Construction of Minimization of Multiplicative Graphs

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Abstract
In this paper , we evince a method to construct minimization of multiplicative graphs and also establish the minimization of multiplicative of the path union of 'n' copies of a cycle to the solution of a system involving equations .
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1.Introduction

Graphs regarded here are finite , undirected and simple. The symbols V(G) and E(G) denote the vertex set and the edge set of a graph G. Most graph labeling methods are derived from the descendants of by Rosa's[2] findings of the year 1967.A graph labeling is an assignment of integer to the vertices or edges or both subject to certain conditions. Labeled graph has many branch out applications such as coding theory , missile guidance , X-ray , crystallography analysis , communication network addressing systems , astronomy , radar ,circuit design , database management etc . Minimization of multiplicative labelings was introduced by Shalini.P et. al.[3] . In this paper , we evince a method to construct minimization of multiplicative graphs and also establish the minimization of multiplicative of the path union of 'n' copies of a cycle to the solution of a system involving equations .

2. Preliminaries

Definition 2.1 [2] : Let G= (V(G), E(G)) be a graph G. A graph G is said to be minimization of multiplicative labeling if there exist a bijective function from the vertices of G to the set {1,2,3,…..p} such that when each edge uv is assigned the label f(uv)=f(u)*f(v) f(v)}, then the resulting edge labels are distinct numbers.

Definition 2.2 [2]: A minimization of multiplicative graph with weight as perfect squares numbers are called perfect minimization of multiplicative graph.

3. Main Results

Rather than defining a map f , we implement firstly the work in obverse order by verifying that the set {f(u)*f(v)-min{f(u),f(v)} are distinct . Since $1\leq f(v)\leq n$, for every $v\in V$, we restrict that the edge labels {f(u)*f(v)-min{f(u),f(v)} are distinct and can vary from 1 to $(p-1)^2$ as well . Now a minimization of multiplicative labeling is purely a choice of pair of distinct integers from n consecutive integers which is defined as E_{ki} = { (k,i) : ki-k , where $2\leq i\leq |V(G)|$ and $1\leq k\leq |V(G)|-1$; k<i} , where $1\leq E_{ki}\leq (p-1)^2$should be distinct . The pairs are chosen according to the structure of the graph .

Theorem 3.1 : Let $v_1, v_2, \ldots v_m\, v_1$ be a cycle and let $\beta_1=2, \beta_2=3, \ldots\, \beta_n=U_1+1$ where U_1 is the number of pendant vertices attached to v_1 and in general U_j is the number of pendant vertices attached to v_j where $1\leq j\leq m$ is a sequence of nonnegative integers. Then the graph G obtained by attaching β_i pendant vertices to v_j , $1\leq j\leq m$ is minimization of multiplicative graph .

Proof : Let E_{ki} be the edge label received from the vertex label pair (k,i) of the vertices v_k and v_i , where k<i. Define E_{ki} = { (k,i) : ki-k , where $2\leq i\leq |V(G)|$ and $1\leq k\leq |V(G)|-1$; k<i} , where $1\leq E_{ki}\leq (p-1)^2$where $1\leq E_{ki}\leq (p-1)^2$.

To start with select the edge pairs $(1, \beta_1), (1, \beta_2), \ldots(1, \beta_n)$ for the pendant edges adjacent v_1 which brings forth an edge label E_{1i} = { (1,i) :i-1 where $i\geq 2$} and identify 1 with v_1 .

Select the edge pairs $(\beta_n+1, \beta_n+2), (\beta_n+1, \beta_n+3), \ldots(\beta_n+1, \beta_n+1+U_2)$ for the pendant edges adjacent v_2 which brings forth an edge label $E\,\beta_n+1$={ $(\beta_n+1)i-\beta_n+1$: (β_n+1,i) where $i\geq \beta_n+2$} and identify β_n+1 with v_2 .Select the pairs $(\beta_n+1+U_2+1, \beta_n+1+U_2+2)$, $((\beta_n+1+U_2+1, \beta_n+1+U_2+3), \ldots\ldots (\beta_n+1+U_2+1, \beta_n+1+U_2+1+U_3)$ and identify $\beta_n+1+U_2+1$with v_3 and repeat the process .Finally select the single edge pair $(\beta_n+1+U_2+U_3+U_4+\ldots+U_{m-1}+1, \beta_n+1+U_2+U_3+U_4+\ldots+U_{m-1}+1+U_m)$ which brings forth an edge label $(\beta_n+1+U_2+U_3+U_4+\ldots+U_{m-1}+1)^2=(p-1)^2$ .Henceforth all the selected pairs results in distinct edge labels . Hence G is a minimization of multiplicative graph.

Theorem 3.2 : $(C_m \times P_2) \odot \overline{K_n}$ is a minimization of multiplicative graph .

Proof : It is obvious from theorem 3.1

Theorem 3.3 : Let G be a firecracker in which all the stars are of different or/and same size. Then G is a minimization of multiplicative graph .

Proof : Let there be n stars each of size $m_1, m_2, m_3, \ldots, m_n$. Let $v_1, v_2, \ldots v_n$ be the pendant vertices of the stars joined to form a path P_n . Let $u_1, u_2, \ldots u_n$ be the central vertices of the stars. Let $m_1+m_2+m_3+\ldots+m_n=m$. The order of the firecracker is n(m + 1). Let E_{ki} be the edge label received from the vertex label pair (k,i) of the vertices v_k and v_i , where k<i. Define E_{ki} = { (k,i) : ki-k , where $2\leq i\leq |V(G)|$ and $1\leq k\leq |V(G)|-1$; k<i} , where $1\leq E_{ki}\leq (p-1)^2$where $1\leq E_{ki}\leq (p-1)^2$.Let x_1 be the number of pendant vertices of the first star say S_1 with the central vertex u_1 .Let x_2 be the number of pendant vertices of the second star say S_2 with the central vertex u_2 .In general , let x_n be the number of pendant vertices of the n^{th} star say S_n with the central vertex u_n. Select the edge pairs $(1,2),(1,3),\ldots(1,x_1+1)$ for the edges E_{12} , $E_{13}, \ldots E_{1(x1+1)}$ and label the central vertex u_1 with 1 . Select the edge pairs (x_1+2,x_1+3),(

$x_1+2,x_1+4),\ldots(x_1+2, x_1+x_2+2)$ for the edges $E_{(x1+2,x1+3)}$),$E_{(x1+2,x1+4)}$, …..$E_{(x1+2, x1+x2+2)}$ and label the central vertex u_2 with x_1+2 .Select the edge pairs $(x_1+x_2+3,x_1+x_2+4),(x_1+x_2+3,x_1+x_2+5),\ldots(x_1+x_2+3,x_1+x_2+x_3+3)$ for the edges $E_{(x1+x2+3,x1+x2+4)}$,$E_{(x1+x2+3,x1+x2+5)}$, …..$E_{(x1+x2+3,x1+x2+x3+3)}$ and label the central vertex u_3 with x_1+x_2+3 .Proceeding like this select the edge pairs $(x_1+x_2+\ldots+x_{n-1}+n,x_1+x_2+\ldots+x_{n-1}+n+1),(x_1+x_2+\ldots+x_{n-1}+n,x_1+x_2+\ldots+x_{n-1}+n+2),\ldots\ldots(x_1+x_2+\ldots+x_{n-1}+n,x_1+x_2+\ldots+x_n+n)$ for the edges $E_{(x1+x2+\ldots+xn-1+n,x1+x2+\ldots+xn-1+n+1)}$,$E_{(x1+x2+\ldots+xn-1+n,x1+x2+\ldots+xn-1+n+2)}$,……$E_{(x1+x2+\ldots+xn-1+n,x1+x2+\ldots+xn+n)}$ and label the central vertex u_n with $x_1+x_2+\ldots+x_{n-1}+n$ .Select the edge pair $(x_1+x_2+\ldots+x_n+n+1,x_1+x_2+\ldots+x_n+n+2)$ for the edge v_1v_2 of the path . Select the edge pair $(x_1+x_2+\ldots+x_n+n+2,x_1+x_2+\ldots+x_n+n+3)$ for the edge v_2v_3 of the path . Select the edge pair $(x_1+x_2+\ldots+x_n+n+1,x_1+x_2+\ldots+x_n+n+2)$ for the edge v_1v_2 of the path . Proceeding like this select the edge pair $(x_1+x_2+\ldots+x_n+n+(n-1),x_1+x_2+\ldots+x_n+n+n)$ for the edge $v_{n-1}v_n$ of the path .By assigning the vertex labels following the method described gives a minimization of multiplicative labeling of the firecracker graph with all the edges are distinct and lies between 1 and $(p-1)^2$.

Theorem 3.4 : C_m is a minimization of multiplicative graph .

Proof : For m =3 , consider the edge pairs (1,2) ,(2,3) ,(3,1) clearly brings forth C_3 is a minimization of multiplication graph . For m=4 , Consider the edge pairs (1,2) , (2,3) ,(3,4) and (4,1) admits minimization of multiplicative graph . For m= 5 ,Selecting the edge pairs as (1,2) , (2,3),(3,5),(4,5) and (4,1) brings forth minimization of multiplicative labeling for C_5 . For m>5 , select the edge pairs as (1,2),(2,3),(3,4)……(m-1,m) ,(m,1) brings forth distinct edge numbers such that $1\leq f(uv)\leq (p-1)^2$ .Let the cycle graph C_m has m vertices $A_1, A_2,A_3,\ldots A_m$ and m edges .

Theorem 3.5 : $C_{ar^{n-1}} \times P_n$, where a=3 , r=2 , n $\geq$ 1 is a minimization of multiplicative graph .

Proof : Let the cycle graph $C_{ar^{n-1}}$ has ar^{n-1} vertices $A_1, A_2,A_3,\ldots A_{ar^{n-1}}$ and ar^{n-1} edges . Let the path P_n has n vertices $B_1,B_2,\ldots B_n$ and n-1 edges . Then $|V(C_{ar^{n-1}} \times P_n)| = ar^{n-1}n$ where a=3 , r=2 , n $\geq$ 1 and $|E(C_{ar^{n-1}} \times P_n)| = ar^{n-1}(n-1)+nar^{n-1}$.Let E_{ki} be the edge label received from the vertex label pair (k,i) of the vertices v_k and v_i , where k<i. Define E_{ki} = { ki-k : (k,i) , where $2\leq i\leq |V(G)|$ and $1\leq k\leq |V(G)|-1$; k<i} , where $1\leq E_{ki}\leq (p-1)^2$where $1\leq E_{ki}\leq (p-1)^2$. Let the cycle graph $C_{ar^{n-1}}$ has ar^{n-1} vertices $A_1, A_2,A_3,\ldots A_{ar^{n-1}}$ and ar^{n-1} edges . Let the path P_n has n vertices $B_1,B_2,\ldots B_n$ and n-1 edges .Then $|V(C_{ar^{n-1}} \times P_n)| = ar^{n-1}n$ where a=3 , r=2 , n $\geq$ 1 and $|E(C_{ar^{n-1}} \times P_n)| = ar^{n-1}(n-1)+nar^{n-1}$.Select the edge pairs $(1,2),(2,3),(3,4),\ldots(ar^{n-1}-1, ar^{n-1}),(1, ar^{n-1})$ for the edges of the base cycle $(A_1B_1, A_2B_1),(A_2B_1, A_3B_1),\ldots(A_{ar^{n-1}-1}B_1, A_{ar^{n-1}}B_1),(1, A_{ar^{n-1}}B_1)$ .Then select the edge pairs as $(ar^{n-1}+1, ar^{n-1}+2),\ldots\ldots(ar^{n-1}+1, nar^{n-1}+1)$ for the edges$(A_1B_{ar^{n-1}}, A_2B_{ar^{n-1}}),(A_2B_{ar^{n-1}}, A_3B_{ar^{n-1}}),\ldots(A_{ar^{n-1}-1}B_{ar^{n-1}}, A_{ar^{n-1}}B_{ar^{n-1}}),(A_1B_{ar^{n-1}},, A_{ar^{n-1}}B_{ar^{n-1}})$ where a=3 , r=2 , n $\geq$ 1 . Select the edge label pairs $(1, ar^{n-1}+1),(ar^{n-1}+1, ar^{n-1}+ar^{n-1}+1),\ldots\ldots\ldots(ar^{n-1}+1,(n-1)ar^{n-1}+1)$ for the edges $(A_1B_1, A_1B_2),(A_1B_2, A_1B_3),\ldots(A_1B_{n-1}, A_1B_n)$. For the edges $(A_2B_1, A_2B_2),(A_2B_2, A_2B_3),\ldots(A_2B_{n-1}, A_2B_n)$ choose the edge label pairs as $(2, ar^{n-1}+2),(ar^{n-1}+2, ar^{n-1}+ar^{n-1}+2),\ldots\ldots\ldots(ar^{n-1}+2,(n-1)ar^{n-1}+2)$.Proceeding like this we arrive at the edge label pairs $(ar^{n-1},ar^{n-1}+ar^{n-1}),(ar^{n-1}+ar^{n-1}, ar^{n-1}+ar^{n-1}+ar^{n-1})\ldots((n-1)ar^{n-1},nar^{n-1})$ for the

edges $(A_{ar^{n-1}}B_1, A_{ar^{n-1}}B_2)$,$(A_{ar^{n-1}}B_2, A_{ar^{n-1}}B_3)$,…$(A_{ar^{n-1}}B_{n-1}, A_{ar^{n-1}}B_n)$ .This process brings forth the minimization of multiplication labeling of $C_{ar^{n-1}} \times P_n$, where a=3 , r=2 , n $\geq$ 1 .

Theorem 3.6 : $C_m \times P_n$, where m$\neq$ ar^{n-1} ; a=3 , r=2 , n $\geq$ 1 is not a minimization of multiplicative graph .

Proof : It is obvious that from theorem 3.5 , that , for $C_m \times P_n$, where m$\neq$ ar^{n-1} ; a=3 , r=2 , n $\geq$ 1 2 edge pairs out of ar^{n-1} (n-1)+nar^{n-1} edge pairs reproduce similar edge labels which is a contradiction to the definition of the minimization of multiplicative labeling

Theorem 3.7 : The graph G = $P_r + \overline{K_s}$ is a minimization of multiplicative graph when s$\geq$ r .

Proof : Let v_1, v_2,…v_r be the path P_r and u_1,u_2,…..u_s be the vertices of $\overline{K_s}$. Clearly G has r+s vertices and rs+r-1 edges . Let E_{ki} be the edge label received from the vertex label pair (k,i) of the vertices v_k and v_i , where k<i. Define E_{ki} = { (k,i):ki-k , where 2≤i≤ $|V(G)|$ and 1≤k≤ $|V(G)|$-1 ; k<i} , where 1≤E_{ki}≤ $(p-1)^2$where 1≤E_{ki}≤ $(p-1)^2$.Select the pairs (1,2), (2,3),(2,5) …..(2,s) for the edges E_{12} , E_{23} ,E_{25} , E_{27},…..E_2s and Identify $\frac{t}{2}$ – (w+1) with v_1 is shown in the table 1.

Values of s = t-r	Corresponding values of 'w'
2	-1
3	0
4	1
5	2
6	3
In general for s	s-3

Table 1

Select the pairs (1,4), (3,4),(4,5) …..(4,s) for the edges E_{14} , E_{34} ,E_{45} , E_{47},…..E_4s . Identify $\frac{t}{2}$ – (w_1+4) with v_2 for t= rs =4,6,8 is shown in the following table 2 and for t= rs$\geq$ 10 Identify $\frac{t}{2}$ +w_2 with v_2 is given in table 3 .

Values of s = t-r	Corresponding values of 'w_1'
2	-6
3	-5
4	-4

Table 2

Values of h	**Values of s = t-r**	**Corresponding values of 'w_2'**
1	5	-1
2	6	-2
3	7	-3
In general h	In general h+4	s –(2h+4)

Table 3

Select the pairs (1,6), (3,6),(5,6) …..(6,s) for the edges E_{16} , E_{36} ,E_{56} , E_{67},…..E_{6}s . Identify $\frac{t}{2}$ +w_3 with v_3 is shown in the following table 4.

Values of s = t-r	**Corresponding values of 'w_3'**
3	3
4	2
5	1
6	0

Table 4

Values of 'h'	**Values of s = t-r**	**Corresponding values of 'w_3'**
1	7	-1
2	8	-2
3	9	-3
In general h	In general h+6	In general s-(s+h)

Table 5

Finally the labeling process will arrive at selecting the edge pairs (1,r), (3,r),(5,r) …..(s,r) for the edges E_{1r} , E_{3r},E_{5r} , E_{7r},…..E_{sr} . Identify $2|V(P_r)|$ with v_r .This construction leads to the minimization of multiplicative labeling of P_r + $\overline{K_s}$ where s= r and s>r such that s=r+1 .

Theorem 3.8 : The graph G= $C_3+\overline{K_n}$ is minimization of multiplicative labeling for all n.

Proof : Let v_1v_2,v_2v_3,v_3v_1 be the cycle C_3 and let u_1,u_2,….u_s be the vertices of $\overline{K_n}$. Here $|E(G)|$=3n+3 . Let E_{ki} be the edge label received from the vertex label pair (k,i) of the vertices v_k and v_i , where k<i. Define E_{ki} = {(k,i) : ki-k, where 2≤i≤ $|V(G)|$ and 1≤k≤ $|V(G)|$-1 ; k<i} , where 1≤E_{ki}≤ $(p-1)^2$where 1≤E_{ki}≤ $(p-1)^2$. Select the edge pairs (1,$\left|V(\overline{K_n})\right|$+2) , ($\left|V(\overline{K_n})\right|$+2, $\left|V(\overline{K_n})\right|$+3),(1, $\left|V(\overline{K_n})\right|$+3) for the cycle edges $E_{1|V(\overline{K_n})|+2}$, $E_{(|V(\overline{K_n})|+2)(|V(\overline{K_n})|+3)}$, $E_{1(|V(\overline{K_n})|+3)}$ and identify the label of the vertices of the cycle as1 with v_1 , $\left|V(\overline{K_n})\right|$+2 with v_2 and $\left|V(\overline{K_n})\right|$+3 with v_3 . Select the edge pairs (1,2), (2, $\left|V(\overline{K_n})\right|$+2),(2, $\left|V(\overline{K_n})\right|$+3) for the edges E_{12} , $E_{1|V(\overline{K_n})|+2}$, $E_{2|V(\overline{K_n})|+3}$ and identify the label of the vertex as 2 with u_1 . Select the edge pairs (1,3),(3, $\left|V(\overline{K_n})\right|$+2),(3, $\left|V(\overline{K_n})\right|$+3) for the edges E_{13} , $E_{3|V(\overline{K_n})|+2}$, $E_{3|V(\overline{K_n})|+3}$ and

identify the label of the vertex as 3 with u_2 . Proceeding like this we will arrive at the selection of edge pairs (1, $|V(\overline{K_n})|$+1),($|V(\overline{K_n})|$+1, $|V(\overline{K_n})|$+2) ,($|V(\overline{K_n})|$+1, $|V(\overline{K_n})|$+3) for the edges $E_{1(|V(\overline{K_n})|+1)}$, $E_{(|V(\overline{K_n})|+1)(|V(\overline{K_n})|+2)}$, $E_{(|V(\overline{K_n})|+1)(|V(\overline{K_n})|+3)}$ and identify the label of the vertex as $|V(\overline{K_n})|$+1 with u_s . Clearly this process brings forth minimization of multiplicative labeling of G= $C_3+\overline{K_n}$ for all 'n'.

Theorem 3.9 : The complete tripartite graph $K_{1,g,h}$ is minimization of multiplicative graph for g=h and g>h such that g=h+1 and g<h such that h=g+1.

Proof : Let {z},{w_1 , w_2 ,…..w_g } ,{x_1 , x_2 ,….x_h } be the vertex set of the tripartite graph .Here $|E(K_{1,g,h})|$ = gh+g+h .

Case 1 : g=h

Select the edge pairs (1,2) , (1,4) ,(1,6),…(1,h) for the edges E_{12} , E_{14} ,….E_{1h} and identify 1 with w_1 . Select the edge pairs (2,3),(3,4),…(3,h) for the edges E_{23}, E_{34} ,…E_{3h} and identify 3 with w_2 .Proceeding like this finally we arrive at the selection of the vertex label for w_g by selecting the edge pairs (2,2g-1),(4,2g-1),…(2g-1,2h) and identify 2g-1 with w_g . Select the edge pairs (1,2),(2,3),…(2,2g-1) for the edges E_{12}, E_{23} ,…$E_{2(2g-1)}$ and identify 2 with x_1 .Select the edge pairs (1,4),(3,4),…(4,2g-1) for the edges
E_{14}, E_{34} ,…$E_{4(2g-1)}$ and identify 4 with x_2 . Proceeding like this finally we arrive at the selection of the vertex label for x_h . Select the edge pairs (1,2h),(3,2h),…(2g-1,2h) and identify a label 2h for the vertex x_h .Then , select the edge pairs (1,g+h+1),(3,g+h+1),(5,g+h+1),…(2g-1,g+h+1),(2,g+h+1),(4,g+h+1),….(2h,g+h+1) and identify g+h+1 with z . All these selected edge pairs brings forth the assignment of the vertex labels and all the edges are distinct and lies between 1 and $(p-1)^2$.

Case 2 : g>h where g=h+1

Proceed the labeling procedure as mentioned in case 1

Case 3 : g<h where h=g+1

Proceed the labeling procedure as mentioned in case 1 except for the vertex x_h , select the edge pairs (1,2h-1),(3,2h-1),(5,2h-1),….(2g-1,2h-1) and identify 2h-1 with x_h .And select the edge pairs (1,g+h+1),(3,g+h+1),(5,g+h+1),…(2g-1,g+h+1),(2,g+h+1),(4,g+h+1),….(2h-1,g+h+1) and identify g+h+1 with z . All these selected edge pairs brings forth the assignment of the vertex labels and all the edges are distinct and lies between 1 and $(p-1)^2$.

4. Minimization of multiplicative of any path union of 'n' heterogeneous or homogeneous copies of a cycle graph to the solution of a system involving equations

Theorem 4.1 : Let $\gamma(v_{1i})$ = i ,where $1\leq i\leq m_1$; $\gamma(v_{2i})$ = i ,where $1+m_1 \leq i\leq m_1+m_2$; $\gamma(v_{3i})$ = i ,where $m_1+m_2+1 \leq i\leq m_1+m_2+m_3$ etc.., $\gamma(v_{ni})$ = i , where $m_1+m_2+\cdots.m_{n-1}+1 \leq i\leq m_1+m_2+m_3+\ldots m_n=p=k$ be the vertex labeling of the G=(V,E) respectively. Let $m_1,m_2,\ldots m_n$ where be the number of vertices in the first cycle , second cycle and so on respectively . Then any path union $C_m(n)$ of 'n' heterogeneous or homogeneous copies of cycle C_m (n≥2 and m≥ 3) is minimization of multiplicative if the following are satisfied:

(i) $\sum_{j=1}^{n} m_j = p$

(ii) $\sum_{i=1}^{m_1}\gamma(v_{1i}) + \sum_{i=m_1+1}^{m_1+m_2}\gamma(v_{2i}) + \ldots + \sum_{m_1+m_2+\cdots m_{n-1}+1}^{m_1+m_2+\cdots m_n}\gamma(v_{ni}) = \frac{p(p+1)}{2}$

(iii) $(i,j) = (m_1, 1)$, $(i,j) = (m_1 + m_2, m_1 + 1)$,….($m_1 + m_2 + \cdots m_n, m_1 + m_2 + \cdots m_n - 1), \gamma(v_{ki}v_{kj}) < [\gamma(v_{ki})]^2$, where $1 \leq k \leq n$ and For $(i,j) = ((1, m_1 + 1), (m_1 + 1, m_1 + m_2 + 1), \ldots \ldots ((n-1)(m_1 + m_2 + \cdots .. m_{n-2} + 1), n(m_1 + m_2 + \cdots m_{n-1} + 1))$, $\gamma(v_{ki}v_{mj}) > [\gamma(v_{ki})]^2$ where
(k,m)=(1,2),(2,3)….(n-1,n) .
(iv) If $\gamma(v_{1j}v_{1k}) = [\gamma(v_{1j})]^2$, where $1 \leq j \leq m_1 - 1$ and $2 \leq k \leq m_1$, $\gamma(v_{2j}v_{2k}) = [\gamma(v_{2j})]^2$, where $m_1 + 1 \leq j \leq m_1 + m_2 - 1$ and $m_1 + 2 \leq j \leq m_1 + m_2$ etc .., $\gamma(v_{nj}v_{nk}) = [\gamma(v_{nj})]^2$, where $m_1 + m_2 + \cdots m_{n-1} + 1 \leq j \leq m_1 + m_2 + m_3 + \cdots m_{n-1} + m_n - 1$ and $m_1 + m_2 + \cdots m_{n-1} + 2 \leq k \leq m_1 + m_2 + m_3 + \cdots m_{n-1} + m_n$ then $|\gamma(v_{ij}v_{ik})| = |E(G)|$ -3 where $1 \leq i \leq n$

Proof : Let G = $C_m(n)$ be the path union of 'n' heterogeneous or homogeneous copies of cycle C_m (n≥2 and m≥ 3). Define a bijection $\gamma : V(G) \to \{1, 2, \ldots, p\}$ as follows.
Let $\gamma(v_{1i}) = i$,where $1 \leq i \leq m_1$; $\gamma(v_{2i}) = i$,where $1 + m_1 \leq i \leq m_1 + m_2$; $\gamma(v_{3i}) = i$,where $m_1 + m_2 + 1 \leq i \leq m_1 + m_2 + m_3$ and so on . $\gamma(v_{ni}) = i$, where $m_1 + m_2 + \cdots . m_{n-1} + 1 \leq i \leq m_1 + m_2 + m_3 + \ldots . m_n = p$. Let E = $\{v_{11}v_{2(m_1+1)}, v_{2(m_1+1)}v_{3(m_1+m_2+1)}, \ldots . v_{(n-1)(m_1+m_2 \ldots m_{n-2}+1)}v_{n(m_1+m_2 \ldots m_{n-1}+1)}\} \cup \{v_{11}v_{12}$, $v_{12}v_{13}$,….. $v_{1m_1-1}v_{1m_1}$, $v_{1m_1}v_{11}$, $v_{2(m_1+1)}v_{2(m_1+2)}, \ldots \ldots v_{2(m_1+m_2-1)}v_{2(m_1+m_2)}, \ldots \ldots$ $v_{n(m_1+m_2+\cdots m_{n-1}+1)}v_{n(m_1+m_2+\cdots m_{n-1}+2)}, \ldots \ldots v_{n(m_1+m_2+m_3+\cdots m_{n-1})}v_{n(m_1+m_2+m_3+\cdots mn)}\}$
(i)Suppose m_j is the number of vertices in the j^{th} cycle has the labels 1,2,…m_j , then we shall label 'n' heterogeneous or homogeneous copies of G in G(n) are labeled with the labels 1,2,…p. Hence $\sum_{j=1}^{n} m_j = p$.
(ii) From (i) the result (ii) is obvious.
(iii) From (ii) and from the definition of minimization of multiplicative labeling , the edge labels $\gamma(v_i v_j)$ where i<j are all distinct and for $(i,j) = (m_1, 1)$, $(i,j) = (m_1 + m_2, m_1 + 1)$,….($m_1 + m_2 + \cdots m_n, m_1 + m_2 + \cdots m_n - 1), \gamma(v_{ki}v_{kj}) < [\gamma(v_{ki})]^2$, where $1 \leq k \leq n$ and for $(i,j) = ((1, m_1 + 1), (m_1 + 1, m_1 + m_2 + 1), \ldots \ldots ((n-1)(m_1 + m_2 + \cdots .. m_{n-2} + 1), n(m_1 + m_2 + \cdots m_{n-1} + 1))$, $\gamma(v_{ki}v_{mj}) > [\gamma(v_{ki})]^2$ where (k,m)=(1,2),(2,3)….(n-1,n) .
(iv)It is clear from (iii) $|\gamma(v_{ij}v_{ik})| = |E(G)|$ -3 where $1 \leq i \leq n$.

Conclusion : In this paper , we evinced a method to construct minimization of multiplicative graphs and also investigated the minimization of multiplicative of the path union of 'n' copies of a cycle to the solution of a system involving equations .

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