Production-Inventory Model with Preservation Technology Investment with Carbon Emission and Partial Backlogging

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Abstract: This study examines how well the Twin concatenation strategy for error-correcting capabilities performs with the BCH code. Low complexity BCH codes are the main focus. The quantity of mistakes introduced into the transmission channel has a significant impact on the quality of digital transmission. The Bose-Chaudhuri-Hocquenghem (BCH) codes are frequently employed in storage and communication systems. In next-generation wireless networks, low-latency communication is one of the most crucial application situations. The time needed for a packet to be transmitted through a channel is a common definition of latency in communication-theoretic studies. However, due to the strict latency requirements and complexity-restricted receivers, the time needed for packet decoding must be taken into account in the overall delay analysis through precise modeling. The new Twin CC algorithm has been proved to perform better than the current approach.

Keywords – BCH Coding, Turbo Coding, FEC, Interleaver.

I. INTRODUCTION

The term degradation refers to decay, damage, evaporation, out of fashion, etc. In our global world of today, it has become very common due to various socioeconomic and environmental changes. Known as deteriorating items, these items slowly degrade over time. Towards inventory control modeling, Ghare and Sharda (1963) introduced the concept of deterioration. Although conventional inventory management assumes that deterioration rate is an uncontrollable variable, in recent years a number of authors have discussed whether or not it is controllable. The deterioration rate can be controlled up to a certain extent through preservation technology investment. One can check out the works of Lea, Dye (2012), and Mishra (2013) for more survey.

The demand rate has a significant impact on inventory control models. According to classical theory, demand for inventory is considered a constant rate; however, the demand for seasonal goods, fashionable items, etc., typically varies over time. Thus, time dependent demand rates can be considered more practical than constant demand rates. A linear trend in demand was first proposed by Resh, Friedman, Barbosa (1976) and Donaldson (1977). Furthermore, many researchers have extended their work, including Giri, Chakraborty, Chaudhuri (2012), Balkhi (2003) and Teng and Yang (2007).
A shortage period will result in customers not waiting for the products and instead buying them from another supplier. The willingness of customers to wait for backlog during shortage periods is also higher when the product is fashionable or high tech. In other words, the longer the waiting time, the smaller the backlog rate, i.e. the backlog rate declines with waiting time. For a more detailed analysis, we can review the work of Chang and Dye (1999), Yang (2010) and Pentico and Drake (2011).

Burning fossil fuels, such as natural gas, crude oil, and coal, is often associated with the release of carbon dioxide into the atmosphere, along with other greenhouse gases. The effects of greenhouse gases are hazardous to the environment and to health. Through the trapping of heat, they alter the climate, creating arid conditions for a variety of species. In addition to extreme weather, wildfires, droughts and food supply disruptions, climate change caused by greenhouse gas emissions also plays a role in the development of extreme weather patterns. Hua et al. (2011) focused on reducing carbon emissions as a means of reducing global warming. They also examined market-based methods for trading carbon emissions. In the study, carbon constraints were numerically verified as a factor affecting a minimum cost. Hu and Zhou (2014) studied the manufacturing company's carbon emission reduction and price policy during the carbon emission trade period. To determine what policy would maximize profits, they examined the impact of carbon emissions. Taleizadeh et al. (2018) investigated the impact of partial backorders and carbon emissions on joint pricing decisions and inventory decisions.

As a result of our investment in preservation technology, we have developed models of production-inventory that are demand sensitive and controlled deterioration rates. It is assumed that demand will be linear with time with partially backlogged shortages. Optimizing operating times and costs of relevant operations is one of our objectives. As we proceed to the next section, we will describe notations, assumptions, and modeling formulations. Afterward, the proposed model is illustrated numerically and sensitivity is assessed.

II. ASSUMPTIONS AND NOTATIONS

Following are the specified assumptions and notations used in forming mathematical models:

A. Assumptions

- The demand for goods depends on the time i.e. $D(t) = \alpha + \beta t$, where $\alpha, \beta > 0$.
- Production rate is determined by demand i.e. $P(t) = \tau D(t) = \tau(\alpha + \beta t)$, where $\tau > 0$.
- It is allowed to have shortages that are partially backlogged with a backlog rate $B = e^{-\rho t}$ where $\rho > 0$.
- Using preservation techniques can control the deterioration rate; however, investment $\xi$ in preservation technology $m(\xi)$ is a strictly inverse function of the level of deterioration where $\lim_{\xi \rightarrow \infty} m(\xi) = 0$
- Inventory planning has an infinite horizon, however only a typical planning schedule of length $T$ is discussed here. The other phases are similar to this.
- In addition, we assumed instant replenishment and zero lead time.
- There is a constant rate of deterioration $\theta$, where, $0 < \theta < 1$.

B. Notations

- $C_p$: Per unit cost of production.
III. MATHEMATICAL MODEL FORMULATION

In the first, we considered a partial backlog inventory problem at the end of the system. In the second model, we consider partial shortages at the beginning of the system. As shown below, both mathematical models are formulated as follows.

A. Case-1

Figure-1 shows the inventory level. Inventory is initially at zero. Due to the combined effect of demand and deterioration during \([0, t_1]\), the production begins at time \(t = 0\) and inventory level gradually decreases. Due to deterioration and demand during the time period \([t_1, t_3]\), production stops at \(t = t_1\) and inventories decrease. Time \(t = t_2\) marks the zero level of inventory. \([T_2, T_3]\) are time periods when shortages occur and partially backlogs occur. As production begins at time \(t = t_3\), and partial shortages are resolved at time \(t = T\), a positive inventory period begins. The deterioration rate is reduced by \(\theta - m(\xi)\) due to the capital investment in preservation technologies. Thus, the following differential equation can be used to calculate inventory depletion.

\[
\frac{dQ(t)}{dt} + \left[\theta - m(\xi)\right]Q(t) = P(t) - D(t); \quad 0 \leq t \leq t_1, \quad Q(0) = 0
\]

\[
\frac{dQ(t)}{dt} + \left[\theta - m(\xi)\right]Q(t) = -D(t); \quad t_1 \leq t \leq t_2, \quad Q(t_2) = 0
\]
\[
\frac{dQ(t)}{dt} = -D(t)e^{-\rho t}; \quad t_2 \leq t \leq t_3, \quad Q(t_2) = 0
\]

\[
\frac{dQ(t)}{dt} = P(t) - D(t); \quad t_3 \leq t \leq T, \quad Q(T) = 0
\]

Solutions of above equations are as follows
\[
Q(t) = \left\{ (\tau - 1)(1 - (\theta - m(\xi))) \right\} \left\{ at + \left( \beta + \alpha(\theta - m(\xi)) \right) \left( \frac{t_2^2}{2} + \beta(\theta - m(\xi)) \frac{t_2^3}{3} \right) \right\}
\]
\[
Q(t) = (1 - (\theta - m(\xi))) \left( \alpha(t_2 - t) + \left( \beta + \alpha(\theta - m(\xi)) \right) \left( \frac{t_2^2 - t^2}{2} \right) + \beta(\theta - m(\xi)) \right) \left( \frac{t_3^2 - t^2}{3} \right) \left( \frac{t_3^3 - t^3}{3} \right)
\]
\[
Q(t) = \alpha(t_2 - t) + \beta(\theta - m(\xi)) \left( \frac{t_2^2 - t^2}{2} \right) - \beta(\theta - m(\xi)) \left( \frac{t_3^2 - t^2}{3} \right)
\]
\[
Q(t) = (\tau - 1) \left( \alpha(T - t) + \beta \left( \frac{T^2 - t^2}{2} \right) \right)
\]

All the expenses incurred by a business in the process of manufacturing a product or providing a service are considered production costs. A number of factors affect production costs, including labor, raw material, consumable manufacturing supplies, and overhead. The production cost can be calculated as
\[
PC = C_p \left\{ \int_{t_1}^{t_3} P(t) dt + \int_{T}^{t_3} P(t) dt \right\}
\]
\[
= C_p \tau \left[ \alpha(T + t_1 - t_3) + \beta \left( \frac{T^2}{2} + t_1^2 - t_3^2 \right) \right]
\]

A major contributor of greenhouse gas emissions to the atmosphere is the manufacturing industry. A 2017 report from the Environmental Protection Agency estimates that 22% of US emissions came from industry. The researchers are therefore more inclined to look for emissions due to the manufacturing. Carbon emission cost associated with the production cost
\[
= C_pe \tau \left[ \alpha(T + t_1 - t_3) + \beta \left( \frac{T^2}{2} + t_1^2 - \frac{t_3^2}{2} \right) \right]
\]
The cost of storing unsold inventory is referred to as holding costs. Storage space, labor, insurance, and maintenance are all included in the cost of holding a business and can be formulated as
\[
HC = h \left\{ \int_{t_1}^{t_3} Q(t) dt + \int_{t_1}^{t_3} Q(t) dt \right\}
\[
= h \left\{ (\tau - 1) \left\{ \frac{\alpha t_1^2}{2} + (\beta + \alpha(\theta - m(\xi))) \frac{t_1^3}{6} + \beta(\theta - m(\xi)) \frac{t_1^4}{12} \right. \\
\left. - (\theta - m(\xi)) \left( \frac{\alpha t_1^3}{3} + (\beta + \alpha(\theta - m(\xi))) \frac{t_1^4}{8} + \beta(\theta - m(\xi)) \frac{t_1^5}{15} \right) \right\} \\
\right. \\
+ \left\{ \alpha \left( \frac{t_2^2}{2} - t_2 t_1 + \frac{t_2^2}{2} \right) + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t_2^3}{3} - \frac{t_1 t_2^2}{2} + \frac{t_2^3}{6} \right) \right\} \\
\left. + (\beta - m(\xi)) \left( \frac{t_2^4}{4} - \frac{t_1 t_2^3}{3} + \frac{t_1^4}{12} \right) - (\theta - m(\xi)) \alpha \left( \frac{t_2^3}{6} - \frac{t_2 t_1^2}{2} + \frac{t_1^3}{3} \right) \right\} \\
\left. + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t_2^5}{8} - \frac{t_1 t_2^4}{4} + \frac{t_1^5}{8} \right) \right\} \\
\left. + \beta(\theta - m(\xi)) \left( \frac{t_2^5}{10} - \frac{t_1^2 t_2^3}{6} + \frac{t_1^5}{15} \right) \right\} \right] 
\]

(10)

Carbon emission cost associated with the holding cost
\[
= h_e \left\{ (\tau - 1) \left\{ \frac{\alpha t_1^2}{2} + (\beta + \alpha(\theta - m(\xi))) \frac{t_1^3}{6} + \beta(\theta - m(\xi)) \frac{t_1^4}{12} \right. \\
\left. - (\theta - m(\xi)) \left( \frac{\alpha t_1^3}{3} + (\beta + \alpha(\theta - m(\xi))) \frac{t_1^4}{8} + \beta(\theta - m(\xi)) \frac{t_1^5}{15} \right) \right\} \\
\right. \\
+ \left\{ \alpha \left( \frac{t_2^2}{2} - t_2 t_1 + \frac{t_2^2}{2} \right) + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t_2^3}{3} - \frac{t_1 t_2^2}{2} + \frac{t_2^3}{6} \right) \right\} \\
\left. + (\beta - m(\xi)) \left( \frac{t_2^4}{4} - \frac{t_1 t_2^3}{3} + \frac{t_1^4}{12} \right) - (\theta - m(\xi)) \alpha \left( \frac{t_2^3}{6} - \frac{t_2 t_1^2}{2} + \frac{t_1^3}{3} \right) \right\} \\
\left. + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t_2^5}{8} - \frac{t_1 t_2^4}{4} + \frac{t_1^5}{8} \right) \right\} \\
\left. + \beta(\theta - m(\xi)) \left( \frac{t_2^5}{10} - \frac{t_1^2 t_2^3}{6} + \frac{t_1^5}{15} \right) \right\} \right\} \right]
\]

According to the definition of degradation costs, these are those that reflect the qualitative degradation of natural resources caused by economic activity. Deterioration Cost is
\[
DC = C_d(\theta - m(\xi)) \left\{ \int_0^{t_1} P(t)dt - \int_0^{t_2} D(t)dt \right\}
\]
\[
DC = C_d(\theta - m(\xi)) \left[ \alpha ((\rho + 1)t_1 - t_2) + \beta \left( (\rho + 1) \frac{t_1^2}{2} - \frac{t_2^2}{2} \right) \right]
\]

(11)

The absence of raw materials results in a business incurring shortage costs, such as time lost while waiting for raw materials. It can be formulated as
\[
SC = C_s \left\{ - \int_{t_2}^{t_3} Q(t)dt - \int_{t_3}^{T} Q(t)dt \right\}
\]
\[ C_s \left\{ \left( \alpha \left( \frac{t_2^2}{2} - t_3 t_2 + \frac{t_3^2}{2} \right) + (\beta - \alpha \rho) \left( \frac{t_3^3}{3} - \frac{t_3 t_2^2}{2} + \frac{t_3^3}{6} \right) - \beta \rho \left( \frac{t_2^4}{4} - \frac{t_3 t_2^3}{3} + \frac{t_3^4}{12} \right) \right) \right. \\
+ (\tau - 1) \left\{ \alpha \left( \frac{T^2}{2} - T t_3 + \frac{t_3^2}{2} \right) + \beta \left( \frac{T^3}{3} - \frac{t_3 T^2}{2} + \frac{t_3^3}{6} \right) \right\} \right]\]

(12)

The lost sales cost (LSC) is the price we pay when we fail to satisfy customer demand.

\[ LC = C_s \left[ - \int_{t_2}^{t_3} (1 - e^{-\rho t}) D(t) dt \right] \]

\[ LC = -\rho C_s \left[ \alpha \left( \frac{t_3^3}{2} - \frac{t_3^2}{2} \right) + \beta \left( \frac{t_3^3}{3} - \frac{t_3^2}{3} \right) \right] \]

(13)

Hence the total relevant cost for the production-inventory system during the cycle \([0, T]\) is

\[ TC(t_1, t_2, t_3, T, \xi) = [PC + HC + DC + SC + LC] \]

(14)

**Case-2**

Inventory level starts out at zero and shortages begin to develop at \(t = 0\). At the onset of the production process \((t = t_1)\), they reach their maximum level of shortages. All partially backlogged shortages have been met at time \(t_2\) and inventory is in positive territory. The inventory level increases and reaches its maximum at \(t = t_3\). As a result of the combined effects of the lower demand and the deterioration during \([t_3, T]\) the production process is suspended and the inventory level drops. Investing in preservation technology reduces deterioration by \(\theta - m(\xi)\), because the cost of the preservation technology \(\xi\) is invested. Thus, inventory depletion can be derived from the differential equation shown below in figure 2.

**Figure: 2 (Case-2)**

\[ \frac{dQ(t)}{dt} = -D(t) e^{-\rho t}; \ 0 \leq t \leq t_1, \ Q(0) = 0 \]  

(15)

\[ \frac{dQ(t)}{dt} = P(t) - D(t); \ t_1 \leq t \leq t_2, \ Q(t_2) = 0 \]  

(16)

\[ \frac{dQ(t)}{dt} + (\theta - m(\xi))Q(t) = P(t) - D(t); \ t_2 \leq t \leq t_3, \ Q(t_2) = 0 \]  

(17)

\[ \frac{dQ(t)}{dt} + (\theta - m(\xi))Q(t) = -D(t); \ t_3 \leq t \leq T, \ Q(T) = 0 \]  

1704

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The solutions of above equations are

\[
Q(t) = - \left[ \alpha(t) + (\beta - \alpha \rho) \left( \frac{t^2}{2} \right) - \beta \rho \left( \frac{t^3}{3} \right) \right], \quad 0 \leq t \leq t_1
\]

\[
Q(t) = \left( \rho - 1 \right) \left( \alpha(t - t_2) + \beta \left( \frac{t^2 - t_2^2}{2} \right) \right), \quad t_1 \leq t \leq t_2
\]

\[
Q(t) = \left[ (1 - (\theta - m(\xi)) t) \left( \alpha(t - t_2) + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t^2 - t_2^2}{2} \right) + \beta(\theta - m(\xi)) \left( \frac{t^2 - t_2^2}{3} \right) \right) \right], \quad t_2 \leq t \leq t_3
\]

\[
Q(t) = \left[ (1 - (\theta - m(\xi)) t) \left( \alpha(T - t) + (\beta + \alpha(\theta - m(\xi))) \left( \frac{T^2 - t^2}{2} \right) + \beta(\theta - m(\xi)) \left( \frac{T^2 - t^2}{3} \right) \right) \right], \quad t_3 \leq t \leq T
\]

Production cost

\[
PC = C_p \left\{ \int_{t_1}^{t_2} P(t) \, dt + \int_{t_2}^{t_3} P(t) \, dt \right\}
\]

\[
PC = C_p \left[ \alpha(t_3 - t_1) + \beta \left( \frac{t_3^2 - t_1^2}{2} \right) \right]
\]

Holding cost is

\[
HC = h \left[ \int_{t_2}^{t_3} Q(t) \, dt + \int_{t_3}^{T} Q(t) \, dt \right]
\]

\[
HC = (\tau - 1) \left\{ \alpha \left( \frac{t_3^2}{2} - t_3 t_2 + \frac{t_2^3}{2} \right) + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t_3^2}{3} - \frac{t_3^2}{2} + \frac{t_2^3}{6} \right) + \beta(\theta - m(\xi)) \left( \frac{t_3^2}{3} - \frac{t_3^2}{2} + \frac{t_2^3}{6} \right) \right\}
\]

Carbon emission cost associated with the holding cost, \(HC\) is

\[
HC = h \left[ (\tau - 1) \left\{ \alpha \left( \frac{t_2^3}{2} - t_3 t_2 + \frac{t_2^3}{2} \right) + (\beta + \alpha(\theta - m(\xi))) \left( \frac{t_2^3}{3} - \frac{t_2^3}{2} + \frac{t_2^3}{6} \right) + \beta(\theta - m(\xi)) \left( \frac{t_2^3}{3} - \frac{t_2^3}{2} + \frac{t_2^3}{6} \right) \right\} \right]
\]
\[
\left(\frac{t^3_2 t^3_3}{6} + \frac{t^5_3}{15}\right) + \left\{\alpha \left(\frac{T^2}{2} - T t_3 + \frac{t^3_2}{2}\right) + \left(\beta + \alpha (\theta - m(\xi))\right)\left(\frac{T^3}{3} - \frac{t^3_2}{2} + \frac{t^5_3}{6}\right) + \beta (\theta - m(\xi))\left(\frac{T^4}{4} - \frac{t^3_3 t^3_2}{3} + \frac{t^5_3}{12}\right) - \left(\theta - m(\xi)\right)\left\{\alpha \left(\frac{T^3}{3} - \frac{t^3_3}{2} + \frac{t^5_3}{3}\right) + \left(\beta + \alpha (\theta - m(\xi))\right)\left(\frac{T^4}{8} - \frac{t^2_2}{4} + \frac{t^4_3}{8}\right) + \beta (\theta - m(\xi))\left(\frac{T^5}{10} - \frac{t^3_3}{2} + \frac{t^5_3}{15}\right)\right\}\right] 
\]

Deterioration Cost is
\[
DC = C_d (\theta - m(\xi)) \left[\int_{t_2}^{T} p(t) dt - \int_{t_2}^{T} D(t) dt\right] 
\]
\[
DC = C_d (\theta - m(\xi)) \left[t \left(\alpha (t_3 - t_2) + \beta \left(\frac{t^3_3}{2} - \frac{t^2_2}{2}\right)\right) - \left(\alpha (T - t_2) + \beta \left(\frac{T^2}{2} - \frac{t^2_2}{2}\right)\right)\right] 
\]

(25)

Shortage cost
\[
SC = C_s \left\{-\int_{0}^{t_1} q(t) dt - \int_{t_1}^{t_2} q(t) dt\right\} 
\]
\[
SC = C_s \left[\left\{\alpha \left(\frac{t^3_1}{2}\right) + (\beta - \alpha \rho) \left(\frac{t^5_1}{6}\right) - \beta \rho \left(\frac{t^7_1}{12}\right)\right\} + (\tau - 1)\left\{\alpha \left(\frac{t^3_2}{2} - t_1 t_2 + \frac{t^5_3}{2}\right) + \beta \left(\frac{t^3_3}{3} - \frac{t_1 t^2_2}{2} + \frac{t^5_3}{6}\right)\right\}\right] 
\]

(26)

Lost sales cost
\[
LC = C_l \left[-\int_{0}^{t_1} (1 - e^{-\delta t}) D(t) dt\right] 
\]
\[
= -\rho C_l \left[\alpha \left(\frac{t^3_2}{2}\right) + \beta \left(\frac{t^5_3}{3}\right)\right] 
\]

(27)

Therefore, the total cost would be
\[
TC(t_1, t_2, t_3, T, \xi) = [C_0 + PC + HC + DC + SC + LC] 
\]

(28)

To minimize the objective function, we need to differentiate the total cost function TC w. r. t to the decision variable \(t_1\) and \(T\). The necessary conditions to get the extreme values are as follows

\[
\frac{\partial TC(t_1, T)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC(t_1, T)}{\partial T} = 0 
\]

In the case of a hessian matrix, the determinant of the principal minor must be positive. This is the Hessian matrix of the total cost function:
\[
\begin{bmatrix}
\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\
\frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2}
\end{bmatrix}
\]

IV. NUMERICAL EXAMPLE

We employ the following inventory system as an illustration of proposed model, which gives values of various parameters in proper units

Example 1 (Case 1): We have considered the values of parameters based on the previous studies, but resalable estimation has been done

Vol. 71 No. 3s 2 (2022)

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1706
$\alpha = 500, \beta = 1.5, C_0 = 250, C_p = 15, C_{pe} = 1.5, a = 0.05, C_d = 5, C_{de} = 0.45, C_s = 5, C_l = 3, h = 1.2, h_e = 0.5, \theta = 0.05, \rho = 0.1, \tau = 2$

Solving the outlined problem provides following solutions

$t_1^* = 1.160988, \ t_2^* = 1.547984, \ t_3^* = 2.321976, \ T^* = 4.643952, \ \xi^* = 70.5918, \ TC^*(t_1, t_2, t_3, T, \xi) = 5531.35$

Example 2 (Case 2): We have considered the values of parameters based on the previous studies, but resalable estimation has been done

$\alpha = 700, \beta = 1.2, C_0 = 250, C_p = 2, C_{pe} = 2, a = 0.05, C_d = 5, C_{de} = 5, C_s = 5, C_l = 3, h = 1.2, h_e = 0.25, \theta = 0.05, \rho = 0.1, \tau = 2.2$

Solving the outlined problem provides following solutions

$t_1^* = 1.160988, \ t_2^* = 1.547984, \ t_3^* = 2.321976, \ T^* = 4.643952, \ \xi^* = 70.5916, \ TC^*(t_1, t_2, t_3, T, \xi) = 5531.43.$

V. SENSITIVITY ANALYSIS

Changes in some parameters such as demand parameters, deterioration rate etc. are studied by performing a sensitivity analysis. Listed below in table 1 and table 2, are the results of the analysis.

### Table 1: Case-1

<table>
<thead>
<tr>
<th>Change in</th>
<th>$\xi^*$</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$T^*$</th>
<th>TC*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in a</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.173216</td>
<td>1.56428</td>
<td>2.34642</td>
<td>4.69284</td>
<td>6315.972</td>
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<td>70.88604</td>
<td>1.167312</td>
<td>1.556408</td>
<td>2.334612</td>
<td>4.669224</td>
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<td>1.538952</td>
<td>2.308428</td>
<td>4.616856</td>
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<td>1.529236</td>
<td>2.293854</td>
<td>4.587708</td>
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</tr>
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<td>Change in b</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.531348</td>
<td>2.297022</td>
<td>4.594044</td>
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</tr>
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**Table II** CASE-2

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Observations
1. From the above table, it is noticed that the small increment in demand parameter ‘a’ resulted in increment in the total cost and cycle length.
2. Meanwhile, optimal value of $\xi^*$ is also reported to be positive sensitive toward the demand rate.
3. It is revealed that the increment in demand parameter ‘b’ resulted in the reduction in cost and optimal cycle length.
4. The results shown that the partial backlogging parameter ‘$\delta$’ is reported to be negative sensitive toward the optimal values of $\xi^*$ and cycle length.
5. With Increment in constant deterioration rate $0$, Optimal value of $\xi^*$ is also increases.

VI. CONCLUSION
The deterioration rate for deteriorated items can be controlled by controlling production inventory and determining the time-dependent demand rate of the deteriorated item inventory. Several factors influence production cost, including demand and carbon emission. In this article we have examined two cases: the first where the shortage occurs at the end of the model and the second when it starts. Throughout the study, preservation is taken into consideration. The minimum value of the total relevant cost has been determined. The model is illustrated numerically and a sensitivity analysis is presented at the end. In addition to
inflation and trade credit, this study can incorporate other inventory control system parameters such as income and sales tax.

REFERENCES


