Mathematical Study Related to Centre of Mass in Modified Newtonian Dynamics

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Abstract
In this study we discussed the general two-body problem within the MOND radius in the gravitational field of MOND like galaxy and derived the expression for the total force between the system of test particles, with small masses and a point mass with a big mass as compare to them. Derived the results for the Centre of mass of acceleration for the isolated particles in purely Newtonian regime, for the isolated particles in the galactic neighbourhood of the Sun but in the influence of the gravitational field of mother system, MOND galaxy and for the non-isolated system which is in the gravitational field of large mass. The derived results always approach Newtonian dynamics in the regions of high accelerations. Emphasis is put on the alternative physics in the framework of MOND.

Key Words: MOND, Newtonian Regime, MOND Regime, Two-body Problem, Centre of Mass acceleration.

I. Introduction: The first successful mathematical theory of gravitation is evolved by English physicist Isaac Newton. Newton's idea of gravity was the result of a Mathematical language of forces and accelerations associated to the empirical observations of Kepler's laws. But it has long been known that the dynamics of the universe cannot be explained only by the Newton gravity force, researchers find many difficulties in describing the universe [1], [2]. When Newtonian laws are applied directly to these systems, it cannot explain the rotation curves of galaxies. Galactic rotation curves shows the unexpected Keplerian falloff. To solve this rotation curve problem of spiral galaxies there are two models one is Dark Matter (DM) and another is Modified Newtonian Dynamics (MOND).

MOND [3, 4, 5,] introduced to eliminate the inference that large quantities of invisible mass exist in galaxies. It is one of the first, and still one of the most successful paradigm which says that the mass discrepancies in galactic systems exist not due to dark matter but to a major departure from Newtonian dynamics and general relativity at low accelerations.

Milgrom in his research papers 1983a,b,c was proposed that if a certain modified form of nonrelativistic dynamics that is MOND is used to describe the motion of objects within galaxies and galaxy clusters then there is no need to assume the existence of large amount of unseen matter (Dark Matter) in these systems. According to him in the outskirts of galaxies the motion of stars can be well explained if the gravitational force is inversely proportional to the distance rather than inversely proportional to the square of the distance as in the Newtonian theory [Milgrom 83]. Milgrom finds that he can explain the flat rotation curves of galaxies and large virial velocities in clusters without adding Dark Matter.

According to MOND, gravity is modified beyond a gravitational acceleration of a_0 [3, 4, 5,]. Milgrom in MOND introduced a new constant with the dimensions of an acceleration, a_0, which defines the boundary between conventional dynamics and a new domain of dynamics, adopting a mean value of $a_0 = 2 \times 10^{-8} cms^{-2}$ same order as $CH_0 = 5 \times 10^{-8} cms^{-2}$($H_0$ being the Hubble constant and C velocity of light particle , $a_0 \sim CH_0 \sim C^2 \Lambda^{1/2} \sim C^2 l_U$ where Λ is the 'cosmological constant', and l_U is a cosmological characteristic length [11] .

In the deep-MOND limit (DML) universe there is a great deviation of MOND from the standard dynamics, where the accelerations is $a \ll a_0$ everywhere. In this limit neither a_0 nor G only $\mathcal{A}_0 = Ga_0$ appears. Occasionally it is useful to express DML findings in terms of MOND mass $M_m = \frac{C^4}{\mathcal{A}_0}$ [10].

The basic idea of modification of Newtonian dynamics is to do away with invisible mass by altering the second law that is by writing the equation of motion as

$$F = m\mu\left(\frac{a}{a_0}\right)a \tag{1.1}$$

instead of the standard Newtonian form $F = ma_N$, where $\mu\left(\frac{a}{a_0}\right)$ is an interpolating, monotonically increasing function of a, it takes two different values for two different regimes as follows,

$\mu\left(\frac{a}{a_0}\right) = 1$ for $a \gg a_0$ to restore Newtonian Dynamics in this limit, and $\mu\left(\frac{a}{a_0}\right) = \frac{a}{a_0}$ for $a \ll a_0$ to obtain flat rotation curves in the opposite limit.

Milgrom showed that this modification of the Newtonian dynamics, could account for the flatness of the rotation curves of spiral galaxies, with no need of introducing any extra unseen matter.

Our work is assembled as follows: In Section **II** we discussed the two body problem in the MOND gravitational field. **III** devoted to the study of Centre of mass acceleration for three different situations. **IV** includes discussions and conclusions.

II. The two-body problem:

Remarkably, the two-body problem turns out to be no more complicated, and indeed equivalent to, the single-body problem.

Equations of motion: Here we are going to discuss the general two-body problem within the DML(deep- MOND Limit) in the presence of gravitational field of some large mass like MOND Mass $M_m = \frac{C^4}{\mathcal{A}_0}$, where $\mathcal{A}_0 = Ga_0$. Let us consider a system of two particles with small masses m_1, m_2 at a large distance R from a large body with big mass M , within a MOND region. As $m_1, m_2 \ll M$ we can consider a point mass, with masses m_1, m_2 as test particles in the gravitational field $g_M(R)$ of M [8] which is given by,

$$g_M(R) = a_0 I^{-1}(MG/a_0 r^2) \tag{2.1}$$

Here I^{-1} is the inverse function of $I(x) = x\mu(x)$ [3].

Let F_{m_1}, F_{m_2} be the forces exerted by point mass M on test particles of masses m_1, m_2respectively.

Then
$$F_{m_1} = m_1 a_0 I^{-1}(MG/a_0 r^2) \tag{2.2}$$
and
$$F_{m_2} = m_1 a_0 I^{-1}(MG/a_0 r^2) \tag{2.3}$$

Hence the total force on the stated system is given by

$$F_{Tot} = F_{m_1} + F_{m_2} \qquad (2.4)$$

that is
$$F_{Tot} = (m_1+m_2)a_0 I^{-1}(MG/a_0 r^2) \qquad (2.5),$$
we derived this expression for the force between the system of test particles with small masses and a point mass with a big mass as compare to them.

III. Centre of Mass Motion:

Centre of mass is nothing but an imaginary point where you can think all the masses are concentrated.

Firstly let us consider $\wp_1$ and $\wp_2$ be two small planets with masses m_1 and m_2 be small enough in the field of Solar system which are isolated from other masses that is only internal forces are present here $\wp_1 - \wp_2$ dynamics follows Newtonian rules and Centre of Mass acceleration will be zero there. This can happen only in linear theory like Newtonian dynamics. Now in another case let us discuss the same $\wp_1 - \wp_2$ dynamics of isolated particles in the galactic neighbourhood of the Sun but this time this subsystem that is Solar system is falling in the gravitational field of mother system, MOND galaxy. These isolated test particles moves according to the MOND rules even if within it the relative accelerations are large enough [6]. In MOND we cannot treat the dynamics of $\wp_1 - \wp_2$ system ignoring the presence of external gravitational field in which system lie, as it happens in the Newtonian dynamics.

Let the position vectors of $\wp_1$ and $\wp_2$ are r_1 and r_2 respectively then the position of centre of mass of $\wp_1 - \wp_2$ system is,

$$r_{CM} \cong \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \qquad (3.1)$$

The acceleration a_{m_1} produced by m_2 alone at the position of m_1 is given by

$$a_{m_1} = m_2^{1/2}(\mathcal{A}_0)^{1/2} r^{-1} \qquad (3.2)$$

and the acceleration a_{m_2} produced by m_1 alone at the position of m_2 is

$$a_{m_2} = -m_1^{1/2}(\mathcal{A}_0)^{1/2} r^{-1} \qquad (3.3)$$

where r the is distance between two test particles.

After differentiating equation (3.1) and using equations (3.2) and (3.3), we get the acceleration of the centre of mass as,

$$a_{CM} = \frac{m_1 m_2^{\frac{1}{2}}(\mathcal{A}_0)^{\frac{1}{2}} r^{-1} - m_2 m_1^{1/2}(\mathcal{A}_0)^{1/2} r^{-1}}{m_1 + m_2}$$

$$a_{CM} = \frac{(\mathcal{A}_0)^{\frac{1}{2}} r^{-1}\left(m_1 m_2^{\frac{1}{2}} - m_2 m_1^{\frac{1}{2}}\right)}{m_1 + m_2} \neq 0\,, m_1 \neq m_2 \qquad (3.4)$$

As in Mondian the two accelerations are not inversely proportional to the masses as in Newtonian dynamics, Newton's third law of motion does not hold henceforth the acceleration of Centre of mass does not vanish as in case of Newton except for the special case $m_1 \approx m_2$ [7].

Consider a third simple case of non-isolated system $\wp_1 - \wp_2$ consisting of two particles with small masses m_1 and m_2 respectively, separated by distance r being placed in the gravitational field of some large mass like MOND Mass $M_m = \frac{C^4}{\mathcal{A}_0}$. System $\wp_1 - \wp_2$ is at large distance R from MOND galaxy. Due to the non-linearity of MOND, which consequently follows from the basic premises of the paradigm, the gravitational field of MOND galaxy affects so much the internal, $\wp_1 - \wp_2$ dynamics. Henceforth the given system with internal accelerations that are Newtonian still behave as MOND in a MONDian external field [9] In above discussed case the system was isolated but now the same system is non-isolated and under the influence of large gravitational field therefore there arises two extra forces F_{1ext} the external force on test particle $\wp_1$ and F_{2ext} the external force on $\wp_2$.

$$m_1 a_{m_1}(m_1/M_m) = F_1 = F_{12} + F_{1ext} \tag{3.5}$$

and

$$m_2 a_{m_2}(m_2/M_m) = F_2 = F_{21} + F_{2ext} \tag{3.6}$$

where F_{12}, F_{21} are the forces by particle $\wp_1$on particle $\wp_2$ and particle $\wp_2$ on particle $\wp_1$ respectively. F_1 and F_2 are total forces on $\wp_1$ and $\wp_1$ respectively $a_{m_1}(m_1/M_m)$ and $a_{m_2}(m_2/M_m)$ are the accelerations produced by m_2 on m_1 and m_1 on m_2 respectively under the gravitational field of mass M_m .

Adding equations (3.5) and (3.6) we get the combined effect of the motion of these two test particles is equal to all the forces.

$$m_1 a_{m_1}(m_1/M_m) + m_2 a_{m_2}(m_2/M_m) = F_{12} + F_{21} + F_{1ext} + F_{2ext} = F_{Tot} \tag{3.7}$$

Let the total mass of the two particle system is $M = m_1 + m_2$, The total force F_{Tot} acting on mass M produces Centre of mass acceleration a_{cm} .

Consequently

$$Ma_{cm} = m_1 a_{m_1}(m_1/M_m) + m_2 a_{m_2}(m_2/M_m) \tag{3.8}$$

Implies

$$a_{cm} = \frac{1}{M}[m_1 a_{m_1}(m_1/M_m) + m_2 a_{m_2}(m_2/M_m)] \tag{3.9}$$

$$a_{CM} = \frac{1}{m_1+m_2}\left[(\mathcal{A}_0)^{\frac{1}{2}}\ r^{-1}\left(m_1 m_2^{\frac{1}{2}} - m_2 m_1^{\frac{1}{2}}\right) + h(m_1/M_m)(m_1 + M_m)^{\frac{3}{2}} R^{-1}(\mathcal{A}_0)^{\frac{1}{2}} + h(m_2/M_m)(m_2 + M_m)^{\frac{3}{2}} R^{-1}(\mathcal{A}_0)^{\frac{1}{2}}\right] \tag{3.10}$$

where h is some characteristic length scale in the galaxy as per the scaling law discussed in [4]. In this expression for external forces we taken into consideration Milgrom 1986[8].

By observing above equation we can say that in MOND region Centre of mass of acceleration of system which is not isolated from other masses, includes both external and internal forces.

Conclusion: MOND aims not only to explain the rotation curve of spirals, but to propose an alternative theory of Gravitation and/or Dynamics. Derived expression for the total force between the system of test particles with small masses and a point mass with a big mass as compare to them, is so different from Newtonian. Expressions for Centre of mass of acceleration for three different regimes are non-identical. It is observed that in purely Newtonian regime COM acceleration of isolated test particles is zero but in the gravitational field of MOND galaxy , COM acceleration of isolated test particles is not zero. It means that in MOND regime one cannot neglect the presence of MOND gravitational field even though the system is isolated. In the case of non-isolated system of particles in the MOND regime the centre of mass of acceleration is equal to the sum of internal and external forces exerted by external field on test particles.

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