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Bulk Viscous String Cosmological Model in a Modified Theory of Gravity

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Abstract

A study of the dynamics of Marder type cosmological model based on bulk viscous strings in the paradigm of $f(R)$ gravity theory is described in this article. The solution for the field equations has been obtained by using the proportionality condition between the shear scalar (σ) and the expansion scalar (θ), that gives a relation betwixt metric potentials, and other being the use Hybrid expansion law (HEL). The kinematic and geometrical properties of our model namely deceleration parameter (DP) (q), "cosmographic parameters jerk (j), snap (s) and lerk (l)", squared speed of sound (v_s^2), $(r-s)$, $(r-q)$ statefinder plane and the energy conditions have been examined. After the graphical analysis of the dynamical features of the constructed model it is found that the model gives the description for the Universe's accelerated expansion and this is significant as these theoretical results are in accordance with the astronomical observations.

Keywords: Marder type space-time; f(R) theory; Bulk viscosity; Cosmological strings.

1 Introduction

The research-based achievements of millennium emphasizes that cosmology and gravitation have been developed during the past few centuries, where the contemporary observational data such as "type 1a supernovae [1–3], baryonic oscillations [4, 5] and large-scale structures [6]", reveals the phenomenon of Universe's accelerated expansion. The root cause for this acceleration is unknown and is supposed to be because of the mysterious component called Dark energy (DE), which contributes about 70% of the total Universe energy-mass. In an approach to explain this cosmic acceleration, one can propose and study various dynamical DE models on the right side of Einstein field equation.And another proposition is to alter left side of the Einstein field equation. A mathematical framework has been well made for describing gravitational field properties by incorporating Riemann geometry into general relativity (GR). Howsoever, this still has shortcomings on large scales, which has been established by current observational data. Some apprehensions regarding the absolute validity of the classical GR has been questioned. As mentioned earlier, the adjustment to Einstein's field equation'sgravitational part, to explain the acceleration, we come down to the modified theories of gravity. Hitherto, various modified theories past GR, such as the $f(R,T)$ gravity [7–10], the $f(T,B)$ gravity [11, 12], the $f(R,G)$ gravity [13, 14] and the $f(R)$ gravity [15–17] have been put into focus.

Amongst these, $f(R)$ theory of gravity (where Lagrangian density is an arbitrary function of Ricciscalar R), is a possibility to explicit the idea of exotic matter and accelerating Universe [18] and canbe modified in a simplest way from the geometrical aspect which do interpret the cosmic accelerationof the Universe in early phase and as well as in the late time accelerated expansion phase. It is clear that the Einstein-Hilbert-Lagrangian can be understood as f(R), is an analytic function of the Ricci scalar R as opposed to the R itself in the forward modification of GR. From the reviews of Clifton etal. [19] Sotiriou and Faraoni [20], Nojiri and Odintsov [21], Nojiri et al. [22] a concise idea of the accomplishments and the demerits of the study of f(R)modification can be concluded. It can be said that the f(R) theory is analogous to the GR theory when $f(R) = R$. There have been diverse models off(R) gravity suggested in the literature which explain the concept of exotic matter [23]. In the f(R) theory of gravity, Santhi and Naidu [24] have "studied strange quark matter cosmological models attached to string cloud". Shah and Samanta [25] have discussed cosmological dynamics of f(R) models in dynamical system analysis. Beesham and Bamba [26] have examined inflationary Universe from anomaly-free f(R) gravity. Santhi et al. [27] have investigated some Bianchi type bulk viscous string cosmological models in f(R) gravity. Ozdemir and Aktas [28] have investigated generalized anisotropic Universe models for magnetized strange quark matter distribution in the framework of f(R) gravitation theory.

An extensive study has been done by many authors [29–31] and they did try to elucidate the inflation in the early stage of the cosmos and its evolution in late times. Eckart [32] was first to formulate the theory of dissipative fluids in relative thermodynamics; and the influence of these dissipative parameters incorporating heat transport, shear viscosity and bulk viscosity has a vital role in the cosmic evolution; which was further modified by Landau and Lifshifz [33]. Deviations from thermodynamic equilibrium of the first order is what described in Eckart theory, whereas Israle and Stewart [34] introduced dissipative thermodynamic theory called as the casual theory of relativistic viscosity. Dissipative variables are used in order to account non-equilibrium states, causing this theory to be a casual and stable theory. In recent years, viscous DE models have been suggested as one way of understanding the growth of the Universe. The LRS Bainchi type-I cosmological model filled with bulk viscous cosmological fluid in f(R) gravity has been studied by Rakesh et al. [35], in the presence of time varying gravitational and cosmological constant. Bulk viscous string cosmological models have been discussed by Mishra and Dua [36] especially in the Saez-Ballester theory of gravity under a time-dependent DP. Archana et al. [37] have investigated anisotropic bulk viscous string cosmological models of the Universe. A spatially homogeneous and anisotropic Kantowski-Sachs space-time, was examined by Prasanthi and Aditya [38], that is filled with bulk viscous fluid, containing one-dimensional cosmic strings within the framework of f(R) modified theory gravity.

A study of the Universe using modern technological tools has revealed that there are a number of strings in the early Universe which were stable topological structures occurring during the phase transition when the temperature dropped below some critical temperatures. The strings containing the stress energy, combined with gravitational fields; exhibits the Universe's anisotropic behavior. Although they are not visible today and don't threaten cosmological models, strings lead to remarkably exhilarating astrophysics results, as contrasting to domain walls and monopolies. Stringsare effective in explaining the nature as well as the fundamental configuration of the earstwhile cosmos, as all the matter and forces are integrated as theory, thus elucidating the formation of the Universe based on strings. The gravitational effect caused by strings may also be worth investigating, since strings can couple to a gravitational field and possess stress energy. Also a reportby GUT (grand unified theories) [39–44], these strings appeared, as the temperatures dropped below critical point after the big bang explosion, because of the symmetry breaking during the phase transition.

Nowadays considerable number of researchers are concerned on the investigation of cosmological models with strings of the cosmos, to have an extensive apprehension on the development of the cosmos.
By inspiring with the above works, we work in the modified theory of gravity. A cosmological model of the Marder type is characterized by a bulk viscous string. The article is fully described as follows: In section 2, we study the mathematical formalism of the model. In section 3, we study the dynamical behaviour of the model. The results are summarized in the last section 4..

2 Mathematical formalism of the model

The emergence of spatially homogeneous and anisotropic cosmological models have recently attracted much attention, because such models can provide important insights into its infancy, the structure of the Universe on a large-scale. We consider a spatially homogeneous Marder type metricof the form (Marder [45])

$$ds^2 = p_1^2 dx^2 + p_2^2 dy^2 + p_3^2 dz^2 - p_1^2 dt^2, \tag{1}$$

where p_1, $p_2$and p_3 are functions of time t only. Aygun et al. [46] have investigated energy momentum of Marder Universe in Teleparallel Gravity. Aktas et al. [47, 48] have discussed behaviors of DE and mesonic scalar field for anisotropic Universe in $f(R)$gravity and the magnetized strange quark matter solutions are obtained for a Marder type Universe using constant DP. Aygun [49] have investigated tachyon and k-essence DE candidates with varying G and Λ for Marder Universe in $f(R, T)$ gravitation theory. Recently, Pawar and Shahare [50] have examined ananisotropic Tilted Marder cosmological model is investigated in the $f(R, T)$ theory of gravity andvery recently, wet dark fluid (WDF) model have been studied by Pawar et al. [51] in an anisotropic homogeneous space-time namely Marder space-time.

The following action gives the field equations of $f(R)$gravity

$$\mathcal{S} = \int \left(\mathcal{L}_m + \frac{f(R)}{16\pi G}\right)\sqrt{-g}\, d^4x, \tag{2}$$

where $f(R)$ is a Ricci scalar general function and the matter Lagrangian is given as $\mathcal{L}_m$. Varyingaction (2) w.r.t. metric gives the following field equations:

$$R_{\mu\nu}F(R) + g_{\mu\nu}\ F(R) - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) = \kappa T_{\mu\nu}, \tag{3}$$

where $F(R) = \frac{df(R)}{dR}$, $T_{\mu\nu}$ is the energy-momentum tensor of matter and $\Box = \nabla^\mu\nabla_\mu$, $\nabla_\mu$is the covariant derivative.
Contracting field equations (3), we get

$$R\,F(R) + 3\Box F(R) - f(R)\,2 = \kappa T. \tag{4}$$

The relation of $f(R)$ and $F(R)$ in Equation (4) can be utilized to evaluate $f(R)$ and for the simplification of the field equations. By multiplying eq.(3) with $g^{\mu\nu}$, we get

$$F(R)R_\mu^\mu - \frac{1}{2}\ f(R)\delta_\mu^\mu - \nabla^\mu\nabla_\mu F(R) + \delta_\mu^{\mu\,F(R)} = \kappa T_\mu^\mu. \tag{5}$$

For a bulk viscous fluid containing a cosmic string in one dimension, tensor energy momentum isdefined as follows:

$$T_{\mu\nu} = (\rho + \overline{p})u_\mu u_\nu + \overline{p}g_{\mu\nu} - \lambda x_\mu x_\nu, \tag{6}$$

$$\overline{p} = p - 3\xi H(= \omega\rho). \tag{7}$$

As mentioned above, the total pressure is denoted ~~by~~ $\overline{p}$ where the the isotropic pressure is included (p). ρ, $\xi(t)$, H and λ are "the rest energy density of the system, the coefficient of bulk viscous pressure, the Hubble parameter of the model and the string tension density respectively". A relation for the isotropic pressure and energy density is given by the equation of state (EoS) parameter as

$$p = \Psi\rho, \tag{8}$$

where the EoS parameter is represented as Ψ and $\omega = \Psi - \zeta$ (constant). "The vacuum dominated, matter dominated, radiation dominated and stiff fluid era are given respectively by the values of EoS $\Psi = -1, 0, 1/3, 1$ [38]." In the following equation, you can represent the anisotropic directions of the string by the four vectors of velocity u^{μ}, and the space-like vector x_{μ} as follows:

$$g^{\mu\nu}u_{\mu}u_{\nu} = -x^{\mu}x_{\nu} = -1, \quad u^{\mu}x_{\mu} = 0. \tag{9}$$

Suppose that the string is on the x-axis. There is an assumption that the strings are loaded with particle and energy density in the form of $\rho_p = \rho - \lambda$. Then the components of energy momentum tensor are

$$T_1^1 = \overline{p} - \lambda; \quad T_2^2 = T_3^3 = \overline{p}; \quad T_4^4 = -\rho. \tag{10}$$

With the help of Eq.(10), field Eq. (5) for metric (1) gives the following equations:

$$\left(\frac{\ddot{p}_1}{p_1} - \frac{\dot{p}_1{}^2}{p_1^2} + \frac{\dot{p}_1\dot{p}_2}{p_1p_2} + \frac{\dot{p}_1\dot{p}_3}{p_1p_3}\right)\frac{F}{p_1^2} + \frac{f(R)}{2} - \left(\frac{\dot{p}_1}{p_1} + \frac{\dot{p}_2}{p_2} + \frac{\dot{p}_3}{p_3}\right)\dot{F} - \ddot{F} = \kappa(\lambda - \overline{p}). \tag{11}$$

$$\left(\frac{\ddot{p}_2}{p_2} + \frac{\dot{p}_2\dot{p}_3}{p_2p_3}\right)\frac{F}{p_1^2} + \frac{f(R)}{2} - \left(\frac{\dot{2}p_1}{p_1} + \frac{\dot{p}_3}{p_3}\right)\dot{F} - \ddot{F} = -\kappa\overline{p}. \tag{12}$$

$$\left(\frac{\ddot{p}_3}{p_3} + \frac{\dot{p}_2\dot{p}_3}{p_2p_3}\right)\frac{F}{p_1^2} + \frac{f(R)}{2} - \left(\frac{\dot{2}p_1}{p_1} + \frac{\dot{p}_2}{p_2}\right)\dot{F} - \ddot{F} = -\kappa\overline{p}. \tag{13}$$

$$\left(\frac{\ddot{p}_1}{p_1} + \frac{\ddot{p}_2}{p_2} + \frac{\ddot{p}_3}{p_3} - \frac{\dot{p}_1{}^2}{p_1^2} - \frac{\dot{p}_2\dot{p}_1}{p_2p_1} - \frac{\dot{p}_3\dot{p}_1}{p_3p_1}\right)\frac{F}{p_1^2} + \frac{f(R)}{2} - \left(\frac{\dot{p}_1}{p_1} + \frac{\dot{p}_2}{p_2} + \frac{\dot{p}_3}{p_3}\right)\dot{F} = \kappa\rho. \tag{14}$$

The over head dot indicates differentiation with respect to t.

(i) A relation among the metric potentials which is brought down by the proportionality conditionbetween the shear scalar σ and the scalar expansion θ as given by Collins et al., [52]

$$p_1 = (p_2p_3)^m, \tag{15}$$

where m > 0 except one and maintaining the non-isotropic nature of the Universe.

(ii) The power law relation previously take by Johri and Sudharsan [53] between the scalar fieldand average scale factor

$$F(R) \propto (a(t))^n, \tag{16}$$

where the arbitrary constant is given by n. Therefore, F & a(t) has a power law relation with a proportionality constant F_0 as,

$$F(R) = F_0[a(t)]^n. \tag{17}$$

(iii) Taking the association of the exponential law and the power law for the average

scale factor
$a(t)$ given by Akarsu et al., [54], as

$$a(t) = t^{\alpha} e^{t\beta}, \tag{18}$$

where $\alpha > 0$, $\beta > 0$ and $a(t)$ is the average scale factor as follows

$$a(t) = V^{\frac{1}{3}} = (p_1^2 p_2 p_3)^{\frac{1}{3}}. \tag{19}$$

Now using Eqs.(15), (17)-(19) we derive the metric potentials and F as

$$p_1(t) = \left(t^{\alpha} e^{t\beta}\right)^{\frac{3m}{2m+1}}, \tag{20}$$

$$p_2(t) = \left(t^{\alpha} e^{t\beta}\right)^{\frac{1}{2m+1}}, \tag{21}$$

$$p_3(t) = \left(t^{\alpha} e^{t\beta}\right)^{\frac{2}{2m+1}}, \tag{22}$$

$$\& \ F(t) = F_0\left(t^{\alpha} e^{t\beta}\right)^{n}. \tag{23}$$

String density is given by

$$\left.\begin{aligned} \lambda = \ & \frac{1}{-(2m+1)^2 t^2 \kappa} \times (6(m-\tfrac{1}{2})(F_0(t^{\alpha}e^{t\beta})^n(\tfrac{-3\alpha^2}{2}) + (-3t\beta + m + \tfrac{1}{2})\alpha - \tfrac{3t^2\beta^2}{2}) \\ & \times (t^{\alpha}e^{t\beta})^{\frac{-6m}{2m+1}} - (m+\tfrac{1}{2})n(\beta t + \alpha)^2 F_0(t^{\alpha}e^{t\beta})^n). \end{aligned}\right\} \tag{24}$$

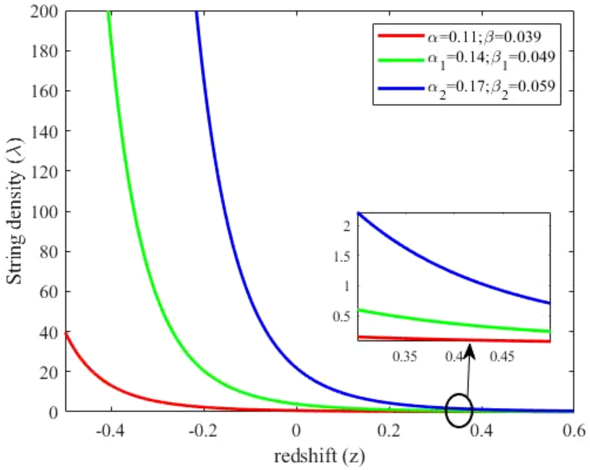

Figure 1: String density (λ) v/s redshift (z)

We get the energy density as

$$\left.\begin{aligned} \rho = \ & \frac{1}{8(\zeta-\Psi-1)\kappa t^2 (m+\frac{1}{2})^2} \times (12F_0(t^{\alpha}e^{t\beta})^{\frac{-6m}{2m+1}+n}((\tfrac{3m}{4} - \tfrac{1}{12})\alpha^2 + (m^2 \\ & +(\tfrac{3t\beta}{2} + 1)m - \tfrac{t\beta}{6} + \tfrac{1}{4})\alpha + \tfrac{3\beta^2}{4}(m - \tfrac{1}{9})t^2) - 8(t^{\alpha}e^{t\beta})^n(((m+\tfrac{1}{2})n \\ & -\tfrac{3m}{2} + \tfrac{1}{4})\alpha^2 + (2t\beta n(m+\tfrac{1}{2}) - 3\beta m t + \tfrac{\beta t}{2} - m - \tfrac{1}{2})\alpha \\ & +t^2\beta^2((m+\tfrac{1}{2})n - \tfrac{3m}{2} + \tfrac{1}{4}))nF_0(m+\tfrac{1}{2})). \end{aligned}\right\} \tag{25}$$

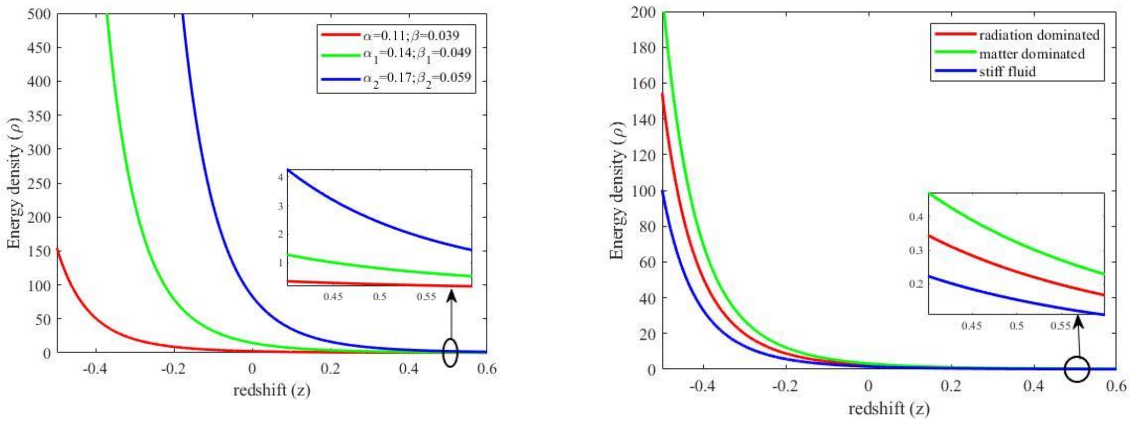

Figure 2: Energy density (ρ) v/s redshift (z)

Effective pressure is given by

$$\begin{aligned}\bar{p} = {} & \frac{3}{2\kappa t^2(\zeta-\Psi-1)(m+\frac{1}{2})^2} \times ((F_0(t^{\alpha}e^{t\beta})^{\frac{-6m+n(2m+1)}{2m+1}}((\frac{3m}{4}-\frac{1}{12})\alpha^2+(m^2 \\ & +(\frac{3t\beta}{2}+1)m-\frac{t\beta}{6}+\frac{1}{4})\alpha+\frac{3\beta^2}{4}(m-\frac{1}{9})t^2)-\frac{2}{3}(t^{\alpha}e^{t\beta})^n(((m+\frac{1}{2})n \\ & -\frac{3m}{2}+\frac{1}{4})\alpha^2+(2t\beta n(m+\frac{1}{2})-3\beta mt+\frac{\beta t}{2}-m-\frac{1}{2})\alpha \\ & +t^2\beta^2((m+\frac{1}{2})n-\frac{3m}{2}+\frac{1}{4}))nF_0(m+\frac{1}{2}))\omega).\end{aligned} \quad (26)$$

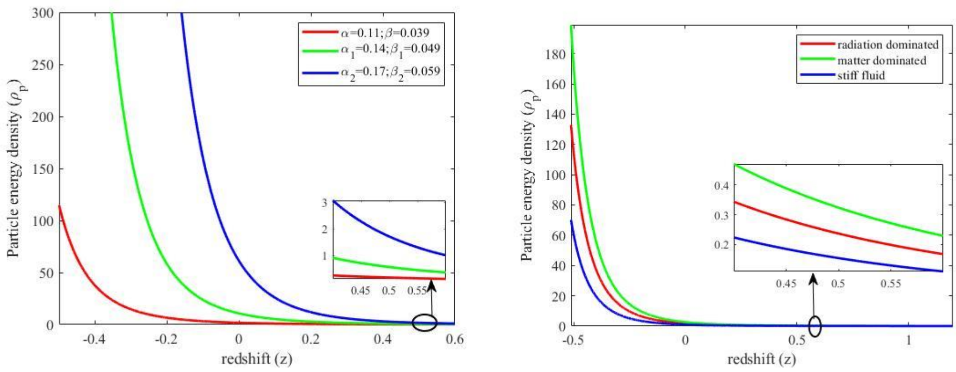

Figure 3: Particle energy density (ρ_p) v/s redshift(z)

ξ, the coefficient of bulk viscosity takes the form as

$$\begin{aligned}\xi = {} & \frac{1}{2\kappa t(\beta t+\alpha)(\zeta-\Psi-1)(m+\frac{1}{2})^2} \times ((F_0(t^{\alpha}e^{t\beta})^{\frac{-6m}{2m+1}+n}((\frac{3m}{4}-\frac{1}{12})\alpha^2+(m^2 \\ & +(\frac{3t\beta}{2}+1)m-\frac{t\beta}{6}+\frac{1}{4})\alpha+\frac{3\beta^2}{4}(m-\frac{1}{9})t^2)-\frac{2}{3}(t^{\alpha}e^{t\beta})^n(((m+\frac{1}{2})n \\ & -\frac{3m}{2}+\frac{1}{4})\alpha^2+(2t\beta n(m+\frac{1}{2})-3\beta mt+\frac{\beta t}{2}-m-\frac{1}{2})\alpha \\ & +t^2\beta^2((m+\frac{1}{2})n-\frac{3m}{2}+\frac{1}{4}))nF_0(m+\frac{1}{2}))(\omega_0-\omega)).\end{aligned} \quad (27)$$

The proper pressure is given by

$$p = \frac{3}{2\kappa t^2(\zeta-\Psi-1)(m+\frac{1}{2})^2} \times \Big((F_0(t^\alpha e^{t\beta})^{\frac{-6m+n(2m+1)}{2m+1}}((\frac{3m}{4}-\frac{1}{12})\alpha^2+(m^2 + (\frac{3t\beta}{2}+1)m-\frac{t\beta}{6}+\frac{1}{4})\alpha+\frac{3\beta^2}{4}(m-\frac{1}{9})t^2)-\frac{2}{3}(t^\alpha e^{t\beta})^n(((m+\frac{1}{2})n - \frac{3m}{2}+\frac{1}{4})\alpha^2+(2t\beta n(m+\frac{1}{2})-3\beta m t+\frac{\beta t}{2}-m-\frac{1}{2})\alpha + t^2\beta^2((m+\frac{1}{2})n-\frac{3m}{2}+\frac{1}{4}))nF_0(m+\frac{1}{2}))\omega_0\Big). \tag{28}$$

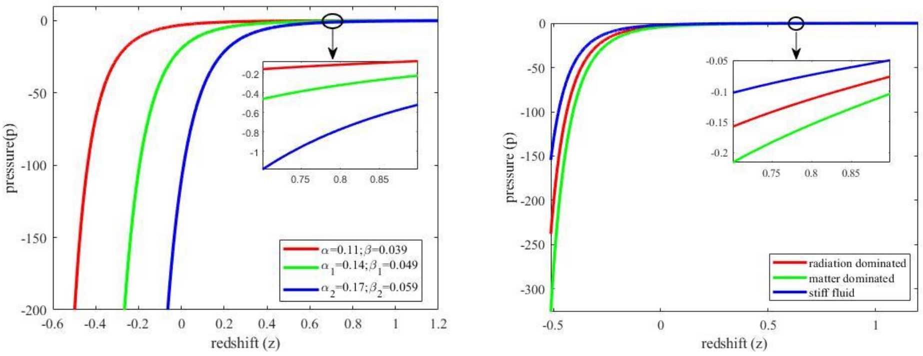

Figure 4: Pressure (p) v/s redshift (z)

Hence, the metric (1) is rewritten as

$$ds^2 = \left(t^\alpha e^{t\beta}\right)^{\frac{6m}{2m+1}}dx^2 + \left(t^\alpha e^{t\beta}\right)^{\frac{2}{2m+1}}dy^2 + \left(t^\alpha e^{t\beta}\right)^{\frac{4}{2m+1}}dz^2 - \left(t^\alpha e^{t\beta}\right)^{\frac{6m}{2m+1}}dt^2. \tag{29}$$

3 Dynamical behavior of the framework

In this segment, we compute the cosmological parameters of the model (29) and present their physicalsignificance.

• Spatial volume(V), average scale factor(a(t)), the mean Hubble's parameter (H), expansion scalar(θ), shear scalar (σ) and anisotropic parameter (A_h) of the model are given as

$$V = \left(t^{3\alpha}e^{3t\beta}\right); \quad a(t) = t^\alpha e^{t\beta}. \tag{30}$$

$$H = \frac{(\alpha+\beta t)}{t}. \tag{31}$$

$$\theta = \frac{3(\alpha+\beta t)\left(t^\alpha e^{t\beta}\right)^{-\frac{3m}{2m+1}}}{t}. \tag{32}$$

$$\sigma^2 = \frac{(\beta t+\alpha)^2(3m^2-3m+1)\left(t^\alpha e^{t\beta}\right)^{\frac{-6m}{2m+1}}}{(2m+1)^2 t^2}. \tag{33}$$

$$\& \ A_h = \frac{(24m^2-12m+2)^2}{3(2m+1)^2}. \tag{34}$$

Here A_h indicates the deviation from isotropic expansion and when $A_h = 0$, there is isotropical expansion of the Universe. Also, Universe's expansion rates in the of x, y and z directions are indicated by the directional Hubble's parameters H_1, H_2, H_3 respectively.

• **Deceleration parameter:** The study of cosmological parameter DP helps us to understand the nature of the expanding Universe. It is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H(t)}\right) - 1 = \frac{\alpha}{(\alpha+\beta t)^2} - 1.$$
(35)

Universe expansion shows the standard way of deceleration phase for the positive values of DP. The range of DP : $-1 \leq q < 0$, $q < -1$, "$q = 0$ and $q = -1$" current Universe has an accelerated expansion, "super exponential expansion", marginal inflation and finally behaves like de-sitter expansion respectively.

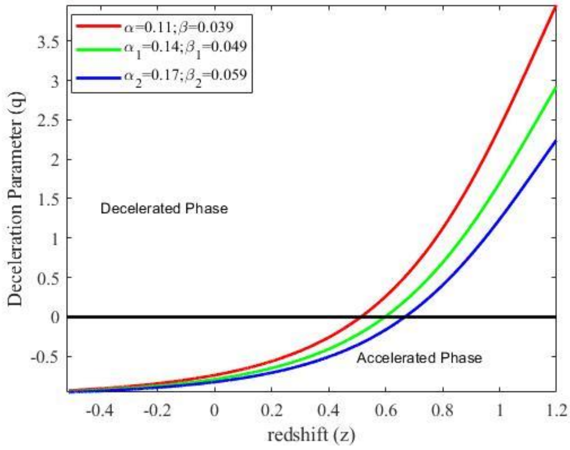

Figure 5: Deceleration parameter (q) v/s redshift (z)

- **Statefinder parameters:** As mentioned earlier in section (1) a mysterious force DE may be responsible for the cosmos to undergo an accelerated expansion at the current era. But as of now there is no adequate information about the DE. Hence, it becomes necessary to identify and understand the various properties of DE and its importance with various models of cosmography. Dvali et al. [55], Armendariz Picon et al. [56], Kamenschik et al. [57] and Ratra and Peebles [58] directed various studies to realize that different DE forms such as quintessence, "Chaplygin gas", k-essence, "brane world models" that give several curves of scale factor a(t). As a way of categorizing the different types of DE, Sahni et al. [59] suggested the 'statefinder pair' that is based upon "the second and third order derivatives of a(t). We have obtained expressions for statefinder pair (r,s) for the models as"

$$r = \frac{(\alpha^3+(3\beta t-3)\alpha^2+(3\beta^2t^2-3\beta t+2)\alpha+t^3\beta^3)}{(\beta t+\alpha)^3}. \quad (36)$$

$$\&\ s = \frac{2\alpha\left(t\beta+\alpha-\frac{2}{3}\right)}{(3\beta^2t^2+6\alpha\beta t+3\alpha^2-2\alpha)(\beta t+\alpha)}. \quad (37)$$

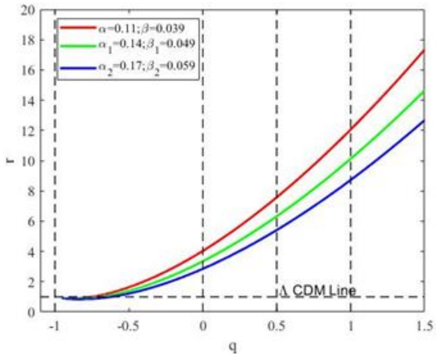

Figure 6: r − s plane

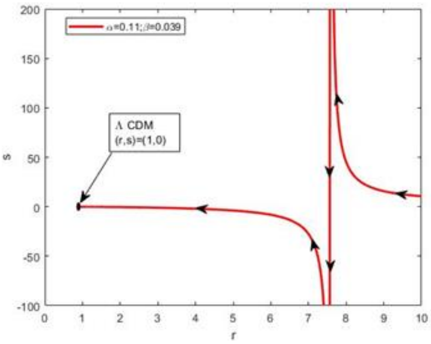

Figure 7: q−r plane

- **Cosmographic parameters:** A prominent number of observations have lead to the advancement in the study of modern cosmology. In other words, many cosmological tests are model dependent, and as such, it would be worthwhile to develop an independent method for analyzing cosmological scenarios, as it would make it possible to distinguish among the best models and set bounds where appropriate. Cosmography, which is the study of a scale factor by expanding it through the Taylor series w.r.t. the cosmic time; or the study of the kinematics of the Universe, is considered to be one of the most fascinating aspects of this discipline. The distance-redshift relation can be determined by this type of expansion, which is independent of the solution of the motion equation in cosmological models. To study cosmography, it is worthintroducing the cosmographic parameters as follows:

$$H = \frac{1}{a}\frac{da}{dt}, \tag{38}$$

$$q = -\frac{1}{a}\frac{d^2a}{dt^2}H^{-2}, \tag{39}$$

$$j = \frac{1}{a}\frac{d^3a}{dt^3}H^{-3}, \tag{40}$$

$$s = \frac{1}{a}\frac{d^4a}{dt^4}H^{-4}, \tag{41}$$

$$l = \frac{1}{a}\frac{d^5a}{dt^5}H^{-5}, \tag{42}$$

where H, q and l are the Hubble, the deceleration and the lerk parameters respectively, and jerk j and snap s form the state finder pair.

$$j = \frac{\alpha^3+(3\beta t-3)\alpha^2+(3\beta^2t^2-3\beta t+2)\alpha+t^3\beta^3}{(\beta t+\alpha)^3}, \quad (43)$$

$$s = \frac{\alpha^4+(4\beta t-6)\alpha^3+(6\beta^2t^2-12\beta t+11)\alpha^2+(4t^3\beta^3-6t^2\beta^2+8t\beta-6)\alpha+t^4\beta^4}{(\beta t+\alpha)^4}, \quad (44)$$

$$\text{and } l = \frac{1}{(\beta t+\alpha)^5} \times (\alpha^5 + (5\beta t - 10)\alpha^4 + (10\beta^2t^2 - 30\beta t + 35)\alpha^3 \quad (45)$$
$$+(10\beta^3t^3 - 30\beta^2t^2 + 55\beta t - 50)\alpha^2 + (5t^4\beta^4 - 10t^3\beta^3 + 20\beta^2t^2 - 30t\beta + 24)\alpha + t^5\beta^5).$$

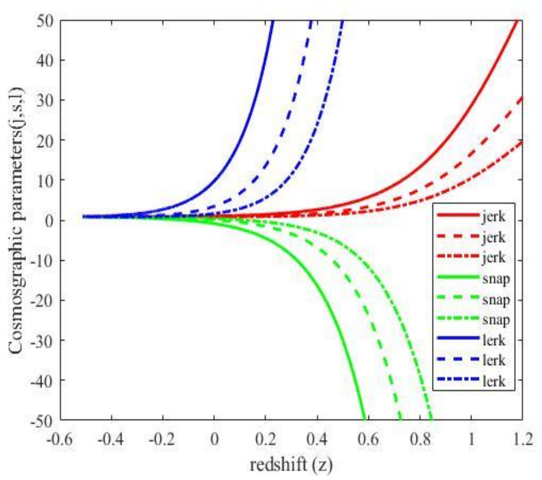

Figure 8: Cosmographic parameters v/s redshift(z)

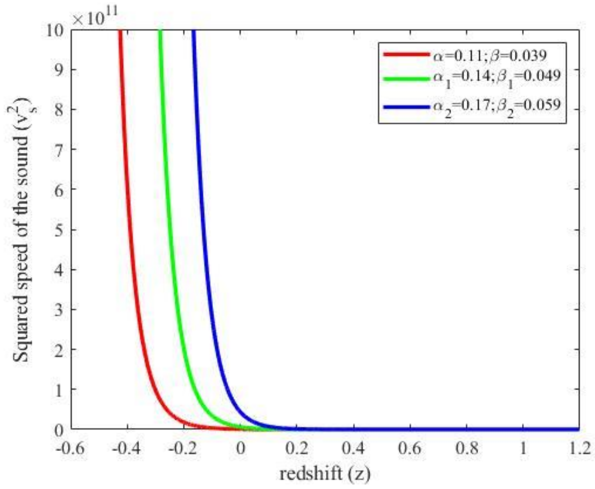

Figure 9: Squared speed sound (v^2) v/s redshift (z)

Model's Stability: The stability of model discussed by means of squared sound speed parameter is defined as

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}}. \quad (46)$$

The stability of the DE models can be ascertained with the help of this parameter, as the positiveaction of this parameter gives a stable model, while on contrary, the negative behavior gives an unstable model. The squared sound speed can be calculated by substituting and simplifying

corresponding expressions for the parameters in the Eq.(46),

$$
\left. \begin{aligned} v_s^2 = \; & -(16(\zeta - \Psi - 1)\kappa t^3 (m + \tfrac{1}{2})^3)/(16nF_0(nt^3\beta^3((n - \tfrac{3}{2})m + \tfrac{n}{2} + \tfrac{1}{4}) \\ & +3n\beta^2((n - \tfrac{3}{2})m + \tfrac{n}{2} + \tfrac{1}{4})\alpha t^2 + 3((1 + \alpha n^2 + (-1 - \tfrac{3\alpha}{2})n)m \\ & -\tfrac{1}{6} + \frac{\alpha n^2}{2} + (\tfrac{\alpha}{4} - \tfrac{1}{2})n)\beta\alpha t + (\alpha n - 2)\alpha((n\alpha - \frac{3\alpha}{2} - 1)m + \frac{n\alpha}{2} \\ & +\frac{\alpha}{4} - \tfrac{1}{2}))(m + \tfrac{1}{2}^{2})(t^\alpha e^{t\beta})^n + 72(t^\alpha e^{t\beta})^{\frac{-6m}{2m+1}} F(\tfrac{3}{4}(m - \tfrac{1}{9})t^3 m\beta^3 \\ & +\frac{9}{4}(m - \tfrac{1}{9})m\beta^2\alpha t^2 + (-\frac{1}{36} + m^3 + (\tfrac{9\alpha}{4} + \tfrac{3}{2})m^2 + (-\tfrac{\alpha}{4} + \tfrac{4}{9})m)\beta\alpha t \\ & +(m^2 + (\tfrac{3\alpha}{4} + 1)m - \frac{\alpha}{12} + \tfrac{1}{4})(\tfrac{1}{3} + (\tfrac{2}{3} + \alpha)m)\alpha)). \end{aligned} \right\}
$$
(47)

Energy conditions: The energy conditions (ECs) are the necessary to understand the geodesics of the Universe. These take the following form that are derived from the familiar Raychaudhury equations [60],

- WEC: $\rho \geq 0$
- NEC: $\rho + p \geq 0$
- DEC: $\rho - p \geq 0$
- SEC: $\rho + 3p \geq 0$

Here, the weak energy condition, null energy condition, dominant energy condition and strong energy condition are denoted by WEC, NEC, DEC and SEC respectively. The main purpose of these energy conditions is to check the expansion of the Universe. Several authors have worked on these energy conditions particularly Salti et al. [61], Sahoo et al. [62], Hegazy and Rahaman [63], Kumar and Singh [64], Bhar et al. [65], Mishra et al. [66, 67], Aziz et al. [68], Mollah and Singh [69].

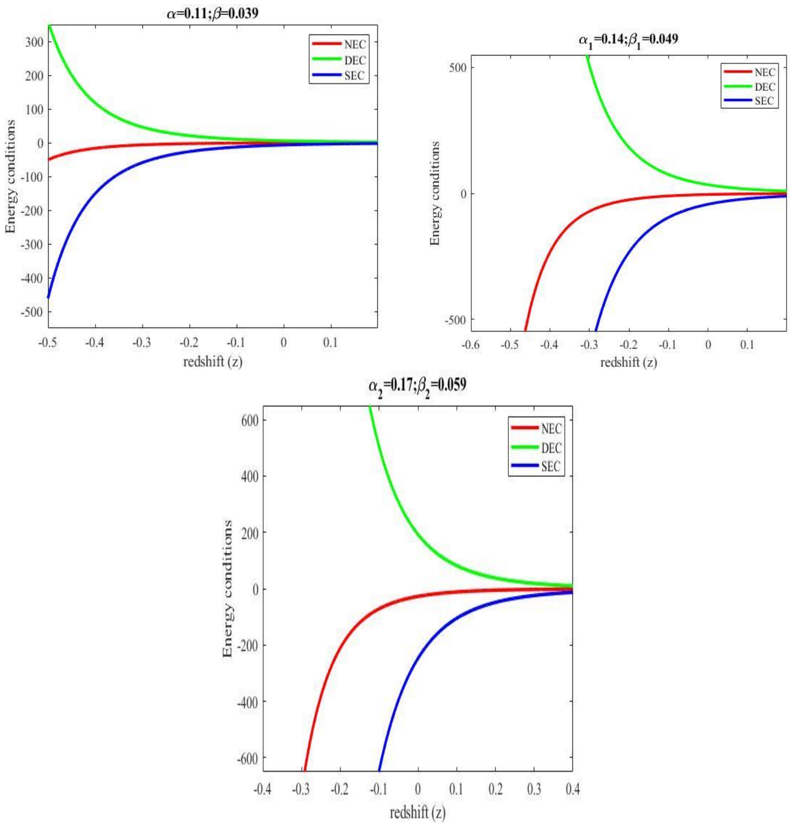

Figure 10: Energy conditions v/s redshift (z)

4 Results and Discussion

There has been a detailed discussion in the article on “the Marder type bulk viscous cosmological model in a modified theory”, with hybrid expansion law suggested by Akarsu et al. [54]. Here we have derived expressions for the some geometric and physical parameters for better understanding of the model. The conclusions of this work are being presented in figures(figure (1)-(10)) with the following values: $m = 2.75$, $\alpha = 0.11, 0.14, 0.17$, $\beta = 0.39, 0.049, 0.059$, $\kappa = 1$, $F_0 = -0.25$, $n = 6.25$,
$\omega = -0.99$, $\omega_0 = -1.33$, $\Psi = 1/3, 0, 1$ and $\zeta = 0.1$. We have the following description of the results, in accordance with the graphical description :

- In figure (1), (2)(left panel),(3)(left panel) represents string density Λ, energy density ρ, particle energy density ρ_p versus redshift(z) respectively and they vary in positive region throughout the evolution of the Universe and increasing against redshift irrespective of the values of α and β. In figures (2)(right panel) and (3)(right panel) represents plots of energy density(ρ) and particle energy density(ρ_p) versus redshift for various periods of cosmos with $\alpha = 0.11$ and $\beta = 0.039$, we observe various eras of cosmos such as matter dominated ($\Psi = 0$), radiation dominated ($\Psi = 1/3$) and stiff fluid ($\Psi = 1$) eras vary in positive regions through out the Universe’s evolution against redshift(z).

- In (4) pressure against redshift(z) has been considered. From (4)(left panel) we observe that pressure decreases against redshift irrespective of the values of α and β. Figure (4))(right panel) shows the same behaviour for different phases of the Universe(for $\Psi = 0, 1/3, 1$) against redshift for α and β.

- DP (q) versus redshift(z) is taken in (5) and in the present model (29) the parameter is time-dependent, exhibiting a transition from previous decelerated phase to present acceleratedphase of the Universe for three different values of α and β. The present values of DP (q) for our obtained model is $q \approx -0.738, -0.7898, -0.8245$ which is consistent with the various astrophysical observations.

- Figure (6) indicates the statefinder plane $r - s$ and we observe that the trajectories of (r, s) begin from chaplygin gas region ($r > 1, s < 0$), turns towards the ΛCDM point ($r = 1, s = 0$) by crossing quintessence and phantom ($r < 1, s > 0$) regions, Hence, the cosmos projects a ΛCDM model for the corresponding model.

- Figure (7) indicates $r - q$ plane’s evolution. In $q - r$ plane, radiation dominated era, matter dominated era and de-Sitter era lines are represented by $q = 1, 0.5, -1$ respectively. Also $r = 1$ represents ΛCDM line. Figure (7) explains the began of matter dominated phase and evolving to de-Sitter phase, showing the phase transition of the cosmos, as there is a signaturechange from positive to negative.

- We have plotted cosmographic parameters versus redshift in figure (8). It is observed that the cosmographic parameters jerk(j) and lerk(l) are varying in positive region against redshift(z) irrespective of the values of α and β. Whereas, the cosmographic parameter snap(s) shows sign change from previous negative values to current positive ones which agrees well with therecent observational data.

- In figure (9) we have plotted squared speed of sound(v_s^2) versus redshift(z). We observe that

the trajectories of v_s^2 are positive throughout the evolution and increasing against

redshift
irrespective of the values of α and β, which indicates the stable behavior of our model.

- As observed from figure (10), the nature of the energy conditions for the constructed model is well satisfied throughout the cosmic evolution for the DEC. However in late times, there is a violation for NEC and SEC. In the modified theory of gravity, the violation of SEC indicates anaccelerate expansion of the Universe.

5 Conclusions

To investigate the phenomenon of accelerated expansion of the cosmos, we have studied bulk viscous string cosmological model in f (R) theory of gravity with anisotropic Marder space-time, by taking hybrid expansion law. In the process we have investigated DP (q) and cosmographic parameters, squared speed of sound (v^2) and cosmic planes like state finder $r - s$ and $r - q$ for the model. For the corresponding model we have also investigated the energy conditions.

We have noticed that the model behaves like point type singularity at $t = 0$, ensuring the divergence of the kinematical and physical parameters, as the spacial volume $V \to 0$ at $t = 0$. At $t \to \infty$, spatial volume becomes infinite establishing the Universe's expansion at constant rate. For the model, the
anisotropic parameter $A_h \neq 0$ implying the anisotropic behavior of the model all over the cosmic evolution. Also, as cosmic time t approaches to ∞, the expansion scalar (θ), shear scalar (σ^2) and the mean Hubble's parameter H converges to a constant value i.e. the Universe is expanding ina constant rate. The energy density(ρ), particle energy density(ρ_m) and string density(λ) are all positive and increasing functions, where as pressure(p) is negative and decreasing function of the model. In correspondence with the current observations, the DP (q) depicts a transition from the initial deceleration phase to the current acceleration phase. The state finder parameters corresponds to ΛCDM limit and besides showing Chaplgyin gas, quintessence and phantom like behavior. The $r - q$ plane of our model shows SCDM in the past and de-Sitter phase of the Cosmos in future.

The squared speed of sound(v^2)show a stable behavior for our model. For our model (29), DEC obeyed, but NEC and SEC are violated. In our model the cosmographic parameters jerk and lerk vary in positive regions whereas snap parameter shows signature flipping nature from previous decelerationto present acceleration.

References

1. Adam G Riess, Alexei V Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M Garnavich, Ron L Gilliland, Craig J Hogan, Saurabh Jha, Robert P Kirshner, et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J., 116(3):1009, 1998.
2. Saul Perlmutter, Goldhaber Aldering, Gerson Goldhaber, RA Knop, Peter Nugent, Patricia G Castro, Susana Deustua, Sebastien Fabbro, Ariel Goobar, Donald E Groom, et al. Measurements of Ω and Λ from 42 high-redshift supernovae. Astrophys. J., 517(2):565, 1999.
3. David N Spergel, R Bean, O Doré, MR Nolta, CL Bennett, Joanna Dunkley, G Hinshaw, N ea Jarosik, E Komatsu, L Page, et al. Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: implications for cosmology. Astrophys. J. Suppl. Ser., 170(2):377, 2007.
4. Daniel J Eisenstein, Idit Zehavi, David W Hogg, Roman Scoccimarro, Michael R

Blanton, Robert C Nichol, Ryan Scranton, Hee-Jong Seo, Max Tegmark, Zheng Zheng, et al. Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies. Astrophys. J., 633(2):560, 2005.

5. Shaun Cole, Will J Percival, John A Peacock, Peder Norberg, Carlton M Baugh, Carlos S Frenk, Ivan Baldry, Joss Bland-Hawthorn, Terry Bridges, Russell Cannon, et al. The 2dF Galaxy Redshift Survey: power-spectrum analysis of the final data set and cosmological implications. Mon. Not. R. Astron. Soc., volume=362, number=2, pages=505–534, year=2005,publisher=The Royal Astronomical Society.
6. Ed Hawkins, Steve Maddox, Shaun Cole, Ofer Lahav, Darren S Madgwick, Peder Norberg, John A Peacock, Ivan K Baldry, Carlton M Baugh, Joss Bland-Hawthorn, et al. The 2dF Galaxy Redshift Survey: correlation functions, peculiar velocities and the matter density of the Universe. Mon. Not. R. Astron. Soc., volume=346, number=1, pages=78–96, year=2003, publisher=Blackwell Science Ltd Oxford, UK.
7. Simran Arora, Xin-he Meng, SKJ Pacif, and PK Sahoo. Effective equation of state in modified gravity and observational constraints. Class. Quantum Gravity, 37(20):205022, 2020.
8. Tiberiu Harko, Francisco SN Lobo, Shin'ichi Nojiri, and Sergei D Odintsov. $f(R,T)$ gravity.
9. Phys. Rev. D, 84(2):024020, 2011.
10. PHRS Moraes and PK Sahoo. Modeling wormholes in $f(R,T)$ gravity. Phys. Rev. D,96(4):044038, 2017.
11. Kadhim, R. R., and M. Y. Kamil. "Evaluation of Machine Learning Models for Breast Cancer Diagnosis Via Histogram of Oriented Gradients Method and Histopathology Images". International Journal on Recent and Innovation Trends in Computing and Communication, vol. 10, no. 4, Apr. 2022, pp. 36-42, doi:10.17762/ijritcc.v10i4.5532.
12. Z Yousaf, MZUH Bhatti, and M Ilyas. Existence of compact structures in $f(R,T)$ gravity. Eur.Phys. J. C, 78(4):1–13, 2018.
13. Sebastian Bahamonde, M Zubair, and G Abbas. Thermodynamics and cosmologicalreconstruction in $f(T,B)$ gravity. Phys. Dark Univ., 19:78–90, 2018.
14. Salvatore Capozziello, Maurizio Capriolo, and Loredana Caso. Weak field limit and gravitational waves in $f(T,B)$ teleparallel gravity. Eur. Phys. J. C, 80(2):1–11, 2020.
15. K Atazadeh and F Darabi. Energy conditions in f (R, G) gravity. Gen. Relativ. Gravit., 46(2):1–14, 2014.
16. Mariafelicia De Laurentis, Mariacristina Paolella, and Salvatore Capozziello. Cosmologicalinflation in F(R,G) gravity. Phys. Rev. D, 91(8):083531, 2015.
17. Hans A Buchdahl. Non-linear Lagrangians and cosmological theory. Mon. Not. R. Astron. Soc.,150(1):1–8, 1970.
18. Salvatore Capozziello and Mariafelicia De Laurentis. Extended theories of gravity. Phys. Rept.,509(4-5):167–321, 2011.
19. Shin'ichi Nojiri and Sergei D Odintsov. Modified gravity with negative and positive powers of curvature: Unification of inflation and cosmic acceleration. Phys. Rev. D, 68(12):123512, 2003.
20. Shin'ichi Nojiri and Sergei D Odintsov. Introduction to modified gravity and gravitational alternative for dark energy. Int.J.Geom.Methods Mod.Phys., 4(01):115–145, 2007.
21. Bulla, P. . "Traffic Sign Detection and Recognition Based on Convolutional Neural Network". International Journal on Recent and Innovation Trends in Computing and Communication, vol. 10, no. 4, Apr. 2022, pp. 43-53, doi:10.17762/ijritcc.v10i4.5533.
22. Timothy Clifton, Pedro G Ferreira, Antonio Padilla, and Constantinos Skordis. Modified

gravityand cosmology. Phys. Rep., 513(1-3):1–189, 2012.
23. Thomas P Sotiriou and Valerio Faraoni. f (R) theories of gravity. Rev. Mod. Phys., 82(1):451,2010.
24. Shin'ichi Nojiri and Sergei D Odintsov. Unified cosmic history in modified gravity: from F(R) theory to Lorentz non-invariant models. Phys. Rep., 505(2-4):59–144, 2011.
25. Chaudhary, D. S. . (2022). Analysis of Concept of Big Data Process, Strategies, Adoption and Implementation. International Journal on Future Revolution in Computer Science &Amp; Communication Engineering, 8(1), 05–08. https://doi.org/10.17762/ijfrcsce.v8i1.2065
26. Sh Nojiri, SD Odintsov, and VK3683913 Oikonomou. Modified gravity theories on a nutshell:inflation, bounce and late-time evolution. Phys. Rep., 692:1–104, 2017.
27. Shin'ichi Nojiri and Sergei D Odintsov. Dark energy, inflation and dark matter from modified
28. F(R) gravity. arXiv preprint arXiv:0807.0685, 2008.
29. M Vijaya Santhi and T Chinnappalanaidu. Strange quark matter cosmological models attached to string cloud in f (R) theory of gravity. Indian J. Phys., 96(3):953–962, 2022.
30. Parth Shah and Gauranga C Samanta. Cosmological dynamics of f (R) models in dynamical system analysis. Int. J. Mod. Phys. A, 35(22):2050124, 2020.
31. Aroonkumar Beesham and Kazuharu Bamba. Inflationary Universe from Anomaly-Free-Gravity. Adv. H. Ene. Phys., 2022, 2022.
32. M Vijaya Santhi, T Ch Naidu, and D Ch Papa Rao. Some Bianchi Type Bulk Viscous String Cosmological Models in f (R) Gravity. In J. Phys. Conf. Series, volume 1344, page 012036. IOP Publishing, 2019.
33. M. Dursun and N. Goker, "Evaluation of Project Management Methodologies Success Factors Using Fuzzy Cognitive Map Method: Waterfall, Agile, And Lean Six Sigma Cases", Int J Intell Syst Appl Eng, vol. 10, no. 1, pp. 35–43, Mar. 2022.
34. Onur Özdemir and Can Aktaş. Anisotropic universe models with magnetized strange quark matter in f (R) gravity theory. Mod. Phys. Lett. A, 35(14):2050111, 2020.
35. John D Barrow. The deflationary universe: an instability of the de Sitter universe. Phys. Lett. B, 180(4):335–339, 1986.
36. I Brevik and SD Odintsov. Cardy-Verlinde entropy formula in viscous cosmology. Phys. Rev. D, 65(6):067302, 2002.
37. I Brevik and O Gorbunova. Dark energy and viscous cosmology. Gen. Relativ. Gravit, 37:2039,2005.
38. C Eckart. The isotropy of the Universe. Phys. Rev, 58:919, 1940.
39. L Landau and E Lifshitz. Fluid mechanics second edition, 1987.
40. W Israel and JM Stewart. Thermodynamics of nonstationary and transient effects in a relativisticgas. Phys. Lett. A, 58(4):213–215, 1976.
41. M. . Parhi, A. . Roul, B. Ghosh, and A. Pati, "IOATS: an Intelligent Online Attendance Tracking System based on Facial Recognition and Edge Computing", Int J Intell Syst Appl Eng, vol. 10, no. 2, pp. 252–259, May 2022.
42. Rakesh Raushan, AK Shukla, R Chaubey, and T Singh. Locally rotationally symmetric Bianchi type-I cosmological model with dynamical Λ and G in f (R) gravity. Pramana – J. Phys., 92(5):1–9, 2019.
43. RK Mishra and Heena Dua. Bulk viscous string cosmological models in Saez-Ballester theoryof gravity. Astrophys Space Sci., 364(11):1–12, 2019.
44. Archana Dixit, Rashid Zia, and Anirudh Pradhan. Anisotropic bulk viscous string cosmological models of the Universe under a time-dependent deceleration parameter. Pramana – J. Phys., 94(1):1–11, 2020.
45. UY Divya Prasanthi and Y Aditya. Kantowski-Sachs bulk viscous string cosmological

model in f (R) theory of gravity. In J.Phys.Conf.Series, volume 1344, page 012041. IOP Publishing, 2019.

46. Vanitha, D. D. . (2022). Comparative Analysis of Power switches MOFET and IGBT Used in Power Applications. International Journal on Recent Technologies in Mechanical and Electrical Engineering, 9(5), 01–09. https://doi.org/10.17762/ijrmee.v9i5.368
47. Yakov Boris Zel'dovich, I Yu Kobzarev, and Lev Borisovich Okun. Cosmological consequences of spontaneous violation of discrete symmetry. Zh. Eksp. Teor. Fiz., 40:3–11, 1974.
48. Thomas WB Kibble. Topology of cosmic domains and strings. J Phys. A:Math.Gen., 9(8):1387, 1976.
49. Tom WB Kibble. Some implications of a cosmological phase transition. Phys. Rept., 67(1):183– 199, 1980.
50. Allen E Everett. Cosmic strings in unified gauge theories. Phys. Rev. D, 24(4):858, 1981.
51. Alexander Vilenkin. Cosmic strings. Phys. Rev. D, 24(8):2082, 1981.
52. Alexander Vilenkin. Gravitational field of vacuum domain walls and strings. Phys. Rev. D, 23(4):852, 1981.
53. L Marder. Gravitational waves in general relativity II. The reflexion of cylindrical waves. Proc. R. Soc. London A, 246(1244):133–143, 1958.
54. Sezgin Aygün, Hüsnü Baysal, and Ismail Tarhan. Energy momentum of Marder universe in
55. teleparallel gravity. Int. J. Theor. Phys., 46(10):2607–2616, 2007.
56. Can Aktaş, Sezgin Aygün, and İhsan Yılmaz. Behaviors of dark energy and mesonic scalar field for anisotropic universe in f (R) gravity. Phys. Lett. B, 707(2):237–242, 2012.
57. Can Aktaş, Sezgin Aygün, and Pradyumn Kumar Sahoo. Relationship between magnetic field and anisotropy parameter in gravitation theories. Mod. Phy. Let. A, 33(24):1850135, 2018.
58. Sezgin Aygün. Various dark energy models with time-varying G and Λ in $f(R,T)$ gravitation theory. Mod. Phy. Let. A, 34(34):1950280, 2019.
59. DD Pawar and SP Shahare. Anisotropic tilted cosmological model in $f(R,T)$ theory of gravity.
60. New Astr., 75:101318, 2020.
61. DD Pawar, SP Shahare, and VJ Dagwal. Homogeneous space time with wet dark fluid. NewAstr., 87:101599, 2021.
62. CB Collins, EN Glass, and DA Wilkinson. Exact spatially homogeneous cosmologies. Gen.Relativ. Gravit., 12(10):805–823, 1980.
63. VB Johri and R Sudharsan. BD-FRW cosmology with bulk viscosity. Aust.J.Phys., 42(2):215–222, 1989.
64. Özgür Akarsu, Suresh Kumar, R Myrzakulov, M Sami, and Lixin Xu. Cosmology with hybrid expansion law: scalar field reconstruction of cosmic history and observational constraints. J. Cosmol. Astropart. Phys., 2014(01):022, 2014.
65. Gia Dvali, Gregory Gabadadze, and Massimo Porrati. 4D gravity on a brane in 5D Minkowskispace. Phys. Lett.B, 485(1-3):208–214, 2000.
66. Christian Armendariz-Picon, V Mukhanov, and Paul J Steinhardt. Dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration. Phys. Rev. Lett., 85(21):4438, 2000.
67. Alexander Kamenshchik, Ugo Moschella, and Vincent Pasquier. An alternative to quintessence.
68. Phys. Lett. B, 511(2-4):265–268, 2001.
69. Bharat Ratra and Philip JE Peebles. Cosmological consequences of a rolling

homogeneous scalar field. Phys. Rev. D., 37(12):3406, 1988.

70. Varun Sahni, Tarun Deep Saini, Alexei A Starobinsky, and Ujjaini Alam. Statefinder—a new geometrical diagnostic of dark energy. J. Exp. Theor. Phys., 77(5):201–206, 2003.
71. Amalkumar Raychaudhuri. Relativistic cosmology. I. Phys. Rev., 98(4):1123, 1955.
72. Mustafa Salti, Oktay Aydogdu, and Irfan Acikgoz. Extended scalar–tensor theory and thermodynamics in teleparallel framework. Mod. Phys. Lett. A, 31(33):1650185, 2016.
73. PHRS Moraes and PK Sahoo. The simplest non-minimal matter–geometry coupling in the
74. $f(R,T)$ cosmology. Eur. Phys. J. C, 77(7):1–8, 2017.
75. EA Hegazy and Farook Rahaman. Bianchi type VI0 cosmological model with electromagnetic field and variable deceleration parameter in general theory of relativity. Indian J. Phys., 94(11):1847–1852, 2020.
76. Ajay Kumar and CP Singh. Viscous Ricci dark energy model with matter creation: Exact solution and observational tests. Pramana J. Phys., 94(1):1–13, 2020.
77. Piyali Bhar, Tuhina Manna, Farook Rahaman, Saibal Ray, and GS Khadekar. Charged perfect fluid sphere in higher-dimensional spacetime. Indian J. Phys., 94(10):1679–1690, 2020.
78. B Mishra, SK Tripathy, and Sankarsan Tarai. Accelerating models with a hybrid scale factor inextended gravity. J. Astrophys. Astr., 42(1):1–15, 2021.
79. B Mishra, F Md Esmeili, and Saibal Ray. Cosmological models with variable anisotropic parameter in $f(R,T)$ gravity. Indian J. Phys., 95(10):2245–2254, 2021.
80. Abdul Aziz, Sourav Roy Chowdhury, Debabrata Deb, Saibal Ray, Farook Rahaman, and BK Guha. Analytic radiation model for perfect fluid under homotopy perturbation method. Indian J. Phys., 95(7):1581–1588, 2021.
81. Mahbubur Rahman Mollah and Kangujam Priyokumar Singh. Behaviour of viscous fluid in string cosmological models in the framework of Lyra geometry. New Astr., 88:101611, 2021.