum of reciprocal of detour distances between such vertices= $\frac{1}{4} \times 2n(n-1) = \frac{n(n-1)}{2}$

Detour Harary index of F_n ,

$\omega H(G) = n + \frac{n}{2} + \frac{n(n-1)}{2} = \frac{n(n+2)}{2}$

Theorem 3.4 If $G \cong K_n S_m$, where $K_n S_m$ represents the starred complete graph with $n \geq 3, m > 3$ then,

$$\omega H(G) = \frac{n}{2} + m + \frac{m(m-1)}{4} + \frac{m}{n}(n-1)$$

Proof:

Consider K_n to be a complete graph having its vertex set as $\{u, u_1, u_2, \ldots, u_{n-1}\}$ and S_m be considered to be a star graph having it's vertex set as $\{u, v_1, v_2, \ldots, v_m\}$ Consider u to be the identified vertex that is common to K_n and S_m

From theorem 3.1 detour harary index of complete graph = $\frac{n}{2}$

Sum of detour harary distance between vertex u and $v_i = m \times \frac{1}{1} = m$

Detour Harary distance between pair of vertices $v_i, v_j = \frac{1}{2}$

Sum of the detour distance between pair of vertices $v_i, v_j = \frac{1}{2} \times \sum_{i=1}^{m-1}(m-i)$

$$= \frac{1}{2} \times [m(m-1) - \frac{(m-1)m}{2}] = \frac{m}{4}(m-1)$$

Detour Harary distance between pair of vertices $u_i, v_j = \frac{1}{n}$

Sum of detour Harary distance between pair of vertices $u_i, v_j = \frac{1}{n} \times (n-1) \times m = \frac{m}{n}(n-1)$

Detour Harary index of $K_n S_m$,

$$\omega H(G) = \frac{n}{2} + m + \frac{m(m-1)}{4} + \frac{m}{n}(n-1)$$

Theorem 3.5 If $G \cong K_{1,2,\ldots,n}$ where, $K_{1,2,\ldots,n}$ represents the n−partite graph having $n > 3$ then,

$$\omega H(G) = \frac{S_n C_2}{S_n - 1}$$

where, S_n represents the sum of elements in each set of $K_{1,2,\ldots,n}$.

Proof: Let $K_{1,2,\ldots,n}$ be an n− partite graph having vertex set

$\{u_{11}, u_{12}, \ldots, u_{1k_1}\} \cup \{u_{21}, u_{22}, \ldots, u_{2k_2}\} \cup \ldots \cup \{u_{n1}, u_{n2}, \ldots, u_{nk_m}\}$ where, $n > 3$.

Detour Harary distance from every u_{nk_i} to every $u_{nk_{i+1}} = \frac{1}{S_n - 1}$

There are $S_n C_2$ such possibilities,

Detour Harary index of $K_{1,2,\ldots,n}$,

$$\omega H(G) = \frac{S_n C_2}{S_n - 1}$$

Theorem 3.6 Let $G \cong K_n \odot P_1$; where K_n represents a complete graph having $n \geq 3$, P_1 is a path with 1 vertex, then,

$\omega H(G) = \frac{n}{2}[5 + \frac{(n-1)}{(n+1)}] - 1$

Proof: Consider K_n to be a complete graph having vertex set $\{u_1, u_2, \dots, u_n\}$. Let $\{v_1, v_2, \dots, v_n\}$ represent outer vertices of P_1.

From theorem 1.9, detour harary index of complete graph = $\frac{n}{2}$

Consider vertex u_1.

Reciprocal of detour distance from u_1 to $v_1 = \frac{1}{n}$

Since there are $n - 1$ such v_1's and n such u_1's

Sum of reciprocal of detour distances between pair of vertices u_i and v_j

$$= \frac{1}{n} \times (n-1) \times n = (n - 1)$$

Consider v_1.

Reciprocal of detour distance from v_1 to any $v_i = \frac{1}{(n+1)}$

Since there are $(n - 1)$ such $v_i's$, total of reciprocal of detour distance between pair of vertices v_i and v_j

$$= \frac{1}{n+1} \times (n-1) \times n = \frac{n(n-1)}{n+1}$$

Reciprocal of detour distance between pair of adjacent vertices $u_i, v_i = \frac{1}{1} = 1$

Since there are n such pairs, sum of reciprocal of detour distances between $u_i, v_i = 1 = n$

Detour Harary index of $G \cong K_n \odot P_1$

$\omega H(G) = \frac{n}{2} + (n-1) + \frac{n(n-1)}{2(n+1)} + n = \frac{n}{2}[5 + \frac{(n-1)}{(n+1)}] - 1$

4 Results

Result 1: In a complete graph, the relation between Detour index and Detour Harary index is

$\omega(K_n) = (n-1)^2[\omega H(K_n)]$

Proof: From theorem(2.1),

$$\omega(K_n) = \frac{n(n-1)^2}{2} \tag{1}$$

From theorem(3.1),

$$\omega H(K_n) = \frac{n}{2} \tag{2}$$

Dividing 1 by 2,

$\frac{\omega(K_n)}{\omega H(K_n)} = \frac{\frac{n}{2}(n-1)^2}{\frac{n}{2}} = (n-1)^2$

Therefore,

$\omega(K_n) = (n - 1)^2 \times \omega H(K_n)$

Result 2: In a wheel graph, the relation between Detour index and Detour Harary index is

$\omega(W_n) = n^2[\omega H(W_n)]$

Proof: From theorem(2.3),

$$\omega(W_n) = \frac{n^2(n+1)}{2} \tag{3}$$

From theorem(3.2),

$$\omega H(W_n) = \frac{n+1}{2} \tag{4}$$

Dividing 3 by 4,

$\frac{\omega(W_n)}{\omega H(W_n)} = \frac{\frac{n^2(n+1)}{2}}{\frac{n+1}{2}} = n^2$

Therefore,

$\omega(W_n) = n^2 \times \omega H(W_n)$

Result 3: In a Friendship graph, the relation between Detour index and Detour Harary index is

$\omega(F_m) = \frac{4(4m-1)}{(m+2)}[\omega H(F_m)]$

where, $m = \frac{n-1}{2}$

Proof: From theorem(2.4),

$$\omega(F_m) = 2m(4m - 1) \tag{5}$$

From theorem(3.3),

$$\omega H(F_m) = \frac{m(m+2)}{2} \tag{6}$$

Dividing 5 by 6,

$\frac{\omega(F_m)}{\omega H(F_m)} = \frac{2m(4m-1}{\frac{m(m+2)}{2}} = \frac{4(4m-1)}{(m+2)}$

Therefore,

$\omega(F_m) = \frac{4(4m-1)}{(m+2)} \times \omega H(F_m)$

Result 4: In an n-partite graph, the relation between Detour index and Detour Harary index is

ω(K1,2,...,n) = (Sn − 1)^2 [ωH(K1,2,...,n)]

Proof: From theorem(2.7),

ω(K1,2,...,n) = (Sn − 1)SnC2 (7)

From theorem(3.5),

$$\omega H(K_{1,2,\ldots,n}) = \frac{S_n C_2}{S_n - 1} \tag{8}$$

Dividing 7 by 8,

$\frac{\omega(K_{1,2,\ldots,n})}{\omega H(K_{1,2,\ldots,n})} = \frac{(S_n-1)S_nC_2}{\frac{S_nC_2}{S_n-1}} = (S_n - 1)^2$

Therefore,

ω(K1,2,...,n) = (Sn − 1)^2 × ωH(K1,2,...,n)

5 Scope for further research

Topological indices for molecular structures can also be established in similar way that we established these indices for the graphs including starred complete and n-partite graph. If there exist correlation between their physical properties and these indices, it can be useful to study such properties.

We can apply the topological indices discussed here to any type of graphs that would model a molecular structure. From these indices, we can analyse the mathematical values and can also investigate certain physio-chemical properties of molecules.

During the past century, chemists started theoretically working on the uses of these topological indices for obtaining information regarding different properties of some organic substances that depends upon its molecular structure.

6 Conclusion

We established the generalised formula to calculate the Detour index and the Detour Harary index for Complete graph, Cycle graph, Wheel graph, Friendship graph, Lollipop graph, Starred complete graph and n-partite graph. Also found formula for both these indices for a graph obtained by doing the graph operation corona and the relation between the Detour and Detour Harary indices have been found for some graph.

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