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Single Server Queueing System with Three Types of Services

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Abstract
In this paper we learn about a single server system that limits server control. In this model the server offers three types of services (one by one service, a bulk service with an accessible collection and a Non-accessible collection) for newcomers. A system of difference differential equation, the probability of a stable position and the expected number of customers in queue is obtained using Laplace transform. Presented numerical results in the form of tables.

Keywords: steady state probability, difference differential equation, Laplace transforms, batch service with accessible batch and Non-accessible batch.

1. Introduction

Sridhar and Allah Pitchai (2015) study an M/M/2 queuing system with two heterogeneous servers and working vacation. They derive the steady-state solution of the model using matrix geometric method. Sridhar and Allah Pitchai (2014) studied two sever queueing system with heterogeneous batch service and the expression for the expected queue length. Sridhar and Allah Pitchai (2013) had consider queuing for two servers with a single and batch service and allow late customers to join lines of ongoing service in server - I when the system size is less than or equal to control limit. In this paper it is considered a single, batch services queueing system with accessibility to the batches and Non-accessible batches and also mean service rates of single, accessible and Non-accessible batches are different. Customers arrive according to the Poisson process with the parameter λ and are served by a single server. The server serves the customers either one at a time or in batches with or without accessibility to the batches. Specifically, if the system size is below or equal to the control limit c_1, then the server serves one customer at a time according to FCFS law and service time is exponentially distributed with mean service rate μ_S if the queue length is more than c_1 the server serves them altogether with a batch and allows subsequent newcomers to join the batch until the batch size is less than c_2, where the service time is exponentially distributed with mean service rate μ_A and if the queue length is more than or equal c_2 then the server serves them altogether with the batch and will not allow newcomers to join the batch, where the service time is exponentially distributed with mean service rate μ_N , independent of the batch size.

After every service completion epoch the server inspects the system and may determine system size(m) is one of three categories $(1)\ 0 \le m \le c_1, (2)\ c_1 + 1 \le m \le c_2 - 1$ and $(3)\ m \ge c_2$.

When $0 \le m \le c_1$, the sever serves the customers one by one. Similarly when $c_1 + 1 \le m \le c_2 - 1$, the server serves the customers altogether in a batch and allows newcomers to join the batch while the server is on progress until the accessible limit $c_2 - 1$ is reached or the service is over whatever happens first, such batch is called an accessible batch (AB). Finally when $m \geq c_2$, the server takes all the units for service and does not allow newcomers to access to the batches, is called a Non-accessible batch (NAB).

A single and bulk queuing system was considered in Baburaj and Manoharan (1999) and they derived the steady state probabilities of system size. Sivasamy(1990) consider the concept of accessibility into the batches while the service is in progress. In this paper we consider the transient behavior of a single, accessible batch and Non-accessible batch services queueing system with three different service rates. The model is analyzed in Section 2 and In Section 3, Laplace transform of the transient probabilities are obtained. In Section 4, the steady state distribution is obtained and In Section 5, Expected Queue Length is obtained, In Section 6, Presented numerical results in the form of tables. Finally, Section 7 concludes the paper.

2. Analysis of the Model

Let $P_s(0,m,t), m = 0,1,2,\dots,c_1$ represents the probability that the server is busy with a single service or idle (when $m = 0$) and there are m customers in the system at a time t, $P_A(1,m,t), m = c_1+1, c_1+2, \dots, c_2-1$ be the probability that the server is busy with AB and there are m customers in the system and $P_N(2,m,t), m \geq 0$ denotes the probability that the sever is busy with a NAB and there are m customers in the waiting line (except those in service) during t. Here the state space of the system can be represented as $S = S_1 \cup S_2 \cup S_3$, where $S_1 = \{(0,m), m = 0,1,2,\dots,c_1\}$, $S_2 = \{(1,m), m = c_1+1, c_1+2, \dots, c_2-1\}$ and $S_3 = \{(2,m), m \geq 0\}$.

The following are the transitions that can be occurred during $(t, t+h]$

Transition during $(t, t+h]$	Probabilities
$(0,m) \to (0,m+1)$	$\lambda h + o(h), 0 \leq m \leq c_1 - 1$
$(0,m) \to (0,m-1)$	$\mu_S h + o(h), 1 \leq m \leq c_1$
$(0,c_1) \to (1,c_1+1)$	$\lambda h + o(h),$
$(1,m) \to (1,m+1)$	$\lambda h + o(h), c_1+1 \leq m \leq c_2-2$
$(1,m) \to (0,0)$	$\mu_A h + o(h), c_1+1 \leq m \leq c_2-1$
$(1,c_2-1) \to (2,0)$	$\lambda h + o(h)$
$(2,m) \to (0,m)$	$\mu_N h + o(h), 0 \leq m \leq c_1$
$(2,m) \to (1,m)$	$\mu_N h + o(h), c_1+1 \leq m \leq c_2-1$
$(2,m) \to (2,0)$	$\mu_N h + o(h), m \geq c_2.$

$P_s(i,j,t)$, $P_A(i,j,t)$ and $P_N(i,j,t)$ satisfy the following system of difference differential equations with the initial condition $P_s(0,0,t) = 1$.

$$\frac{d}{dt}P_s(0,0,t) = -\lambda P_s(0,0,t) + \mu_S P_s(0,1,t) + \mu_A \sum_{m=c_1+1}^{c_2-1} P_A(1,m,t) + \mu_N P_N(2,0,t) \tag{1}$$

$$\frac{d}{dt}P_s(0,1,t) = -(\lambda+\mu_S)P_s(0,1,t) + \lambda P_s(0,0,t) + \mu_S P_s(0,2,t) + \mu_N P_N(2,1,t) \tag{2}$$

$$\frac{d}{dt}P_s(0,m,t) = -(\lambda+\mu_S)P_s(0,m,t) + \lambda P_s(0,m-1,t) + +\mu_S P_s(0,m+1,t) + \mu_N P_N(2,m,t), \quad 2 \leq m \leq c_1 - 1 \tag{3}$$

$$\frac{d}{dt}P_s(0,c_1,t) = -(\lambda+\mu_S)P_s(0,c_1,t) + \lambda P_s(0,c_1-1,t) + \mu_N P_N(2,c_1,t) \tag{4}$$

$\frac{d}{dt}P_A(1, c_1+1, t) = -(\lambda+\mu_A)P_A(1, c_1+1, t) + \lambda P_s(0, c_1, t) + \mu_N P_N(2, c_1+1, t)$ (5)

$\frac{d}{dt}P_A(1, m, t) = -(\lambda+\mu_A)P_A(1, m, t) + \lambda P_A(1, m-1, t) + \mu_N P_N(2, m, t),$
$c_1+2 \le m \le c_2-1$
(6)

$\frac{d}{dt}P_N(2,0,t) = -(\lambda+\mu_N)P_N(2,0,t) + \lambda P_A(1, c_2-1, t) + \mu_N \sum_{m=c_2}^{\infty} P_N(2, m, t)$ (7)

$\frac{d}{dt}P_N(2, m, t) = -(\lambda+\mu_N)P_N(2, m, t) + \lambda P_N(2, m-1, t)$, $m \ge 1.$ (8)

3. Main Results

Let $P_L(i, m, s), i = 0,1,2$ are the Laplace transforms of $P_s(0, m, t), P_A(1, m, t)$ and $P_N(2, m, t)$ respectively and taking Laplace Transform from the equations (1) to (8), we obtain the following system of transient probabilities.

$(s+\lambda)\, P_L(0,0,s) - 1 = \mu_S P_L(0,1,s) + \mu_A \sum_{m=c_1+1}^{c_2-1} P_L(1, m, s) + \mu_N\, P_L(2,0,s)$ (9)

$(s+\lambda+\mu_S)\, P_L(0,1,s) = \lambda P_L(0,0,s) + \mu_S P_L(0,2,s) + \mu_N\, P_L(2,1,s)$ (10)

$(s+\lambda+\mu_S)\, P_L(0, m, s) = \lambda P_L(0, m-1, s) + +\mu_S P_L(0, m+1, s) + \mu_N P_L(2, m, s)$
$2 \le m \le c_1 - 1$ (11)

$(s+\lambda+\mu_S)\, P_L(0, c_1, s) = \lambda P_L(0, c_1-1, s) + \mu_N\, P_L(2, c_1, s)$ (12)

$(s+\lambda+\mu_A)\, P_L(1, c_1+1, t) = \lambda P_L(0, c_1, s) + \mu_N\, P_L(2, c_1+1, s)$ (13)

$(s+\lambda+\mu_A)P_L(1, m, s) = \lambda P_L(1, m-1, s) + \mu_N\, P_L(2, m, s)\;\; c_1+2 \le m \le c_2-1$ (14)

$(s+\lambda+\mu_N)P_L(2,0,s) = \lambda P_L(1, c_2-1, s) + \mu_N \sum_{m=c_2}^{\infty} P_L(2, m, s)$ (15)

$(s+\lambda+\mu_N)P_L(2, m, s) = \lambda P_L(2, m-1, s)$, $m \ge 1.$ (16)

Resolve (16) repeatedly we find,

$$P_L(2, m, s) = P_L(2,0,s)\, \varepsilon_1^m, \text{ Where } \varepsilon_1 = \frac{\lambda}{(s+\lambda+\mu_N)}$$

From (15)

$$P_L(1, c_2-1, s) = P_L(2,0,s)\left\{\frac{1}{\varepsilon_1} - \frac{\mu_N \varepsilon_1^m}{\lambda(1-\varepsilon_1)}\right\} \quad (17)$$

Solving (11) as the difference equation in $P_L(0, m, s)$, we find,

$$P_L(0, m, s) = B.R^m - P_L(2,0,s)\frac{\mu_N \varepsilon_1^m}{G(\varepsilon_1)},$$

Where B=B(s) is a constant for s and R= R(s) The actual positive root is less than the numerical unity of equation $G(z) = \mu_S z^2 - (s+\lambda+\mu_S)z + \lambda = 0$. Placing $m = c_1 - 1$ and applying this result to (12) we get,

$$P_L(0, c_1, s) = B\varepsilon_2 R^{c_1-1} + P_L(2,0,s)[\varepsilon_3 \varepsilon_1^{c_1} - \frac{\mu_N \varepsilon_2 \varepsilon_1^{c_1-1}}{G(\varepsilon_1)}],$$

Where $\varepsilon_2 = \frac{\lambda}{s+\lambda+\mu_S}$ and $\varepsilon_3 = \frac{\mu_N}{s+\lambda+\mu_S}$.

From (13)

$$P_L(1, c_1+1, s) = B\varepsilon_2\varepsilon_5 R^{c_1-1} + P_L(2,0,s)[(\varepsilon_1\varepsilon_2 + \varepsilon_3\varepsilon_5)\varepsilon_1^{c_1} - \frac{\mu_N \varepsilon_2\varepsilon_5\varepsilon_1^{c_1-1}}{G(\varepsilon_1)}],$$

Where $\varepsilon_4 = \frac{\mu_N}{s+\lambda+\mu_A}$ and $\varepsilon_5 = \frac{\lambda}{s+\lambda+\mu_A}$

From (14)

$$P_L(1, m, s) = B\varepsilon_2\varepsilon_5^{m-c_1} R^{c_1-1} + P_L(2,0,s)\{\ [(\varepsilon_1\varepsilon_4 + \varepsilon_3\varepsilon_5)\varepsilon_1^{c_1-1} - \frac{\mu_N \varepsilon_2\varepsilon_5\varepsilon_1^{c_1-2}}{G(\varepsilon_1)}]\varepsilon_1^{m-c_1-1}$$

$$+(m-c_1-1)\varepsilon_4\varepsilon_1^m\},\ \ c_1+2\le m\le c_2-1$$

Putting $m=c_2-1$ and comparing with (17) we get

$$B= P_L(2,0,s).E_1,$$

Where

$$E_1=\frac{1}{\varepsilon_2\varepsilon_5^{c_2-c_1-1}R^{c_1-1}}\left\{\frac{1}{\varepsilon_1}-\frac{\mu_N\varepsilon_1^{c_2}}{\lambda(1-\varepsilon_1)}-\left[(\varepsilon_1\varepsilon_4+\varepsilon_3\varepsilon_5)\varepsilon_1^{c_2-3}-\frac{\mu_N\varepsilon_2\varepsilon_5\varepsilon_1^{c_2-4}}{G(\varepsilon_1)}\right]\right.$$
$$\left.-(c_2-c_1-2)\varepsilon_4\varepsilon_1^{c_2-2}\right\}.$$

Using these results in (10) we get,

$$P_L(0,1,s)=\varepsilon_2P_L(0,0,s)+P_L(2,0,s)\left\{\varepsilon_6\left[E_1R^2-\frac{\mu_N\varepsilon_1^2}{G(\varepsilon_1)}\right]+\varepsilon_1\varepsilon_3\right\}$$

Where $\varepsilon_6=\frac{\mu_S}{s+\lambda+\mu_S}$.

And from (9)

$$P_L(0,0,s)=P_L(2,0,s).E_2+\frac{(s+\lambda+\mu_S)}{(s+\lambda)^2+s\mu_S},$$

Where $$E_2=\frac{(s+\lambda)(s+\lambda+\mu_S)}{(s+\lambda)^2+s\mu_S}\left\{\left[\ \varepsilon_6\left[E_1R^2-\frac{\mu_N\varepsilon_1^2}{G(\varepsilon_1)}\right]+\varepsilon_1\varepsilon_3\right]\frac{\mu_1}{s+\lambda}\right.$$
$$+\frac{\mu_A}{s+\lambda}\left[E_1\varepsilon_2R^{c_1-1}\left(\frac{\varepsilon_1-\varepsilon_1{}^{c_2-c_1}}{1-\varepsilon_1}\right)+\left[(\varepsilon_1\varepsilon_4+\varepsilon_3\varepsilon_5)\varepsilon_1^{c_1-1}-\frac{\mu_N\varepsilon_2\varepsilon_5\varepsilon_1^{c_1-2}}{G(\varepsilon_1)}\right]\frac{1-\varepsilon_1^{c_2-c_1-1}}{1-\varepsilon_1}\right.$$
$$\left.\left.+\varepsilon_4\left[f(\varepsilon_1)-(c_1+1)\frac{\varepsilon_1{}^{c_1+1}-\varepsilon_1{}^{c_2}}{1-\varepsilon_1}\right]\right]+\frac{\mu_N}{s+\lambda}\right\}$$

and $f(\varepsilon_1)=\sum_{m=c_1+1}^{c_2-1}m.\varepsilon_1^m=(1-\varepsilon_1)^{-2}\left(\varepsilon_1^{c_1+1}-\varepsilon_1^{c_2}\right)+(1-\varepsilon_1)^{-1}\left(c_1\varepsilon_1^{c_1+1}-(c_2-1)\varepsilon_1^{c_2}\right)$.

So we get Laplace transformation for the transient probabilities like,

$$P_L(0,0,s)=P_L(2,0,s).E_2+\frac{(s+\lambda+\mu_S)}{(s+\lambda)^2+s\mu_S} \tag{18}$$

$$P_L(0,1,s)=P_L(2,0,s).E_3+\frac{\lambda}{(s+\lambda)^2+s\mu_S} \tag{19}$$

Where $E_3=\varepsilon_2E_2+\ \varepsilon_6\left[E_1R^2-\frac{\mu_N\varepsilon_1^2}{G(\varepsilon_1)}\right]+\varepsilon_1\varepsilon_3,$

$$P_L(0,m,s)=P_L(2,0,s)\left\{E_1R^m-\frac{\mu_N\varepsilon_1^n}{G(\varepsilon_1)}\right\},\ 2\le m\le c_1-1 \tag{20}$$

$$P_L(0,c_1,s)=P_L(2,0,s).E_4 \tag{21}$$

Where $E_4=E_1\varepsilon_2R^{c_1-1}+\varepsilon_3\varepsilon_1^{c_1}-\frac{\mu_N\varepsilon_2\varepsilon_1^{c_1-1}}{G(\varepsilon_1)}$,

$$P_L(1,c_1+1,s)=P_L(2,0,s).E_5 \tag{22}$$

$$P_L(1,m,s)=P_L(2,0,s)\{E_1\varepsilon_2\varepsilon_5^{m-c_1}R^{c_1-1}+\left[(\varepsilon_1\varepsilon_4+\varepsilon_3\varepsilon_5)\varepsilon_1^{m-2}-\frac{\mu_N\varepsilon_2\varepsilon_5\varepsilon_1^{m-3}}{G(\varepsilon_1)}\right]$$
$$+(m-c_1-1)\varepsilon_4\varepsilon_1^m\},\quad c_1+2\le m\le c_2-1 \tag{23}$$

and $$P_L(2,m,s)=P_L(2,0,s)\ \varepsilon_1^m, m\ge 1 \tag{24}$$

Where $E_5=E_1\varepsilon_2\varepsilon_5R^{c_1-1}+\left[\left(\varepsilon_1\varepsilon_4+\varepsilon_3\varepsilon_5\right)\varepsilon_1^{c_1}-\frac{\mu_N\varepsilon_2\varepsilon_5\varepsilon_1^{c_1-1}}{G(\varepsilon_1)}\right]$.

Then using the normalizing condition

$$\sum_{m=0}^{c_1}P_L(0,m,s)+\sum_{m=c_1+1}^{c_1-c_2-1}P_L(1,m,s)+\sum_{m\ge0}P_L(2,m,s)=\frac{1}{s}$$

We get

$$P_L(2,0,s)=\frac{\lambda^2}{s[(s+\lambda)^2+s\mu_S]}\{E_2+E_3+E_4+E_5+E_1\frac{R^2-R^{c_1}}{1-R}-\frac{\mu_N(\varepsilon_1^2-\varepsilon_1^{c_1})}{G(\varepsilon_1)(1-\varepsilon_1)}$$

$$+E_1\varepsilon_2 R^{c_1-1}\frac{\varepsilon_5^2-\varepsilon_5^{c_2-c_1}}{1-\varepsilon_5}+(\varepsilon_1\varepsilon_4+\varepsilon_3\varepsilon_5)\frac{\varepsilon_1^{c_1}-\varepsilon_1^{c_2-2}}{1-\varepsilon_1}-\frac{\mu_N\varepsilon_2\varepsilon_5}{G(\varepsilon_1)}[\ \varepsilon_1^{c_1-1}+\frac{\left(\varepsilon_1^{c_1}-\varepsilon_1^{c_2-3}\right)}{(1-\varepsilon_1)}]$$

$$+\varepsilon_4\left[f(\varepsilon_1)-(c_1+1)\frac{{\varepsilon_1}^{c_1+2}-{\varepsilon_1}^{c_2}}{1-\varepsilon_1}\right]+\frac{\varepsilon_1}{1-\varepsilon_1}+1\ \}^{-1}.$$

4. Steady State Probabilities

The steady state probabilities can be obtained using Laplace's final value theorem,

$$P_L(i\,,j)=\lim_{t\to\infty}P_L(i,j,t)=\lim_{s\to 0}sP_L(i,j,s)$$

Then $\eta=\lim_{s\to 0}R$, $\eta_1=\lim_{s\to 0}\varepsilon_1$, $\eta_2=\lim_{s\to 0}\varepsilon_2$, $\eta_3=\lim_{s\to 0}\varepsilon_3$, $\eta_4=\lim_{s\to 0}\varepsilon_4$, $\eta_5=\lim_{s\to 0}\varepsilon_5$, $\eta_6=\lim_{s\to 0}\varepsilon_6$,

and $\eta_1=\frac{\lambda}{\lambda+\mu_N}$, $\eta_2=\frac{\lambda}{\lambda+\mu_S}$, $\eta_3=\frac{\mu_N}{\lambda+\mu_S}$, $\eta_4=\frac{\mu_A}{\lambda+\mu_A}$, $\eta_5=\frac{\lambda}{\lambda+\mu_A}$, $\eta_6=\frac{\mu_S}{\lambda+\mu_S}$, $\eta=\frac{\lambda}{\mu_S}=R$

and then from the equations (18) to (24) **,** we find steady state probabilities like

$$P_L(0{,}0)=P_L(2{,}0,).V_2; \tag{25}$$

Where $V_2=\frac{\lambda(\lambda+\mu_S)}{(\lambda)^2}\{\ [\eta_6\left[V_1\eta^2-\frac{\mu_N\eta_1^2}{b(\eta_1)}\right]+\eta_1\,\eta_3]\frac{\mu_S}{\lambda}\ +\frac{\mu_A}{\lambda}[V_1\eta_2\eta^{c_1-1}\left(\frac{\eta_1-{\eta_1}^{c_2-c_1}}{1-\eta_1}\right)$

$+[(\eta_1\eta_4+\ \eta_3\,\eta_5)\eta_1^{c_1\,-1}-\frac{\mu_N\eta_2\,\eta_5\eta_1^{c_1-2}}{b(\eta_1)}]\frac{1-\eta_1^{c_2-c_1-1}}{1-\eta_1}\quad+\eta_4\left[f(\eta_1)-(c_1+1)\frac{{\eta_1}^{c_1+1}-{\eta_1}^{c_2}}{1-\eta_1}\right]]+\frac{\mu_N}{\lambda}\}$,

and $V_1=\frac{1}{\eta_2\eta_5^{c_2-c_1-1}\eta^{c_1-1}}\{\frac{1}{\eta_1}-\frac{\mu_N\eta_1^{c_2}}{\lambda(1-\eta_1)}-[\ (\eta_1\eta_4+\ \eta_3\,\eta_5)\eta_1^{c_2-3}-\frac{\mu_N\eta_2\,\eta_5\eta_1^{c_2-4}}{b(\eta_1)}]$

$-(\ c_2-c_1-2)\ \eta_4\eta_1^{c_2-2}\}$

$$P_L(0{,}1)=P_L(2{,}0,).V_3; \tag{26}$$

Where $V_3=\eta_2V_2+\eta_6\left[V_1\eta^2-\frac{\mu_N\eta_1^2}{b(\eta_1)}\right]+\eta_1\,\eta_3$

$$P_L(0,m)=P_L(2{,}0)\left\{V_1\eta^m-\frac{\mu_N\eta_1^m}{b(\eta_1)}\right\},\ 2\le m\le c_1-1 \tag{27}$$

$$P(0,c_1)=P(2{,}0).V_4 \tag{28}$$

Where $V_4=V_1\eta_3\eta^{c_1-1}+\ \eta_3\eta_1^{c_1}\ -\frac{\mu_N\eta_2\eta_1^{c_1-1}}{b(\eta_1)}$,

$$P_L(1,c_1+1)=P_L(2{,}0).V_5 \tag{29}$$

Where $V_5=V_1\eta_2\,\eta_5\eta^{c_1-1}+[(\eta_1\eta_4+\ \eta_3\,\eta_5)\eta_1^{c_1}-\frac{\mu_N\eta_2\,\eta_5\eta_1^{c_1-1}}{b(\eta_1)}]$;

$$P_L(1,m)=P_L(2{,}0)\{V_1\eta_2\eta_5^{m-c_1}\eta^{c_1-1}+[(\eta_1\eta_4+\ \eta_3\,\eta_5)\eta_1^{m-2}-\frac{\mu_N\eta_2\,\eta_5\eta_1^{m-3}}{b(\eta_1)}]$$
$$+(\ m-c_1-1)\eta_4\eta_1^m\},\quad c_1+2\le m\le c_2-1 \tag{30}$$

$$P_L(2,m)=P_L(2{,}0)\ \eta_1^m,m\ge 1 \tag{31}$$

and

$$P_L(2\,,0)=\ \lim_{s\to 0}\ sP_L(2{,}0,s)$$
$$=\{V_2+V_3+V_4+V_5+V_1\frac{\eta^2-\eta^{c_1}}{1-\eta}-\frac{\mu_N\left(\eta_1^2-\eta_1^{c_1}\right)}{b(\eta_1)(1-\eta_1)}$$

$$+V_1\eta_2\eta^{c_1-1}\frac{\eta_5^2-\eta_5^{c_2-c_1}}{1-\eta_5}+(\eta_1\eta_4+\eta_3\,\eta_5)\frac{\eta_1^{c_1}-\eta_1^{c_2-2}}{1-\eta_1}-\frac{\mu_N\eta_2\eta_5}{b(\eta_1)}[\,\eta_1^{c-1}+\frac{\left(\eta_1^{c_1}-\eta_1^{c_2-3}\right)}{(1-\eta_1)}$$

$$+\eta_4\left[f(\eta_1)-(c_1+1)\frac{\eta_1{}^{c_1+2}-\eta_1{}^{c_2}}{1-\eta_1}\right]+\frac{\eta_1}{1-\eta_1}+1\ \}^{-1}. \tag{32}$$

5. Expected Queue Length

If the number of customers m in the system $(1 \le m \le c_1)$, the size of queue will in the system
be $m-1$ and server is busy with single service and the probability is $P_L(0,m)$. If the size of
queue will in the system be greater or equal to c_1 and less than c_2 $(c_1+1 \le m \le c_2-1)$, the server is busy with AB and probability is $P_L(1,m)$. If size of queue will in the system be greater than or equal to $c_2(m \ge c_2)$ then the server is busy with NAB and probability is $P_L(2,m)$,
so arrival customers will to wait for service until NAB service completes.

$$L_q=\sum_{m=2}^{c_1}(m-1)P_L(0,m)+\sum_{m\ge 1}mP_L(2,m)$$

Utilizing the equations (27), (31) and (32), we find,

$$L_q=P_L(2,0)\left[V_1\eta\{(1-c_1\eta^{c_1-1})(1-\eta)^{-2}\}-\frac{\mu_N}{b(\eta_1)}\left\{\frac{\eta_1^2-\eta_1^{c_1}}{(1-\eta_1)}\right\}+(c_1-1)V_4+\eta_1(1-\eta_1)^{-2}\right].$$

6. NUMERICAL ILLUSTRATIONS

Take the variables values as $\lambda=11,$, $\mu_S=15$, $\mu_A=8$, $\mu_N=6$, $c_1=6$, and $c_2=15$ for the numerical computations.

From the equations (25) to (31) are used to evaluate the steady state probabilities and table 1 shows the numerical results

Table 1. The steady state probabilities numerical results

m	$P_L(\mathbf{0},\mathbf{m})$	**m**	$P_L(\mathbf{1},\mathbf{m})$	**m**	$P_L(\mathbf{2},\mathbf{m})$
0	0.6616	1	0.1278	1	0.2316
1	0.4195	2	0.1283	2	0.1479
2	0.2680	3	0.1123	3	0.0949
3	0.1720	4	0.0916	4	0.0611
4	0.1107	5	0.0715	5	0.0394
5	0.0714	6	0.0542	6	0.0254
6	0.0461	7	0.0402	7	0.0164
		8	0.0293	8	0.0106
		9	0.0211	9	0.0069
		10	0.0151	10	0.0044
		11	0.0107	11	0.0029

12	0.0075	12	0.0019
13	0.0052	13	0.0012
14	0.0036	14	0.0008
		15	0.0005
		16	0.0003
		17	0.0002
		18	0.0001
		19	0.0001
		20	0.0001

On the queue length, we investigate the consequence of the parameters λ , μ_S , μ_A , and μ_N.The result on λ explicates in tables 2, 3 and 4, the outcome on μ_S expounds in tables 5 and 6, the output on μ_A clarifies in tables 7 and 8, the outturn on μ_N elucidates in tables 9 and 10.

Table 2 Effects on λ

$\lambda = 11, , \mu_S = 15, \mu_A = 8 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	1.0334	1.0233	1.0160	1.0107	1.0069	1.0043
3	1.4975	1.4963	1.4929	1.4892	1.4857	1.4828
4	1.8292	1.8827	1.9096	1.9224	1.9279	1.9297
5	---	2.0349	2.1688	2.2425	2.2825	2.3038
6	---	---	---	2.3318	2.4739	2.5539
7	---	---	---	---	---	2.5950

Table 3 Effects on λ

$\lambda = 17, , \mu_S = 15, \mu_A = 8 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	1.7364	1.6867	1.6492	1.6208	1.5990	1.5823
3	2.3821	2.3216	2.2752	2.2390	2.2105	2.1880
4	2.8333	2.8415	2.8393	2.8309	2.8195	2.8071
5	---	---	2.9312	3.1151	3.2311	3.3030
6	---	---	---	---	---	---
7	---	---	---	---	---	---

Table 4 Effects on λ

$\lambda = 23, , \mu_S = 15, \mu_A = 8 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	2.4820	2.3744	2.2914	2.2263	2.1747	2.1334
3	3.2659	3.1180	3.0055	2.9179	2.8485	2.7929
4	3.7366	3.6412	3.5710	3.5155	3.4696	3.4304
5	---	---	---	---	---	3.5920

6	---	---	---	---	---	---
7	---	---	---	---	---	---

Table 5 Effects on μ_S

$\lambda = 11, , \mu_S = 16, \mu_A = 8 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	1.0951	1.0840	1.0760	1.0701	1.0659	1.0629
3	1.5412	1.5400	1.5365	1.5324	1.5287	1.5256
4	1.8499	1.9059	1.9340	1.9473	1.9529	1.9548
5	---	2.0348	2.1725	2.2482	2.2892	2.3111
6	---	---	---	2.3189	2.4634	2.5448
7	---	---	---	---	---	2.5721

Table 6 Effects on μ_S

$\lambda = 11, , \mu_S = 17, \mu_A = 8 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	1.1443	1.1324	1.1237	1.1174	1.1128	1.1096
3	1.5750	1.5736	1.5698	1.5654	1.5614	1.5580
4	1.8658	1.9230	1.9515	1.9648	1.9704	1.9722
5	---	2.0352	2.1744	2.2506	2.2919	2.3139
6	---	---	---	2.3088	2.4536	2.5351
7	---	---	---	---	---	2.5534

Table 7 Effects on μ_A

$\lambda = 11, , \mu_S = 15, \mu_A = 9 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	1.0334	1.0233	1.0160	1.0107	1.0069	1.0043
3	1.4975	1.4963	1.4929	1.4892	1.4857	1.4828
4	1.8292	1.8827	1.9096	1.9224	1.9279	1.9297
5	---	2.0349	2.1688	2.2425	2.2825	2.3038
6	---	---	---	2.3318	2.4739	2.5539
7	---	---	---	---	---	2.5950

Table 8 Effects on μ_A

$\lambda = 11, , \mu_S = 15, \mu_A = 7 , \mu_N = 6$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	0.9942	0.9859	0.9799	0.9755	0.9723	0.9701
3	1.4282	1.4331	1.4344	1.4339	1.4328	1.4315
4	1.7093	1.7784	1.8179	1.8401	1.8524	1.8590
5	---	1.8570	2.0194	2.1159	2.1729	2.2065
6	---	---	---	2.1232	2.3033	2.4117
7	---	---	---	---	---	---

Table 9 Effects on μ_N

$\lambda = 11, , \mu_S = 15, \mu_A = 8 , \mu_N = 7$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	0.7050	0.7042	0.7035	0.7028	0.7022	0.7017
3	1.1402	1.1541	1.1614	1.1652	1.1671	1.1679
4	1.4328	1.5017	1.5407	1.5625	1.5745	1.5812
5	---	1.6395	1.7665	1.8392	1.8805	1.9040
6	---	---	1.7364	1.9312	2.0433	2.1076
7	---	---	---	---	---	2.1637

Table 10 Effects on μ_N

$\lambda = 11, , \mu_S = 15, \mu_A = 8 , \mu_N = 5$

$c_1 \downarrow / c_2 \rightarrow$	10	11	12	13	14	15
2	1.5540	1.5236	1.5011	1.4844	1.4722	1.4632
3	2.0881	2.0578	2.0325	2.0121	1.9962	1.9839
4	2.4978	2.5226	2.5255	2.5188	2.5090	2.4988
5	---	2.6984	2.8424	2.9141	2.9475	2.9609
6	---	---	---	2.9742	3.1725	3.2797
7	---	---	---	---	---	---

7. Conclusion

In this paper, the length of the queue is obtained for different values of arrival rate λ, service rates μ_S, μ_A and μ_N,the control limits c_1 and c_2. Further we can find customer spend the time in the system and queue, busy period of the server. The activities of the system for distinct service rate are represented in some numerical results.

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