MBJ-Neutrosophic Implicative LI-Ideals in Lattice Implication Algebras

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ABSTRACT
We consider the idea of MBJ-Neutrosophic implicative LI ideals and by establishing the characterizations of a MBJ-Neutrosophic implicative LI-Ideal we depict the rules how a MBJ-Neutrosophic LI ideal acts as a MBJ-Neutrosophic implicative LI ideal and we are able to derive the relation between them.

Keywords: Lattice implication algebra (LIA); MBJ-Neutrosophic L I (MBJ-NLI) ideal; MBJ-Neutrosophic implicative LI (MBJ-NILI) ideal.

Introduction

"The idea of Neutrosophic set (Nset) developed by Smarandache in [3], which encompasses the standard set (intuitionistic) fuzzy set, and interval-valued fuzzy (intuitionistic) set". The N set was defined after the neutrosophic elements T, I, F were innovate and indicates the truth-membership, indeterminancy, and false-membership values individually, where [0, 1] is the non-standard unit interval.

"Jun, his colleagues examined, properties and relations of BCK/BCI-algebras using the idea of Neutrosophic set theory" in [2]. "The interval Neutrosophic set was then inaugurate" in [3]. Furthermore, "Interval Neutrosophic ideals were interpreted and several features examined" in [4]. In [5], they potrayed many types of interval Neutrosophic ideals, analysed some of their characteristics, and discovered a relationship between them.

"Jun also proposed the concepts of LI-ideals, in [6], and analysed few of their features as well as the relationship between them". And the concept of neutrosophic set theory was used to LIA.

The abstraction of MBJ set is innovate as another generality of N set, MBJ-NLI ideal and MBJ-NILI ideal of a LIA are introduced, and numerous properties are examined. We talk about how an MBJ-NLI ideal and a MBJ-NILI ideal are related. An MBJ-NLI ideal becomes an MBJ-NILI ideal if certain requirements are met. And also the charaterizations of a MBJ-NILI ideal have discussed.

Preliminaries

Definition-2.1. ([8]). Lattice Implication Algebra (LIA) is described as a Lattice (L, V, Λ, c , 0, 1) with bounds (0 , 1), order-reversing involution " c " comforting propositions given below under a binary operation " → " are

(I1) $\varsigma \to (\varrho \to \varphi) = \varrho \to (\varsigma \to \varphi)$,

(I2) $\varsigma \to \varsigma = 1$,

(I3) $\varsigma \to \varrho = \varrho c \to \varsigma c$,

(I4) $\varsigma \to \varrho = \varrho \to \varsigma = 1 \Rightarrow \varsigma = \varrho$,

(I5) $(\varsigma \to \varrho) \to \varrho = (\varrho \to \varsigma) \to \varsigma$,

(L1) $(\varsigma \vee \varrho) \to \varphi = (\varsigma \to \varphi) \wedge (\varrho \to \varphi)$,

(L2) $(\varsigma \wedge \varrho) \to \varphi = (\varsigma \to \varphi) \vee (\varrho \to \varphi)$,

For all $\varsigma, \varrho, \varphi \in L$ is Lattice H-Implication Algebra, if a LIA 'L' satisfies the propositon

$(\forall \varsigma, \varrho, \varphi \in L)\ (\varsigma \vee \varrho \vee ((\varsigma \wedge \varrho) \to \varphi) = 1)$ (1)

A partial ordering ≤ on 'L' defined by the condition $\varsigma \leq \varrho$ iff $\varsigma \to \varrho = 1$.

A LIA 'L' satisfies the propositions cited below (see [12]):

(a1) $0 \to \varsigma = 1, 1 \to \varsigma = \varsigma$ & $\varsigma \to 1 = 1$.

(a2) $\varsigma \to \varrho \leq (\varrho \to \varphi) \to (\varsigma \to \varrho)$.

(a3) $\varsigma \leq \varrho$ implies $\varrho \to \varphi \leq \varsigma \to \varphi$ and $\varphi \to \varsigma \leq \varphi \to \varrho$.

(a4) $\varsigma c = \varsigma \to 0$.

(a5) $\varsigma \vee \varrho = (\varsigma \to \varrho) \to \varrho$.

(a6) $((\varrho \to \varsigma) \to \varsigma c)c = \varsigma \wedge \varrho = ((\varsigma \to \varrho) \to \varsigma c)c$.

(a7) $\varsigma \leq (\varsigma \to \varrho) \to \varrho$.

(a8) $((\varsigma \to \varrho) \to \varrho) \to \varrho = \varsigma \to \varrho$

Definition 2.2. ([6]). A LI-ideal of 'L'is a nonempty subset χ of 'L' holds the propositions given by

(L3) $0 \in \chi$.

(L4) $(\forall \varsigma \in$ 'L'$)\ (\forall \varrho \in \chi)\ ((\varsigma \to \varrho)c \in \chi \Rightarrow \varsigma \in \chi)$.

Lemma 2.3. ([7]). Any LI-ideal χ of 'L' accomplish the following rule

$(\forall \varsigma \in \chi)\ (\forall \varrho \in L)\ (\varrho \leq \varsigma \Rightarrow \varrho \in \chi)$.

Let L be a non-empty set. A MBJ-Nset in 'L' ([11]) is structurized as:

$A = \{\varsigma ; MA(\varsigma), \tilde{B}A(\varsigma), JA(\varsigma) / \varsigma \in L\}$,

where a truth membership function defined by $MA : L \to [0, 1]$, be an indeterminate membership function defined by $\tilde{B}A : L \to [0, 1]$, and a false membership function defined by $JA : L \to [0, 1]$ For a convenience, we use the symbol $A = (MA, \tilde{B}A, JA)$ to representthe MBJ-N set A .

i.e, $A = \{(\varsigma ; MA(\varsigma), \tilde{B}A(\varsigma), JA(\varsigma)) / \varsigma \in L\}$.

Given a MBJ-N set $A = (MA, \tilde{B}A, JA)$ in a LIA 'L'. Then we prove the succeeding sets

$L(MA ; \alpha) := \{\varsigma \in L / MA(\varsigma) \geq \alpha\}$,

$L(\tilde{B}A ; \beta) := \{\varsigma \in L / \tilde{B}A(\varsigma) \geq \beta\}$,

L(JA ; γ) := {ς ∈ L / JA (ς) ≤ γ}.
Known as MBJ-Neutrosophic level subsets of 'L'.

MBJ-Neutrosophic LI-ideals:
We suppose 'L' as LIA except as otherwise indicated.

Definition 3.1. AMBJ-NLI-ideal of 'L' is an MBJ-Nset
A = (MA, B̃A, JA) in 'L' if the following attributes are true.
(∀ ς ∈ L) (MA (0) ≥ MA(ς), B̃A(0) ≥ B̃A(ς), JA (0) ≤ JA (ς)) (2)

$$\text{and } (\forall\, \varsigma, \varrho \in L) \begin{pmatrix} M_A(\varsigma) \geq \min\{M_A((\varsigma \to \varrho)^c), M_A(\varrho)\} \\ \tilde{B}_A(\varsigma) \geq \operatorname{rmin}\{\tilde{B}_A((\varsigma \to \varrho)^c), \tilde{B}_A(\varrho)\} \\ J_A(\varsigma) \leq \max\{J_A((\varsigma \to \varrho)^c), J_A(\varrho)\} \end{pmatrix} \quad (3)$$

Theorem3.2. A MBJ-Nset A = (MA, B̃A, JA) of 'L'is a MBJ-Neutrosophic LI-ideal iff it accomplishes the rule

$$\forall \varsigma, \varrho, \varphi \in L,\ (\varphi \to \varsigma)c \leq \varrho \Rightarrow \begin{pmatrix} M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho)\} \\ \tilde{B}_A(\varphi) \geq \operatorname{rmin}\{\tilde{B}_A(\varsigma), \tilde{B}_A(\varrho)\} \\ J_A(\varphi) \leq \max\{J_A(\varsigma), J_A(\varrho)\} \end{pmatrix} \quad (4)$$

Proof: Consider 'A' as a MBJ-NLI Ideal of 'L'.
Let ς, ϱ, φ ∈ L be ∋ (φ → ς)c ≤ ϱ.
Since every MBJ-NLI ideal is order reversing it ensures from (2) that
MA(φ) ≥ min{MA((φ → ς)c), MA(ς)}≥ min{MA(ς), MA(ϱ)}
B̃A(φ) ≥ r min{B̃A((φ → ς)c), B̃A(ς)}≥ r min{B̃A(ς), B̃A(ϱ)}
JA(φ) ≤ max{JA((φ → ς)c), JA(ς)}≤ max {JA(ς), JA(ϱ)}
Reversely, take that'A' holds the given condition. As (0 → ς)c ≤ ς , ∀ ς ∈ 'L', we have
(MA (0) ≥ MA(ς), B̃A(0) ≥ B̃A(ς), JA (0) ≤ JA (ς))
Note that (ς→ ϱ)c ≤ ς → ϱ)c for all ς, ϱ ∈ L. Hence, by assumption, we get
MA(ς) ≥ min{ MA(ϱ), MA(ς → ϱ)c)}
B̃A(ς) ≥ r min{ B̃A(ϱ), B̃A(ς → ϱ)c)}
JA(ς) ≤ max{ JA(ϱ), JA(ς → ϱ)c)}
Therefore A is an MBJ-Neutrosophic LI-ideal of 'L'.

Definition 3.3. An implicative LI-Ideal of 'L' is a nonempty subset A of 'L' whic satisfies 0∈L & ∀ς, ϱ, φ ∈ L, (((ς → ϱ)c →ϱ)c → φ)c ∈ A, φ ∈ A ⇒ (ς → ϱ)c ∈ A.

Proposition 3.4. Let I1 be an LI-Ideal of 'L'. Then the implicative rules given below are equivalent.
(1) I1 is an NILI ideal of 'L'.
(2) ∀ς, ϱ ∈ L, (ς→ (ϱ →ς)c)c ∈ I1 ⇒ ς∈ I1.
(3) ∀ς,ϱ∈ L, ((ς → ϱ)c → ϱ)c ∈ I1 ⇒ (ς → ϱ)c ∈ I1.

Definition 3.5. An MBJ-Nset 'A' of 'L' is known as an MBJ-NILI ideal of 'L' if it holds the inequalities given below for all $\varsigma \in L$, we have
from (1) ($MA(0) \geq MA(\varsigma)$, $\tilde{B}A(0) \geq \tilde{B}A(\varsigma)$, $JA(0) \leq JA(\varsigma)$), and $\forall \varsigma, \varrho, \varphi \in L$ &

$$\text{(L5)}\quad MA((\varsigma \to \varrho)c) \geq \min\{MA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), MA(\varphi)\},$$
$$\tilde{B}A((\varsigma \to \varrho)c) \geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), \tilde{B}A(\varphi)\}, \tag{5}$$
$$JA((\varsigma \to \varrho)c) \leq \max\{JA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), JA(\varphi)\}.$$

Example 3.6. Let the Cayley table of a lattice $L = \{0, a, b, 1\}$ as follows:
For all $\varsigma \in L$

ς	ςc
0	1
a	b
b	a
1	0

The operators V and Λ on a Lattice 'L' are defined as given below:

$$\varsigma V \varrho = (\varsigma \to \varrho) \to \varrho,\ \varsigma \Lambda \varrho = ((\varsigma c \to \varrho c) \to \varrho c)c \tag{6}$$

for all $\varsigma, \varrho \in L$. Then $(L, V, \Lambda, \to, c)$ is a LIA. Let 'A' be an MBJ-Nset in 'L' defined by $MA(0) = 0.9$ and $MA(a) = MA(b) = MA(1) = 0.4$. Now clearly, A is an MBJ-NILI ideal of 'L'. Also an MBJ-N set B in 'L' taken as $MB(0) = MB(a) = 0.7$ and $MB(b) = MB(1) = 0.3$ is an MBJ-NILI ideal of 'L'. (Conditions for $\tilde{B}A$ & JA, $\tilde{B}B$ & JB, also be checked).
How an MBJ-NLI ideal and MBJ-NILI ideal are related to each other can be shown in the following theorem.
Theorem 3.7. Every MBJ-NILI ideal of 'L' is an MBJ-NLI ideal of 'L'.
Proof : Say, ' A' be a MBJ-NILI ideal of 'L', put $\varrho = 0$ & $\varphi = \varrho$ in (5)
Then we get

$$MA(\varsigma) = MA((\varsigma \to 0)c)$$
$$\geq \min\{ MA((((\varsigma \to 0)c \to 0)c \to \varrho)c),\ MA(\varrho)\}$$
$$= \min\{ MA((\varsigma \to \varrho)c),\ MA(\varrho)\}.$$
$$\tilde{B}A(\varsigma) = \tilde{B}A((\varsigma \to 0)c)$$
$$\geq r\min\{ \tilde{B}A((((\varsigma \to 0)c \to 0)c \to \varrho)c),\ \tilde{B}A(\varrho)\}$$
$$= r\min\{ \tilde{B}A((\varsigma \to \varrho)c), \tilde{B}A(\varrho)\}.$$
$$JA(\varsigma) = JA((\varsigma \to 0)c)$$
$$\leq \max\{ JA((((\varsigma \to 0)c \to 0)c \to \varrho)c),\ JA(\varrho)\}$$
$$= \max\{ JA((\varsigma \to \varrho)c), JA(\varrho)\}.$$

Therefore 'A' is an MBJ-NLI ideal of 'L'.
The reversing of Theorem 3.7 attain falsity, that can be observed in the example given below.

Example 3.8. Let the Cayley table of a lattice $L = \{0, a, b, 1\}$ as follows:

For all $\varsigma \in L$

ς	ςc
0	1
a	b
b	a
1	0

The operators V and Λ on a Lattice 'L' are defined as given below:
$\varsigma \vee \varrho = (\varsigma \to \varrho) \to \varrho$, $\varsigma \wedge \varrho = ((\varsigma c \to \varrho c) \to \varrho c)c$
for all $\varsigma, \varrho \in L$. Then $(L, \vee, \wedge, \to, c)$ is a LIA . Suppose A be an MBJ-N set in 'L' defined by MA(0) = 0.7 and MA(a) = MA(b) = MA(1)=0.4. And so A is a MBJ-NLI ideal of 'L'. But it is not an MBJ-NILI ideal of 'L' since
$MA((b \to a)c) \ngeq \min\{MA((((b \to a)c \to a)c \to 0)c), MA(0)\}$.
Under what condition it is true we will see in the following theorem.
Theorem 3.9. Every MBJ-NLI ideal is an MBJ-NILI ideal under the condition of choosing 'L' as a lattice H-implication algebra.
Proof. Suppose that A be an MBJ-NLI ideal of a lattice H-Implication Algebra 'L'.
we have by using (I3), (L5) and (3)

$$MA((\varsigma \to \varrho)c) = MA((\varrho c \to \varsigma c)c) = MA((\varrho c \to ((\varrho c \to \varsigma c))c)$$
$$= MA((\varrho c \to (\varsigma \to \varrho))c) = MA(((\varsigma \to \varrho)c \to \varrho)c)$$
$$\geq \min\{MA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), MA(\varphi)\}$$
$$\tilde{B}A((\varsigma \to \varrho)c) = \tilde{B}A((\varrho c \to \varsigma c)c) = \tilde{B}A((\varrho c \to ((\varrho c \to \varsigma c))c)$$
$$= \tilde{B}A((\varrho c \to (\varsigma \to \varrho))c) = \tilde{B}A(((\varsigma \to \varrho)c \to \varrho)c)$$
$$\geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), \tilde{B}A(\varphi)\}$$
$$JA((\varsigma \to \varrho)c) = JA((\varrho c \to \varsigma c)c) = JA((\varrho c \to ((\varrho c \to \varsigma c))c)$$
$$= JA((\varrho c \to (\varsigma \to \varrho))c) = JA(((\varsigma \to \varrho)c \to \varrho)c)$$
$$\leq \max\{JA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), JA(\varphi)\}$$

for all $\varsigma, \varrho, \varphi \in L$, which proves (L5). Then clearly 'A' is a MBJ-Neutrosophic implicative LI-Ideal of 'L'.
Proposition 3.10. Let 'A' be a MBJ-Neutrosophic set in 'L' satisfying (L5) and
(L6) $MA(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \geq \min\{MA((((\varsigma \to \varrho)c \to \varphi)c \to w)c), MA(w)\}$
$\tilde{B}A(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varphi)c \to w)c), \tilde{B}A(w)\}$
$JA(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \leq \max\{JA((((\varsigma \to \varrho)c \to \varphi)c \to w)c), JA(w)\}$
for all $\varsigma, \varrho, \varphi, w \in L$. Then A is an MBJ-NILI ideal of 'L'.
Proof: For any $\varsigma, \varrho, \varphi \in L$, we have

$MA((\varsigma \to \varrho)) = MA((1 \to (\varsigma \to \varrho))c)$ by (a1)
$= MA((0c \to (\varsigma \to \varrho))c)$ by (I2) and (a4)
$= MA(((\varsigma \to \varrho)c \to 0)c)$ by (I3)
$= MA(((\varsigma \to \varrho)c \to 1c)c)$ by (a1) and (a4)
$= MA(((\varsigma \to \varrho)c \to (\varrho \to \varrho)))$ by (I2)
$\geq \min\{MA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), MA(\varphi)\}$ by (a4)

$\tilde{B}A((\varsigma \to \varrho)) = \tilde{B}A((1 \to (\varsigma \to \varrho))c)$ by (a1)
$= \tilde{B}A((0c \to (\varsigma \to \varrho))c)$ by (I2) and (a4)
$= \tilde{B}A(((\varsigma \to \varrho)c \to 0)c)$ by (I3)
$= \tilde{B}A(((\varsigma \to \varrho)c \to 1c)c)$ by (a1) and (a4)
$= \tilde{B}A(((\varsigma \to \varrho)c \to (\varrho \to \varrho)))$ by (I2)
$\geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), \tilde{B}A(\varphi)\}$ by (a4)
$JA((\varsigma \to \varrho)) = JA((1 \to (\varsigma \to \varrho))c)$ by (a1)
$= JA((0c \to (\varsigma \to \varrho))c)$ by (I2) and (a4)
$= JA(((\varsigma \to \varrho)c \to 0)c)$ by (I3)
$= JA(((\varsigma \to \varrho)c \to 1c)c)$ by (a1) and (a4)
$= JA(((\varsigma \to \varrho)c \to (\varrho \to \varrho)))$ by (I2)
$\leq \max\{JA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), JA(\varphi)\}$ by (a4)
That concludes (L5). Then clearly A is a MBJ-NILI ideal of ‘L’.

Proposition 3.11. Suppose, A be an MBJ-NLI ideal of L holds

(L7) $MA(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \geq MA(((\varsigma \to \varrho)c \to \varphi)c)$
$\tilde{B}A(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \geq \tilde{B}A(((\varsigma \to \varrho)c \to \varphi)c)$
$JA(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \leq JA(((\varsigma \to \varrho)c \to \varphi)c)$

for all $\varsigma, \varrho, \varphi \in L$, It is clear that ‘A’ is an MBJ-NILI ideal of ‘L’.

Proof: Let $\varsigma, \varrho, \varphi, w \in L$, Using (a5) and (a2), we have
$MA(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \geq MA(((\varsigma \to \varrho)c \to \varphi)c)$
$\geq \min\{MA((((\varsigma \to \varrho)c \to \varphi)c \to w)c), MA(w)\}$.
$\tilde{B}A(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \geq \tilde{B}A(((\varsigma \to \varrho)c \to \varphi)c)$
$\geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varphi)c \to u)c), \tilde{B}A(w)\}$.
$JA(((\varsigma \to \varphi)c \to (\varrho \to \varphi)c)c) \leq JA(((\varsigma \to \varrho)c \to \varphi)c)$
$\leq \max\{JA((((\varsigma \to \varrho)c \to \varphi)c \to w)c), JA(w)\}$.
Then it is clear that ‘A’ is an MBJ-NILI ideal of ‘L’ by the Proposition 3.10 .

Proposition 3.12. Let A be a MBJ-NLI ideal of ‘L’ satisfying

(L8) $MA((\varsigma \to \varrho)c) \geq MA(((\varsigma \to \varrho)c \to \varrho)c)$
$\tilde{B}A((\varsigma \to \varrho)c) \geq \tilde{B}A(((\varsigma \to \varrho)c \to \varrho)c)$
$JA((\varsigma \to \varrho)c) \leq JA(((\varsigma \to \varrho)c \to \varrho)c)$

for all $\varsigma, \varrho \in L$. Then A is an MBJ-NILI ideal of ‘L’.

Proof. Let $\varsigma, \varrho, \varphi \in L$, we get
$MA((\varsigma \to \varrho)c) \geq MA(((\varsigma \to \varrho)c \to \varrho)c)$
$\geq \min\{MA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), MA(\varphi)\}$, by (L5)
$\tilde{B}A((\varsigma \to \varrho)c) \geq \tilde{B}A(((\varsigma \to \varrho)c \to \varrho)c)$
$\geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), \tilde{B}A(\varphi)\}$, by (L5)
$JA((\varsigma \to \varrho)c) \leq JA(((\varsigma \to \varrho)c \to \varrho)c)$
$\leq \max\{JA((((\varsigma \to \varrho)c \to \varrho)c \to \varphi)c), JA(\varphi)\}$, by (L5)
Clearly ‘A’ is an MBJ-NILI ideal of ‘L’.

Proposition 3.13. All MBJ-NILI ideals of ‘L’ satisfies the implication rule (L7).

Proof. Choose any $\varsigma, \varrho \in L$, let A be MBJ-NILI ideal then we get

$MA((\varsigma \to \varrho)c) \geq \min\{MA((((\varsigma \to \varrho)c \to \varrho)c \to 0)c), MA(0)\}$
$= MA((((\varsigma \to \varrho)c \to \varrho)c \to 0)c)$
$= MA((0c \to (\varsigma \to \varrho)c \to \varrho)c)$
$= MA((\varsigma \to \varrho)c \to \varrho)c)$
$\tilde{B}A((\varsigma \to \varrho)c) \geq r\min\{\tilde{B}A((((\varsigma \to \varrho)c \to \varrho)c \to 0)c), \tilde{B}A(0)\}$
$= \tilde{B}A((((\varsigma \to \varrho)c \to \varrho)c \to 0)c)$
$= \tilde{B}A((0c \to (\varsigma \to \varrho)c \to \varrho)c)$
$= \tilde{B}A((\varsigma \to \varrho)c \to \varrho)c)$

$JA((\varsigma \to \varrho)c) \leq \max\{JA((((\varsigma \to \varrho)c \to \varrho)c \to 0)c), JA(0)\}$
$= JA((((\varsigma \to \varrho)c \to \varrho)c \to 0)c)$
$= JA((0c \to (\varsigma \to \varrho)c \to \varrho)c)$
$= JA((\varsigma \to \varrho)c \to \varrho)c)$
we give a depiction of an MBJ-NILI ideal by uniting the above Propositions.

Theorem 3.14. Suppose A be an MBJ-NLI ideal of 'L' then we observe the equivalence of the statements as shown below
A is a MBJ-NILI ideal of 'L'.
A holds (L6).
A holds (L7).
A holds (L8).
Theorem 3.15. If A is a MBJ-NLI ideal of 'L' then we observe the equivalence of the following statements.
A is a MBJ-NILI ideal of 'L'.
$MA(\varsigma) \geq MA((\varsigma \to (\varrho \to \varsigma)c)c), \forall \varsigma, \varrho \in L.$
$\tilde{B}A(\varsigma) \geq \tilde{B}A((\varsigma \to (\varrho \to \varsigma)c)c), \forall \varsigma, \varrho \in L.$ (7)
$JA(\varsigma) \leq JA((\varsigma \to (\varrho \to \varsigma)c)c), \forall \varsigma, \varrho \in L.$
iii. $MA(\varsigma) \geq \min\{MA(((\varsigma \to (\varrho \to \varsigma)c)c \to \varphi)c), MA(\varphi)\}, \forall \varsigma, \varrho, \varphi \in L.$
$\tilde{B}A(\varsigma) \geq r\min\{\tilde{B}A(((\varsigma \to (\varrho \to \varsigma)c)c \to \varphi)c), \tilde{B}A(\varphi)\}, \forall \varsigma, \varrho, \varphi \in L.$
$JA(\varsigma) \leq \max\{JA(((\varsigma \to (\varrho \to \varsigma)c)c \to \varphi)c), JA(\varphi)\}, \forall \varsigma, \varrho, \varphi \in L.$
Proof. (i) $\Rightarrow$ (ii).
Choose A is a MBJ-NILI ideal of 'L' and let $\varsigma, \varrho, \varphi \in L$.
Note that
$((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)c \to (\varsigma \to (\varrho \to \varsigma)c)c$

$=$	$(\varsigma \to (\varrho \to \varsigma)c) \to ((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)$	by (I3)
$\geq$	$(\varrho \to (\varrho \to \varsigma)c)c \to \varsigma$	by (a2)
$=$	$\varsigma c \to (\varrho \to (\varrho \to \varsigma)c)$	by (I3)
$=$	$\varsigma c \to ((\varrho \to \varsigma) \to \varrho c)$	by (I3)
$=$	$(\varrho \to \varsigma) \to (\varsigma c \to \varrho c)$	by (I1)
$=$	$(\varrho \to \varsigma) \to (\varrho \to \varsigma) = 1.$	by (I2) and (I3)

as $\varsigma \leq 1$ for all $\varsigma \in L$, from (I4) we get that
$((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)c \to (\varsigma \to (\varrho \to \varsigma)c)c = 1,$
i.e., $((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)c \to (\varsigma \to (\varrho \to \varsigma)c)c.$

Since every MBJ-NLI ideal is order reversing, we have

$MA((\varsigma \to (\varrho \to \varsigma)c)c) \geq MA(((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)c)$

$\geq MA((\varrho \to (\varrho \to \varsigma)c)c)$ by Proposition 3.13

$\tilde{B}A((\varsigma \to (\varrho \to \varsigma)c)c) \geq \tilde{B}A(((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)c)$

$\geq \tilde{B}A((\varrho \to (\varrho \to \varsigma)c)c)$ by Proposition 3.13

$JA((\varsigma \to (\varrho \to \varsigma)c)c) \leq JA(((\varrho \to (\varrho \to \varsigma)c)c \to (\varrho \to \varsigma)c)c)$

$\leq JA((\varrho \to (\varrho \to \varsigma)c)c)$ by Proposition 3.13

Note that

$(\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c = \varsigma \to ((\varrho \to \varsigma) \to \varrho c)c)c$ by (I3)

$= (\varsigma \to ((\varsigma c \to \varrho c) \to \varrho c)c)c$ by (I3)

$= (((\varsigma c \to \varrho c) \to \varrho c) \to \varsigma c)c$ by (I3)

$= (((\varrho c \to \varsigma c) \to \varsigma c) \to \varsigma c)c$ by (I5)

$= (\varrho c \to \varsigma c)c$ by (a8)

$= (\varsigma \to \varrho)c$ by (I3)

and

$(\varsigma \to \varrho)c \to (\varsigma \to (\varrho \to \varsigma)c)c = (\varsigma \to (\varrho \to \varsigma)c) \to (\varsigma \to \varrho)$ by (I3)

$\geq (\varrho \to \varsigma)c \to \varrho$ by (I1) and (a2)

$= \varrho c \to (\varrho \to \varsigma)$ by (I3)

$= (\varrho \to 0) \to (\varrho \to \varsigma)$ by (a4)

$\geq 0 \to \varsigma = 1.$ by (I1), a2) and (a1)

Therefore $(\varsigma \to \varrho)c \to (\varsigma \to (\varrho \to \varsigma)c)c = 1$, i.e., $(\varsigma \to \varrho)c \leq (\varsigma \to (\varrho \to \varsigma)c)c$, and so

$MA((\varsigma \to (\varrho \to \varsigma)c)c) \geq MA((\varsigma \to \varrho)c)$

$= MA((\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c)$.

It follows from(L5) that

$MA(\varsigma) \geq \min\{MA((\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c), MA((\varrho \to (\varrho \to \varsigma)c)c)\}$

$\geq MA((\varrho \to (\varrho \to \varsigma)c)c)$,

$\tilde{B}A((\varsigma \to (\varrho \to \varsigma)c)c) \geq \tilde{B}A((\varsigma \to \varrho)c)$

$= \tilde{B}A((\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c)$.

It follows from(L5)that

$\tilde{B}A(\varsigma) \geq r\min\{\tilde{B}A((\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c), \tilde{B}A((\varrho \to (\varrho \to \varsigma)c)c)\}$

$\geq \tilde{B}A((\varrho \to (\varrho \to \varsigma)c)c)$,

$JA((\varsigma \to (\varrho \to \varsigma)c)c) \leq JA((\varsigma \to \varrho)c)$

$= JA((\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c)$.

It follows from (L5) that

$JA(\varsigma) \leq \max\{JA((\varsigma \to (\varrho \to (\varrho \to \varsigma)c)c)c), JA((\varrho \to (\varrho \to \varsigma)c)c)\}$

$\leq JA(((\varsigma \to (\varrho \to \varsigma)c)c)$,

which proves (ii).

(ii) $\Rightarrow$ (iii). It follows by (L5)

(iii) $\Rightarrow$ (i). suppose that A follows the condition (iii) and choose any $\varsigma, \varrho \in L$. put $\varphi = 0$ in

(iii) and using (L5) and (a4), we get

$MA(\varsigma) \geq \min\{MA(((\varsigma \to (\varrho \to \varsigma)c)c \to 0)c), MA(0)\}$

$= MA(((\varsigma \to (\varrho \to \varsigma)c)c \to 0)c)$

$= \quad MA ((((\varsigma \to (\varrho \to \varsigma)c)c)c)c)$
$= \quad ((\varsigma \to (\varrho \to \varsigma)c)c).$
$\tilde{B}A(\varsigma) \quad \geq \quad r\min\{\tilde{B}A (((\varsigma \to (\varrho \to \varsigma)c)c \to 0)c), \tilde{B}A(0)\}$

$= \quad \tilde{B}A (((\varsigma \to (\varrho \to \varsigma)c)c \to 0)c)$
$= \quad \tilde{B}A ((((\varsigma \to (\varrho \to \varsigma)c)c)c)c)$
$= \quad ((\varsigma \to (\varrho \to \varsigma)c)c).$
$JA(\varsigma) \quad \leq \quad \max\{JA (((\varsigma \to (\varrho \to \varsigma)c)c \to 0)c), JA(0)\}$
$= \quad JA (((\varsigma \to (\varrho \to \varsigma)c)c \to 0)c)$
$= \quad JA ((((\varsigma \to (\varrho \to \varsigma)c)c)c)c)$
$= \quad ((\varsigma \to (\varrho \to \varsigma)c)c).$
Note that $((\varsigma \to \varrho)c \to (\varsigma \to (\varsigma \to \varrho)c)c)c$
$= \quad ((\varsigma \to (\varsigma \to \varrho)c) \to (\varsigma \to \varrho))c$ by (I3)
$= \quad (((\varsigma \to \varrho) \to \varsigma c) \to (\varrho c \to \varsigma c))c$ by (I3)
$= \quad (\varrho c \to (((\varsigma \to \varrho) \to \varsigma c) \to \varsigma c))c$ by (I1)
$= \quad (\varrho c \to ((\varsigma c \to (\varsigma \to \varrho)) \to (\varsigma \to \varrho)))c$ by (I5)
$= \quad (\varrho c \to ((\varsigma c \to (\varrho c \to \varsigma c)) \to (\varrho c \to \varsigma c)))c$ by (I3)
$= \quad ((\varsigma c \to (\varrho c \to \varsigma c)) \to (\varrho c \to (\varrho c \to \varsigma c)))c$ by (I1)
$= \quad ((\varrho c \to (\varsigma c \to \varsigma c)) \to (\varrho c \to (\varrho c \to \varsigma c)))c$ by (I1)
$= \quad ((\varrho c \to 1) \to (\varrho \to (\varrho c \to \varsigma c)))c$ by (I2)
$= \quad (\varrho c \to (\varrho c \to \varsigma c))c$ by (a1)
$= \quad ((\varsigma \to \varrho)c \to \varrho)c.$ by (I3)
Using (7), we have
$MA((\varsigma \to \varrho)c) \geq MA (((\varsigma \to \varrho)c \to (\varsigma \to (\varsigma \to \varrho)c)c)c)$
$= MA (((\varsigma \to \varrho)c \to \varrho)c),$
$\tilde{B}A((\varsigma \to \varrho)c) \geq \tilde{B}A (((\varsigma \to \varrho)c \to (\varsigma \to (\varsigma \to \varrho)c)c)c)$
$= \tilde{B}A (((\varsigma \to \varrho)c \to \varrho)c),$
$JA((\varsigma \to \varrho)c) \leq JA (((\varsigma \to \varrho)c \to (\varsigma \to (\varsigma \to \varrho)c)c)c)$
$= JA (((\varsigma \to \varrho)c \to \varrho)c)$
Then it is clear that 'A' is an MBJ-NILI ideal of 'L' from the Proposition 3.13. And hence the proof.

Conclusion:
A MBJ-Neutrosophic implicative LI ideal concept has introduced and when a MBJ-Neutrosophic LI ideal acts as a MBJ-Neutrosophic implicative LI ideal has discussed by establishing the characterizations of an MBJ-Neutrosophic implicative LI Ideal from which we can able to understood the relation between them.

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