Image Deblurring based on Weiner Deconvolution Method using Markov Basis

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Abstract: The purpose of this paper is to enhance our understanding of the mathematics underneath image blurring and deblurring. Numerous references and materials such as books have been raised in order to gain a better understanding of underlying processes involved in deblurring. Tests were carried out to determine the efficacy of the fundamental methods. Through this research, we were capable of improving our knowledge of the sensitivity of the error observed in a few of the naive disturbing solutions that were investigated. Furthermore, we gained a greater understanding of spectral filtering techniques' potential to reduce the impact of noise on deblurring strategies. In this report, we discuss a way of deblurring digital images using HB filters that generate from Average filter by using HB Markov basis. Using these filters, we have deburred the motion-blurred images. The results indicate that HB filters perform better in terms of peak signal-to-noise and Root Mean Square Error.

Keywords: Digital Images, Motion Blur, Deblurring, Markov basis, Weiner Deconvolution

I. INTRODUCTION

Mathematics plays a vital role when police are attempting to solve a crime. Police must determine what occurred at the crime scene and interpret data that must be saved as well as mined for information. It is well known that data can be stored and interpreted using wavelets, probability, and statistics [1,2,3]. Secure transmission can be done using prime numbers and cryptography. However, first and foremost, the police must locate the information hidden within the data. For example, let’s take the scenario where a person has robbed a bank. Even though he attempted to flee in a car, the police were able to photograph the vehicle's registration number. But unluckily, the photo was blurred. This is a situation where mathematics can help us. We can mathematically model the blurring process of the blurred image and detach a few of the blurs to have a better image of the number plate. From this, we can understand that mathematics is the weapon used behind deblurring number plates, reconstructing accidents from skid marks, and in many other crime spots. A digital image can be broken down into pixels, with each pixel having a magnitude that defines its intensity. Every image of the scene recorded by the camera is more or less blurry.

Blurring can occur when scene information overflows to neighbouring pixels. Motion blur occurs when there is motion between the camera and image objects during photography. Mathematical models of the blurring process are used to recover motion-blurred images [4,5]. The convolution of the Point Spread Function with the image represented by its intensities is called motion blur. Image deblurring is the process of recovering the original image by using a mathematical model of the blurring process. Many research scholars developed algorithms to eliminate the blurs such as Gaussian filter (GF), Mean filter MF (or Average filter). We can make the image appear to move by introducing a blur in a given direction to the image. This is called Motion Blur Effect Filter. Markov basis HB is H-invariant to a generated six types of $\frac{n^2-3n}{3}$ X 3 X $\frac{n}{3}$- contingency tables with fixed two-dimensional marginals [6]. Markov basis HB is used to introduce filters from the average filter to remove the motion blur of images, which is denoted by HB-filters.

A. Background:

This topic provides some background on image deblurring concepts

- Causes of Blurring
- Deblurring Function

B. Causes of Blurring:

Many factors can contribute to blurring or image reduction:

- Movement between the process of capturing an image, with the camera or, when using long exposure moments, with the subject.
- Out from emphasis, the use of a wide-angle lens, wind turbulence, or a shorter time of exposure, all of which limit the number of photons collected.
- Distortion of scattered light through a confocal microscope.

C. Deblurring Function:

They are four deblurring functions used to deblur a image listed here:

(1) deconvwnr : Initiating deblurring use using the Wiener filter.
(2) deconvreg : Initiating deblurring using a regularized filter.
(3) deconvlucy: Initiating deblurring using the Lucy-Richardson algorithm.
(4) deconvblind: Initiating deblurring using the blind deconvolution algorithm.

II. EXISTING SOLUTION-GENERAL METHOD OF DEBLURRING

A typical technique of deblurring a image is a straightforward way to get deblurred or blurred image from the blurred image or original image respectively. The most funadmental method is simple matrix multiplication of the rows and columns of blurring matrices with the actual image. A simplified representation of the blurring image is

$$A_c X A_r^T = B$$

where A_c in $\mathbb{R}^{m\times m}$ is the column blurring matrices, A_r in $\mathbb{R}^{n\times n}$ is the row blurring matrices, A_r^T is the transpose, B in $\mathbb{R}^{m\times n}$ is the Blurred image, X in $\mathbb{R}^{m\times n}$ is the original image. We might consider X=$A^{-1}BA_r^{-T}$ to be so, but a quick analogy demonstrates that the result output is not the same as the actual image. Because of things like noise as well as imprecision, knowing the blurry matrix of X is not enough to restore the original. As a result, the error term must be incorporated into the equation $B = E + A_C X A_r^T$, because the value of noise is unknown, the aim is to develop quite sophisticated methods to reduce the impact of inverted noise. The logic that sound makes a lost picture to be unknown because its high frequency components are amplified when little amount of blurred matrices are inverted. A simple way to mitigate this impact is to generate rank-k versions of column and row matrices from their single-value decompositions.

$$A_k^l = [v_1 \dots v_k][\sigma_1 \dots \sigma_k]^{-1} \begin{bmatrix} U1 \\ \vdots \\ Uk \end{bmatrix}$$

Although this procedure provides a higher resolution than the earlier solution, the noise has had a significant impact on the output image when using

$$\mathrm{vec}(x)=A^{-1}{}_k \quad \mathrm{vec}(B)$$

However, without further adoption, this simple approach is not enough to rebuild a blurred image.

PRELIMINARY CONCEPTS:

We review the preliminary concepts about Markov Basis HB, moves, convolution, and Fourier Transforms.

Definition 1[7]: The move is an n-dimensional column vector of integers $z = \{z_i\}_{i\epsilon 1} \epsilon Z^n$ where it is in kernel of A, that is Az = 0. Here, A: $z^n \rightarrow z^d$ is a linear transformation, and d is the number of contingency table x.

Definition 2[8]: A set of finite moves C is called Markov basis if for all x, $A^{-1}[x]$ constitutes one C equivalence class. Here, $A^{-1}[x] = \{d \epsilon N^n: Ad = x\}$. We get 18 elements as per Markov basis in the following set [9].

$Z_1 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $Z_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$;

$Z_3 = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$; $Z_4 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$;

$Z_5 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$; $Z_6 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$;

$Z_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$; $Z_8 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$;

$Z_9 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$; $Z_{10} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$;

$Z_{11} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$; $Z_{12} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$;

$Z_{13} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$; $Z_{14} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$;

$Z_{15} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$; $Z_{16} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$;

$Z_{17} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}$; $Z_{18} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$.

DEFINITION 3: CONVOLUTION 2-D

Suppose we have two discrete dimensional images h(x,y) and f(x,y). The image g(x,y) is their folded (convolved) sum. The convolution of these two functions can be described as:

$$g(x, y) = f(x, y) \otimes h(x, y)$$

$f(x, y) \otimes h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n)\, h(x - m, y - n)$ …(1)

For, $0 \leq y$, $n \leq N-1$ and $0 \leq x$, $m \leq M-1$, where M X N is size of h(x,y).

DEFINITION 4: 2-D DISCRETE FOURIER TRANSFORM

The 2-D Discrete Fourier Transform (DFT) of image function f(x,y) is defined as

$$F(u, v) \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})} \ \ldots (2)$$

where f(x,y) of size M X N is a digital image, and discrete variables u and v are in u=0,1,2,3,..,M-1 and v= 0,1,2,3,4,…,N-1 range. We can obtain f(x,y) by using inverse Discrete Fourier Transform if the transform F(u,v) is given by:

$$f(x, y) \frac{1}{\mathrm{MN}} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})} \quad \ldots (3)$$

The translation properties of the Fourier Transform pair are satisfied by direct substitution into equations (2) and (3):

$$f(x - m, y - n) \Leftrightarrow \qquad \mathrm{F}(u, v) e^{-i2\pi(\frac{um}{M}+\frac{vn}{N})} \ \ldots (4)$$

Now, Fourier Transform of equation (1) is found by

$$\mathfrak{F}\big(\mathrm{f(x, y)} \otimes h(x, y)\big) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) \mathrm{h}(x - m, y - n)]\, e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

By equation (4) we have,

$$\mathfrak{F}\big(\mathrm{f(x, y)} \otimes \mathrm{h}(x, y)\big) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n)\, H(u, v) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})} = F(u, v) H(u, v)$$

Result of Convolution theorem is

$$\mathrm{f(x, y)} \otimes h(x, y) \Leftrightarrow F(u, v) H(u, v) \ \ldots (5)$$

Inverse filter is the conversion of the actual image which is found by partitioning the degraded image's transform G(u,v) by the degradation function H(u,v).

$$\hat{F}(u, v) = \frac{G(u,v)}{H(u,v)} \ \ldots (6)$$

The 2-D DFT in polar form is given by $F(u, v) = |F(u, v)| e^{-i\emptyset(u,v)}$,whose magnitude is given by

$$|F(u, v)| = [R^2(u, v) + I^2(u, v)]^{\frac{1}{2}} \ \ldots (7)$$

is called the Fourier spectrum. Here, I and R are the imaginary and real parts of F(u,v) and u values ranges from u=0,1,2,3,…, M-1 and values of v ranges from v=0,1,2,3,…,N-1. Thus, |F(u,v)|, Ø(u,v) are arrays of size M X N.

III. WEINER DECONVOLUTION FOR IMAGE RESTORATION

In this method the noise and images are considered as random variables. The goal is to find an estimate $\hat{f}$ of an uncorrupted image f with the smallest Mean Square Error between them. The error measure is described by:

$$e^2 = E\{(f - \hat{f})^2\} \ \ldots (8)$$

In the frequency domain, the minimum of error function in (8) is given by

$$\widehat{F}\,(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2+S_\eta(u,v)}\right]G(u,v) \quad = \left[\frac{1}{H(u,v)}\,\frac{|H(u,v)|^2}{|H(u,v)|^2+S_\eta(u,v)/S_f(u,v)}\right]\,G(u,v)\ \ldots (9)$$

where H(u,v) is the degradation function, H*(u,v) is the complex conjugate of H(u,v), $|H(u,v)|^2$ is $H^*(u,v)H(u,v)$, $S_\eta(u,v) = |N(u,v)|^2$ is the Power spectrum of noise, $S_f(u,v) = |F(u,v)|^2$ is the Power spectrum of original image. G(u,v) is the transform of degraded image. When noise is zero, the Weiner filter cuts down to an inverse filter because the noise power spectrum vanishes.

IV. COMPARISON OF DEBLURRING TECHNIQUES:

In most of the deblurring technique approaches, the unknown blur kernel is hard to estimate and is known that way. Different types of texture blurring can be easily identified in a low subspace analysis. However, it also had a problem as for not solving images with similar textures. Previous strategies such as using a binary, straight line pattern Ternary patterns, linear phase quantization etc. have been used.

Aspect / Method	Accuracy	Different type of Blurs
Subspace Analysis	Medium	Low
Image Statistics	High	Medium
Local Phase Quantization	High	Medium
Set Theoretic Approach	High	High

PROPOSED APPROACH:

We use Weiner deconvolution method to deblur the blurred images which are blurred using Markov basis HB that generates HB-filters by adding the average filter with each element in the HB. As a result, we have some HB-filters with 3 X 3 dimensions, each with a different variety of blur. We remove this blur by using the Weiner method.

DEBLURRING:

We have expressed the proposed deblurring method.

A. Procedure:

Weiner deconvolution for the matrices g(x,y) and h(x,y)is

$$f\hat{}(x,\mathrm{y}) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+\mathrm{S_\eta(u,v)/S_F(u.v)}}\right]\mathrm{G(u,v)}$$

Suppose there is no noise $\frac{\mathrm{S_\eta(u,v)}}{\mathrm{S_F(u.v)}}$=0, the noise in the power spectrum disappears. As shown below, the Weiner filter is reduced to the invers filter ,$\hat{\mathrm{F}}(\mathrm{u,v})$=$\frac{\mathrm{G(u,v)}}{\mathrm{H(u,v)}}$.

Step 1: Determine the Fourier transform of the g(x,y)r-by-c dimensions blurred matrix.

$$\mathrm{G(u,v)} = \sum_{x=1}^{m}\sum_{y=1}^{n}\mathrm{g(x,y)}e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

Step 2: Determine the Fourier transform of the HB-filter h(x,y).

$$H(u,v) = \sum_{x=1}^{m}\sum_{y=1}^{n} h(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

If the dimension of h(x,y) is less than the dimension of g, we will add zeros for h(x,y). We do this before performing the transform to develop an image matrix with the same dimension as the image matrix g(x,y), with the result being m x n in dimension.

Step 3: Calculate the transform of estimated image $\hat{F}(u,v)$.

Step 4: Find the estimated image f (u,v) by performing the inverse Fourier transform of F (u,v) as shown below.

$$\hat{f}(x,y) = \frac{1}{MN}\sum_{u=1}^{m}\sum_{v=1}^{n} \hat{F}(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Step 5: Eliminate as many zeros from f(x,y) as (p-1)/2 of the last rows and columns, where the source image matrix f(x,y) dimensions equal the resulting dimensions.

Example: Here, we convert our blurred image to deblurred image

$$g(x,y)=\begin{bmatrix} 46.4444 & 20 & 36.5556 & 10 & 6.6667 \\ 0 & 63.5556 & 94.8889 & 31.8889 & 10 \\ 45.4444 & 43.4444 & 72.556 & 37 & 12.2222 \\ 0 & 30.7778 & 33.2222 & 21.4444 & 5.5556 \\ 11.1111 & 16.2222 & 18.444 & 7.3333 & 2.2222 \end{bmatrix}$$

with HB-filter given by h(x , y)=$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ /9. From the Weiner equation, if $\frac{S_\eta(u,v)}{S_F(u.v)}$=0, the Weiner filter then reduces to an inverse filter as $\hat{F}(u,v)=\frac{G(u,v)}{H(u,v)}$.

Step 1: Calculate the Fourier transform of matrix g(x,y) as

$$G(u,v) = \sum_{x=1}^{m}\sum_{y=1}^{n} g(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

Now,

$$G(1,1) = \sum_{x=1}^{5}\sum_{y=1}^{5} g(x,y)e^{-j2\pi(\frac{x}{5}+\frac{y}{5})}$$

Here $u = v = 1$. Then

$\left(g(1,1)e^{-j2\pi\left(\frac{1}{5}+\frac{1}{5}\right)}\right)$+$\left(g(1,2)e^{-j2\pi\left(\frac{1}{5}+\frac{2}{5}\right)}\right)$+$\left(g(1,3)e^{-j2\pi\left(\frac{1}{5}+\frac{3}{5}\right)}\right)$ +
$\left(g(1,4)e^{-j2\pi\left(\frac{1}{5}+\frac{4}{5}\right)}\right)$+$\left(g(1,5)e^{-j2\pi\left(\frac{1}{5}+\frac{5}{5}\right)}\right)$+$\left(g(2,1)e^{-j2\pi\left(\frac{2}{5}+\frac{1}{5}\right)}\right)$+$\left(g(2,2)e^{-j2\pi\left(\frac{2}{5}+\frac{2}{5}\right)}\right)$+……+
$\left(g(5,5)e^{-j2\pi\left(\frac{5}{5}+\frac{5}{5}\right)}\right)$=46.4444$e^{-j\left(\frac{4}{5}\right)\pi}$+20$e^{-j\left(\frac{6}{5}\right)\pi}$+36.5556$e^{-j\left(\frac{8}{5}\right)\pi}$+10$e^{-j2\pi}$+……......+2.2222 $e^{-j4\pi}$=632+0j

Now, if (u, v)=(1,2)

$G(1,2) = \sum_{x=1}^{5}\sum_{y=1}^{5} g(x,y)e^{-j2\pi(\frac{x}{5}+\frac{2y}{5})}$ = -89.44-191.15j

Next, $G(1,3) = \sum_{x=1}^{5}\sum_{y=1}^{5} g(x,y)e^{-j2\pi(\frac{x}{5}+\frac{3y}{5})}$ =30.94+17.24j

As we proceed in the same way, finally we arrive at

$G(5,5) = \sum_{x=1}^{5}\sum_{y=1}^{5} g(x,y)e^{-j2\pi(\frac{5x}{5}+\frac{5y}{5})}$ =-1.13-45.84j

STEP 2: Because h(x,y) has a smaller dimension than g(x,y), add a zero to h(x,y) to make it the same size as the image matrix g(x,y),so we have $h(x,y) = \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ /9

Now, we can calculate the Fourier transform of h(x,y) by

$$\mathrm{H(u,v)} = \sum_{x=1}^{m}\sum_{y=1}^{n} \mathrm{h(x,y)e}^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Now $\mathrm{H(1,1)} = \sum_{x=1}^{5}\sum_{y=1}^{5} \mathrm{h(x,y)e}^{-j2\pi(\frac{x}{5}+\frac{y}{5})}$. Here $\mathrm{u = v = 1}$. Then,
$\left(\mathrm{h(1,1)e}^{-j2\pi\left(\frac{1}{5}+\frac{1}{5}\right)}\right)+\left(\mathrm{h(1,2)e}^{-j2\pi\left(\frac{1}{5}+\frac{2}{5}\right)}\right)+\left(\mathrm{h(1,3)e}^{-j2\pi\left(\frac{1}{5}+\frac{3}{5}\right)}\right) +$
$\left(\mathrm{h(1,4)e}^{-j2\pi\left(\frac{1}{5}+\frac{4}{5}\right)}\right)+\left(\mathrm{h(1,5)e}^{-j2\pi\left(\frac{1}{5}+\frac{5}{5}\right)}\right)+\left(\mathrm{h(2,1)e}^{-j2\pi\left(\frac{2}{5}+\frac{1}{5}\right)}\right)+\left(\mathrm{h(2,2)e}^{-j2\pi\left(\frac{2}{5}+\frac{2}{5}\right)}\right)+$…………
…………..+$\left(\mathrm{h(5,5)e}^{-j2\pi\left(\frac{5}{5}+\frac{5}{5}\right)}\right)$=2$\mathrm{e}^{-j\left(\frac{4}{5}\right)\pi}$+20$\mathrm{e}^{-j\left(\frac{6}{5}\right)\pi}$+1$\mathrm{e}^{-j\left(\frac{8}{5}\right)\pi}$+0$\mathrm{e}^{-j2\pi}$+……......+0$\mathrm{e}^{-j4\pi}$=1+0 j.

Next, if (u, v)=(1,2), then $\mathrm{H(1,2)} = \sum_{x=1}^{5}\sum_{y=1}^{5} \mathrm{h(x,y)e}^{-j2\pi(\frac{x}{5}+\frac{2y}{5})}$ = 0.1667-0.5129J

Next, $\mathrm{H(1,3)} = \sum_{x=1}^{5}\sum_{y=1}^{5} \mathrm{h(x,y)e}^{-j2\pi(\frac{x}{5}+\frac{3y}{5})}$ =0.1667+0.1211j

And so, finally,

$$\mathrm{H(5,5)} = \sum_{x=1}^{5}\sum_{y=1}^{5} \mathrm{h(x,y)e}^{-j2\pi(\frac{5x}{5}+\frac{5y}{5})} = -0.2828+0.0249j$$

The final matrix of H(u,v) be

$$\begin{bmatrix} 1+0j & 0.1667-0.5129j & 0.1667+0.1211j & 0.1667-0.1211j & 0.1667+0.5129j \\ 0.1667-0.5129j & -0.2828-0.0249j & 0.1667+0.171j & 0.1667+0.0404j & 0.4444+0j \\ 0.1667+0.1211j & 0.1667+0.171j & 0.3383+0.2767j & 0.4444+0j & 0.1667-00404j \\ 0.1667-0.1211j & 0.1667+0.0404j & 0.4444+0j & 0.3383-0.2767j & 0.1667-0.171j \\ 0.1667+0.5129j & 0.4444-0j & 0.1667-0.0404j & 0.1667-0.171j & -0.2828+0.0249j \end{bmatrix}$$

1) STEP 3: Find the Fourier transform of estimated image

$\hat{\mathrm{F}}\mathrm{(u,v)}=\frac{\mathrm{G(u,v)}}{\mathrm{H(u,v)}}$

$$= \begin{bmatrix} 632+0j & 285.83-267.23j & 170.67-20.58j & 170.67+20.58j & 285.83+267.23j \\ 257.77-199.34j & -10.23-161.21j & 61.93+15.37j & 103.17+4.41j & 227.85+46.87j \\ 323.73+94.98j & 218.33+29.01j & 216.73+111.98j & 221.15+81.54j & 219.57+158.02j \\ 323.73-94.98j & 219.57-158.02j & 221.15-81.54j & 216.73-111.98j & 218.33-29.01j \\ 257.77+199.34j & 227.85-46.87j & 103.17-4.41j & 61.93-15.37j & -10.23+161.21j \end{bmatrix}$$

1) STEP 4: Calculate the inverse Fourier transform (IFT) with only real numbers
$\hat{f}(x,y) of\ an\ array\ \hat{F}(u,v)$

$$\hat{f}(x,\mathrm{y}) = \frac{1}{MN}\sum_{u=1}^{m}\sum_{v=1}^{n} \hat{F}\mathrm{(u,v)e}^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Now it becomes,

$\hat{f}(1,1) = \frac{1}{5\times5}\sum_{u=1}^{m}\sum_{v=1}^{n} \hat{F}\mathrm{(u,v)e}^{j2\pi\left(\frac{u}{5}+\frac{v}{5}\right)} = \frac{1}{5\times5}\left(\hat{F}(1,1)\mathrm{e}^{j2\pi\left(\frac{1}{5}+\frac{1}{5}\right)} + \hat{F}(1,2)\mathrm{e}^{j2\pi\left(\frac{1}{5}+\frac{2}{5}\right)} + \hat{F}(1,3)\mathrm{e}^{j2\pi\left(\frac{1}{5}+\frac{3}{5}\right)} + \hat{F}(1,4)\mathrm{e}^{j2\pi\left(\frac{1}{5}+\frac{4}{5}\right)} + \cdots + \hat{F}(5,5)\mathrm{e}^{j2\pi\left(\frac{5}{5}+\frac{5}{5}\right)}\right)$

$= \frac{1}{25}$ ((632+0j) $e^{j\left(\frac{4}{5}\right)\pi}$+(285.83-267.23) $e^{j\left(\frac{6}{5}\right)\pi}$+(170.67-20.58j) $e^{j\left(\frac{8}{5}\right)\pi}$+(170.67+20.58j)$e^{j(2)\pi}$ + $(285.83 + 267.23j)e^{j\left(\frac{12}{5}\right)\pi}$ + $(257.77 - 199.34j)e^{j\left(\frac{6}{5}\right)\pi}$ + $(-10.23 - 161.21j)e^{j\left(\frac{8}{5}\right)\pi}$ + $(61.93 + 15.37j)e^{j(2)\pi}$ + $(103.17 + 4.41j)e^{j\left(\frac{12}{5}\right)\pi}$ + $(227.85 + 46.87j)e^{j\left(\frac{14}{5}\right)\pi}$ + $(323.73 + 94.98j)e^{j\left(\frac{8}{5}\right)\pi}$ + $(218.33 + 29.01j)e^{j(2)\pi}$ + $(216.73 + 111.98j)e^{j\left(\frac{12}{5}\right)\pi}$ + $(221.15 + 81.54j)e^{j\left(\frac{14}{5}\right)\pi}$ + $(219.57 + 18.02j)e^{j\left(\frac{16}{5}\right)\pi}$ + $(323.73 - 94.98j)e^{j(2)\pi}$ + $(219.57 - 158.02j)e^{j\left(\frac{12}{5}\right)\pi}$ + $(221.15 - 81.54j)e^{j\left(\frac{14}{5}\right)\pi}$ + $(216.73 - 111.98j)e^{j\left(\frac{16}{5}\right)\pi}$ + $(218.33 - 290.1j)e^{j\left(\frac{18}{5}\right)\pi}$ + $\begin{pmatrix}257.77 + \\ 199.34j\end{pmatrix}e^{j\left(\frac{12}{5}\right)\pi}$ + $(227.85 - 46.87j)e^{j\left(\frac{14}{5}\right)\pi}$ + $(103.17 - 4.41j)e^{j\left(\frac{16}{5}\right)\pi}$+$(61.93 - 15.37j)e^{j\left(\frac{18}{5}\right)\pi}$+(-10.23+161.12j) $e^{j4\pi}$ =209.

Similarly, $\hat{f}(1,2)$=90,$\hat{f}(1,3)$=60,$\hat{f}(2,1)$=0,…,$\hat{f}(5,5)$=0.

The final estimated image $\hat{f}(x, y)$ *is* given by,

$$\hat{f}(x, y) = \begin{bmatrix} 209 & 90 & 60 & 0 & 0 \\ 0 & 77 & 30 & 0 & 0 \\ 100 & 46 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) STEP 5: Remove the last two rows and columns of zeros $\hat{f}(x, y)$:

$$\hat{f}(x, y) = \begin{bmatrix} 209 & 90 & 60 \\ 0 & 77 & 30 \\ 100 & 46 & 20 \end{bmatrix}$$

The original matrix f(x,y) is

$$f(x,y)= \begin{bmatrix} 209 & 90 & 60 \\ 0 & 77 & 30 \\ 100 & 46 & 20 \end{bmatrix}$$

V. IMPLEMENTATION- MATLAB CODING:

Here we have written the code for Weiner deconvolution in MATLAB.

```
Ioriginal = imread('car.tif.png');
PSF = fspecial('motion',21,11);
Idouble = im2double(Ioriginal);
blurred = imfilter(Idouble,PSF,'conv','circular');
imshow(blurred)
title('Blurred Image')
wnr1 = deconvwnr(blurred,PSF);
imshow(wnr1)
title(' DeBlurred Image')
```

VI. EXPERIMENTAL RESULTS:

The input image to the algorithm is displayed in Fig.1 and the final output (Deblurred image) is displayed in Fig.2.

Fig: 1 Input image

Fig 2: Deblurred Image

VII. ADVANTAGES OF DEBLURRING

- Image enhancement is required to reduce the amount of blur from the image.
- Image deblurring is a technique used to minimize the amount of blurring on a blurred image and make the image as a sharper and clearer image.
- Image deblurring has a broad array of applications in consumer photography. e.g., consider removing motion blur caused by camera shake, radar imaging and the tomography.

CONCLUSION

Image deblurring methodologies will be beneficial in the long run for both the advent of newer photography and the rehabilitation of images and videos which are not as sharp as they could be. The ability to eliminate noise from images taken in immensely technical fields such as astronomy and medicine is crucial to enhance the capabilities of professionals in the industry to perform their jobs to the best of their abilities. This offers a compelling cause for further research into enhancing the precision and class of image deblurring in both corporate and academic environments. Blur has been eliminated from digital images using HB-filters. Even with varying degrees of blur, the HB-filters work well enough for grayscale, binary, and colour images.

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