Production-Inventory Model with Preservation Technology Investment with Carbon Emission and Partial Backlogging

[1]Ashok Kumar, [2]Jitendra Kumar, [3]Arvind Kumar*
[1]Department of Mathematics, Meerut College, Meerut, UP, INDIA, 250001; Email-drakkashyap@hotmail.com
[2]Department of Mathematics, Swami Vivekanand Subharti University, Meerut, UP, INDIA, 250005; Email- jitte.dm@gmail.com
[3]Department of Mathematics, Meerut College, Meerut, UP, INDIA, 250001; Email-arvind.mcat@gmail.com
*Corresponding Authors

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Abstract: This study examines how well the Twin concatenation strategy for error-correcting capabilities performs with the BCH code. Low complexity BCH codes are the main focus. The quantity of mistakes introduced into the transmission channel has a significant impact on the quality of digital transmission. The Bose-Chaudhuri-Hocquenghem (BCH) codes are frequently employed in storage and communication systems. In next-generation wireless networks, low-latency communication is one of the most crucial application situations. The time needed for a packet to be transmitted through a channel is a common definition of latency in communication-theoretic studies. However, due to the strict latency requirements and complexity-restricted receivers, the time needed for packet decoding must be taken into account in the overall delay analysis through precise modeling. The new Twin CC algorithm has been proved to perform better than the current approach.
Keywords – BCH Coding, Turbo Coding, FEC, Interleaver.

I. INTRODUCTION

The term degradation refers to decay, damage, evaporation, out of fashion, etc. In our global world of today, it has become very common due to various socioeconomic and environmental changes. Known as deteriorating items, these items slowly degrade over time. Towards inventory control modeling, Ghare and Sharder (1963) introduced the concept of deterioration. Although conventional inventory management assumes that deterioration rate is an uncontrollable variable, in recent years a number of authors have discussed whether or not it is controllable. The deterioration rate can be controlled up to a certain extent through preservation technology investment. One can check out the works of Leea, Dye (2012), and Mishra (2013) for more survey.

The demand rate has a significant impact on inventory control models. According to classical theory, demand for inventory is considered a constant rate; however, the demand for seasonal goods, fashionable items, etc., typically varies over time. Thus, time dependent demand rates can be considered more practical than constant demand rates. A linear trend in demand was first proposed by Resh, Friedman, Barbosa (1976) and Donaldson (1977). Furthermore, many researchers have extended their work, including Giri, Chakrabarty, Chaudhuri (2012), Balkhi (2003) and Teng and Yang (2007).

A shortage period will result in customers not waiting for the products and instead buying them from another supplier. The willingness of customers to wait for backlog during shortage periods is also higher when the product is fashionable or high tech. In other words, the longer the waiting time, the smaller the backlog rate, i.e. the backlog rate declines with waiting time. For a more detailed analysis, we can review the work of Chang and Dye (1999), Yang (2010) and Pentico and Drake (2011).

Burning fossil fuels, such as natural gas, crude oil, and coal, is often associated with the release of carbon dioxide into the atmosphere, along with other greenhouse gases. The effects of greenhouse gases are hazardous to the environment and to health. Through the trapping of heat, they alter the climate, creating arid conditions for a variety of species. In addition to extreme weather, wildfires, droughts and food supply disruptions, climate change caused by greenhouse gas emissions also plays a role in the development of extreme weather patterns. Hua et al. (2011) focused on reducing carbon emissions as a means of reducing global warming. They also examined market-based methods for trading carbon emissions. In the study, carbon constraints were numerically verified as a factor affecting a minimum cost. Hu and Zhou (2014) studied the manufacturing company's carbon emission reduction and price policy during the carbon emission trade period. To determine what policy would maximize profits, they examined the impact of carbon emissions. Taleizadeh et al. (2018) investigated the impact of partial backorders and carbon emissions on joint pricing decisions and inventory decisions.

As a result of our investment in preservation technology, we have developed models of production- inventory that are demand sensitive and controlled deterioration rates. It is assumed that demand will be linear with time with partially backlogged shortages. Optimizing operating times and costs of relevant operations is one of our objectives. As we proceed to the next section, we will describe notations, assumptions, and modeling formulations. Afterward, the proposed model is illustrated numerically and sensitivity is assessed.

II. ASSUMPTIONS AND NOTATIONS

Following are the specified assumptions and notations used in forming mathematical models:

A. *Assumptions*

- The demand for goods depends on the time i.e. $D(t) = \alpha + \beta t,$ where $\alpha, \beta > 0$.
- Production rate is determined by demand i.e. $P(t) = \tau D(t) = \tau(\alpha + \beta t),$ where $\tau > 0$.
- It is allowed to have shortages that are partially backlogged with a backlog rate $B = e^{-\rho t}$ where $\rho > 0$.
- Using preservation techniques can control the deterioration rate; however, investment ξ in preservation technology m (ξ) is a strictly inverse function of the level of deterioration where $\lim_{\xi T \infty} m(\xi) = \theta$
- Inventory planning has an infinite horizon, however only a typical planning schedule of length T is discussed here. The other phases are similar to this.
- In addition, we assumed instant replenishment and zero lead time.
- There is a constant rate of deterioration θ, where, $0 < \theta < 1$.

B. *Notations*

- C_p: Per unit cost of production.

- A: Setup costs per set up.
- C_d: A cost per unit for deterioration.
- C_s: Per unit of time, the cost of unit shortage.
- h: The cost of carrying inventory per unit of time.
- $m(\xi_j)$: Reduction in deterioration rate due to the use of preservation technology and $m(\xi)= \theta(1- e^{-a\xi})$ where a>0.
- ξ: The Preservation Technology (PT) costs to preserve the product, which also reduces deterioration rate, where ξ >0.
- C_l: In order to avoid loss of generality, we assume that (C_l>C_p) for lost sales per unit time.
- t_1: The point at which production ceases.
- t_2: the time period when inventory levels reach zero and shortages have been partially met.
- t_3: the time period when production starts to satisfy partially backlogged orders.

III. MATHEMATICAL MODEL FORMULATION

In the first, we considered a partial backlog inventory problem at the end of the system. In the second model, we consider partial shortages at the beginning of the system. As shown below, both mathematical models are formulated as follows.

A. *Case-1*

Figure-1 shows the inventory level. Inventory is initially at zero. Due to the combined effect of demand and deterioration during [0, t_1], the production begins at time t = 0 and inventory level gradually decreases. Due to deterioration and demand during the time period [t_1, t_2], production stops at t = t_1 and inventories decrease. Time t = t_2 marks the zero level of inventory. [T_2, T_3] are time periods when shortages occur and partially backlogs occur. As production begins at time t = t_3, and partial shortages are resolved at time t = T, a positive inventory period begins. The deterioration rate is reduced by θ - m (ξ) due to the capital investment in preservation technologies. Thus, the following differential equation can be used to calculate inventory depletion.

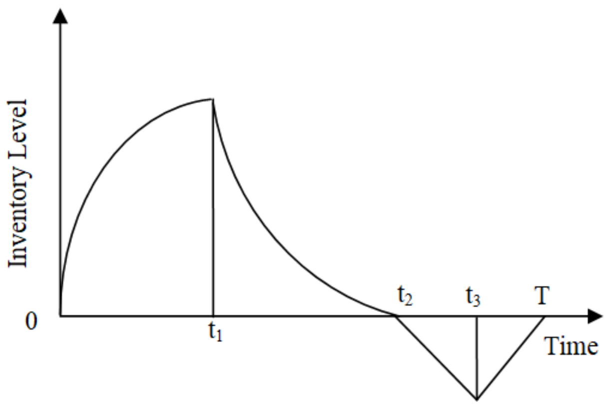

Figure: 1

The differential equations are as follows

$$\frac{dQ(t)}{dt} + [\theta - m(\xi)]Q(t) = P(t) - D(t); 0 \le t \le t_1,\ Q(0) = 0 \tag{1}$$

$$\frac{dQ(t)}{dt} + [\theta - m(\xi)]Q(t) = -D(t);\ t_1 \le t \le t_2,\ \ Q(t_2) = 0$$

(2)

$\frac{dQ(t)}{dt} = -D(t)e^{-\rho t}$; $t_2 \le t \le t_3$, $Q(t_2) = 0$

(3)

$\frac{dQ(t)}{dt} = P(t) - D(t)$; $t_3 \le t \le T$, $Q(T) = 0$

(4)

Solutions of above equations are as follows

$$Q(t) = \left\{(\tau-1)(1-(\theta-m(\xi))t)\left\{\alpha t + \left(\beta+\alpha\big(\theta-m(\xi)\big)\right)\frac{t^2}{2} + \beta\big(\theta-m(\xi)\big)\frac{t^3}{3}\right\}\right\} \quad (5)$$

$$Q(t) = \big(1-\big(\theta-m(\xi)\big)t\big)\left(\alpha(t_2-t) + \left(\beta+\alpha\big(\theta-m(\xi)\big)\right)\left(\frac{{t_2}^2-t^2}{2}\right) + \beta\big(\theta - m(\xi)\big)\left(\frac{{t_2}^3-t^3}{3}\right)\right) \quad (6)$$

$$Q(t) = \alpha(t_2-t) + (\beta-\alpha\rho)\left(\frac{{t_2}^2-t^2}{2}\right) - \beta\rho\left(\frac{{t_2}^3-t^3}{3}\right) \quad (7)$$

$$Q(t) = (\tau-1)\left(\alpha(t-T) + \beta\left(\frac{t^2-T^2}{2}\right)\right) \quad (8)$$

All the expenses incurred by a business in the process of manufacturing a product or providing a service are considered production costs. A number of factors affect production costs, including labor, raw material, consumable manufacturing supplies, and overhead. The production cost can be calculated as

$$PC = C_p\left\{\int_0^{t_1} P(t)dt + \int_{t_3}^{T} P(t)dt]\right\}$$
$$= C_p\tau[\alpha(T+t_1-t_3) + \beta\left(\frac{T^2}{2}+\frac{t_1^2}{2}-\frac{t_3^2}{2}\right)] \quad (9)$$

A major contributor of greenhouse gas emissions to the atmosphere is the manufacturing industry. A 2017 report from the Environmental Protection Agency estimates that 22% of US emissions came from industry. The researchers are therefore more inclined to look for emissions due to the manufacturing. Carbon emission cost associated with the production cost

$$= C_{pe}\tau[\alpha(T+t_1-t_3) + \beta\left(\frac{T^2}{2}+\frac{t_1^2}{2}-\frac{t_3^2}{2}\right)]$$

The cost of storing unsold inventory is referred to as holding costs. Storage space, labor, insurance, and maintenance are all included in the cost of holding a business and can be formulated as $HC = h\left\{\int_0^{t_1} Q(t)dt + \int_{t_1}^{t_2} Q(t)dt\right\}$

$$
\begin{aligned}
&= h\Bigg\{(\tau-1)\left\{\alpha\frac{t_1^2}{2}+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\frac{t_1^3}{6}+\beta\big(\theta-m(\xi)\big)\frac{t_1^4}{12}\right.\\
&\qquad\left.-\big(\theta-m(\xi)\big)\left(\alpha\frac{t_1^3}{3}+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\frac{t_1^4}{8}+\beta\big(\theta-m(\xi)\big)\frac{t_1^5}{15}\right)\right\}\\
&\qquad+\left\{\alpha\left(\frac{t_2^2}{2}-t_2t_1+\frac{t_1^2}{2}\right)+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\left(\frac{t_2^3}{3}-\frac{t_1t_2^2}{2}+\frac{t_1^3}{6}\right)\right.\\
&\qquad+\beta\big(\theta-m(\xi)\big)\left(\frac{t_2^4}{4}-\frac{t_1t_2^3}{3}+\frac{t_1^4}{12}\right)-\big(\theta-m(\xi)\big)\alpha\left(\frac{t_2^3}{6}-\frac{t_2{t_1}^2}{2}+\frac{t_1^3}{3}\right)\\
&\qquad+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\left(\frac{t_2^4}{8}-\frac{{t_1}^2t_2^2}{4}+\frac{t_1^4}{8}\right)\\
&\qquad\left.+\beta\big(\theta-m(\xi)\big)\left(\frac{t_2^5}{10}-\frac{{t_1}^2t_2^3}{6}+\frac{t_1^5}{15}\right)\right\}\Bigg\}
\end{aligned}
\tag{10}
$$

Carbon emission cost associated with the holding cost

$$
\begin{aligned}
&= h_e\Bigg\{(\tau-1)\left\{\alpha\frac{t_1^2}{2}+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\frac{t_1^3}{6}+\beta\big(\theta-m(\xi)\big)\frac{t_1^4}{12}\right.\\
&\qquad\left.-\big(\theta-m(\xi)\big)\left(\alpha\frac{t_1^3}{3}+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\frac{t_1^4}{8}+\beta\big(\theta-m(\xi)\big)\frac{t_1^5}{15}\right)\right\}\\
&\qquad+\left\{\alpha\left(\frac{t_2^2}{2}-t_2t_1+\frac{t_1^2}{2}\right)+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\left(\frac{t_2^3}{3}-\frac{t_1t_2^2}{2}+\frac{t_1^3}{6}\right)\right.\\
&\qquad+\beta\big(\theta-m(\xi)\big)\left(\frac{t_2^4}{4}-\frac{t_1t_2^3}{3}+\frac{t_1^4}{12}\right)-\big(\theta-m(\xi)\big)\alpha\left(\frac{t_2^3}{6}-\frac{t_2{t_1}^2}{2}+\frac{t_1^3}{3}\right)\\
&\qquad+\Big(\beta+\alpha\big(\theta-m(\xi)\big)\Big)\left(\frac{t_2^4}{8}-\frac{{t_1}^2t_2^2}{4}+\frac{t_1^4}{8}\right)\\
&\qquad\left.+\beta\big(\theta-m(\xi)\big)\left(\frac{t_2^5}{10}-\frac{{t_1}^2t_2^3}{6}+\frac{t_1^5}{15}\right)\right\}\Bigg\}
\end{aligned}
$$

According to the definition of degradation costs, these are those that reflect the qualitative degradation of natural resources caused by economic activity. Deterioration Cost is

$$DC = C_d(\theta-m(\xi))\left\{\int_0^{t_1}P(t)dt-\int_0^{t_2}D(t)dt\right\}$$

$$DC = C_d\big(\theta-m(\xi)\big)\left[\alpha\big((\rho+1)t_1-t_2\big)+\beta\left((\rho+1)\frac{{t_1}^2}{2}-\frac{{t_2}^2}{2}\right)\right] \tag{11}$$

The absence of raw materials results in a business incurring shortage costs, such as time lost while waiting for raw materials. It can be formulated as

$$SC = C_s\left\{-\int_{t_2}^{t_3}Q(t)dt-\int_{t_3}^{T}Q(t)dt\right\}$$

$$= C_s \left[\left\{ \alpha \left(\frac{t_2^2}{2} - t_3 t_2 + \frac{t_3^2}{2} \right) + (\beta - \alpha\rho) \left(\frac{t_2^3}{3} - \frac{t_3 t_2^2}{2} + \frac{t_3^3}{6} \right) - \beta\rho \left(\frac{t_2^4}{4} - \frac{t_3 t_2^3}{3} + \frac{t_3^4}{12} \right) \right\} + (\tau - 1) \left\{ \alpha \left(\frac{T^2}{2} - T t_3 + \frac{t_3^2}{2} \right) + \beta \left(\frac{T^3}{3} - \frac{t_3 T^2}{2} + \frac{t_3^3}{6} \right) \right\} \right] \tag{12}$$

The lost sales cost (LSC) is the price we pay when we fail to satisfy customer demand.

$$LC = C_l[- \int_{t_2}^{t_3} (1 - e^{-\rho t}) D(t) dt]$$

$$LC = -\rho C_l \left[\alpha \left(\frac{t_3^2}{2} - \frac{t_2^2}{2} \right) + \beta \left(\frac{t_3^3}{3} - \frac{t_2^3}{3} \right) \right] \tag{13}$$

Hence the total relevant cost for the production-inventory system during the cycle [0, T] is

$$\mathrm{TC}(\mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3, \mathrm{T}, \xi) = [\mathrm{PC} + \mathrm{HC} + \mathrm{DC} + \mathrm{SC} + \mathrm{LC}] \tag{14}$$

Case-2

Inventory level starts out at zero and shortages begin to develop at t = 0. At the onset of the production process ($t = t_1$), they reach their maximum level of shortages. All partially backlogged shortages have been met at time t2 and inventory is in positive territory. The inventory level increases and reaches its maximum at $t = t_3$. As a result of the combined effects of the lower demand and the deterioration during $[t_3, T]$ the production process is suspended and the inventory level drops. Investing in preservation technology reduces deterioration by θ - m (ξ), because the cost of the preservation technology ξ is invested. Thus, inventory depletion can be derived from the differential equation shown below in figure 2.

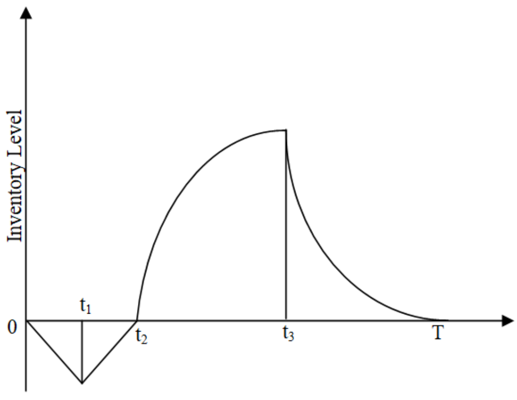

Figure: 2 (Case-2)

$$\frac{dQ(t)}{dt} = -D(t)e^{-\rho t};\ 0 \leq t \leq t_1,\ Q(0) = 0 \tag{15}$$

$$\frac{dQ(t)}{dt} = P(t) - D(t);\ t_1 \leq t \leq t_2,\ Q(t_2) = 0 \tag{16}$$

$$\frac{dQ(t)}{dt} + (\theta - m(\xi))Q(t) = P(t) - D(t);\ t_2 \leq t \leq t_3,\ Q(t_2) = 0$$

$$(17)$$

$$\frac{dQ(t)}{dt} + (\theta - m(\xi))Q(t) = -D(t);\ t_3 \leq t \leq T,\ Q(T) = 0$$

(18)

The solutions of above equations are

$Q(t) = -\left[\alpha(t) + (\beta - \alpha\rho)\left(\frac{t^2}{2}\right) - \beta\rho\left(\frac{t^3}{3}\right)\right]$, $0 \le t \le t_1$

(19)

$Q(t) = \left[(\rho - 1)\left(\alpha(t - t_2) + \beta\left(\frac{t^2 - {t_2}^2}{2}\right)\right)\right]$, $t_1 \le t \le t_2$

(20)

$Q(t) = [\left(1 - (\theta - m(\xi))t\right)\left(\alpha(t - t_2) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{t^2 - {t_2}^2}{2}\right) + \beta(\theta - m(\xi))\left(\frac{t^3 - {t_2}^3}{3}\right)\right)$, $t_2 \le t \le t_3$

(21)

$Q(t) = \left[\left(1 - (\theta - m(\xi))t\right)\left(\alpha(T - t) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{T^2 - t^2}{2}\right) + \beta(\theta - m(\xi))\left(\frac{T^3 - t^3}{3}\right)\right)\right]$, $t_3 \le t \le T$

(22)

Production cost $PC = C_p\left\{\int_{t_1}^{t_2} P(t)dt + \int_{t_2}^{t_3} P(t)dt\right\}$

$$PC = C_p\tau[\alpha(t_3 - t_1) + \beta\left(\frac{t_3^2}{2} - \frac{t_1^2}{2}\right)]$$

(23)

Holding cost is $HC = h[\int_{t_2}^{t_3} Q(t)dt + \int_{t_3}^{T} Q(t)dt]$

$HC = h\left[(\tau - 1)\left\{\alpha\left(\frac{t_3^2}{2} - t_3t_2 + \frac{t_2^2}{2}\right) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{t_2^3}{3} - \frac{t_3t_2^2}{2} + \frac{t_3^3}{6}\right) + \beta(\theta - m(\xi))\left(\frac{t_2^4}{4} - \frac{t_3t_2^3}{3} + \frac{t_3^4}{12}\right) - (\theta - m(\xi))\left(\alpha\left(\frac{t_2^3}{6} - \frac{t_2{t_3}^2}{2} + \frac{t_3^3}{3}\right) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{t_2^4}{8} - \frac{{t_3}^2t_2^2}{4} + \frac{t_3^4}{8}\right) + \beta(\theta - m(\xi))\left(\frac{t_2^5}{10} - \frac{{t_3}^2t_2^3}{6} + \frac{t_3^5}{15}\right)\right)\right\} + \left\{\alpha\left(\frac{T^2}{2} - Tt_3 + \frac{t_3^2}{2}\right) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{T^3}{3} - \frac{t_3T^2}{2} + \frac{t_3^3}{6}\right) + \beta(\theta - m(\xi))\left(\frac{T^4}{4} - \frac{t_3T^3}{3} + \frac{t_3^4}{12}\right) - (\theta - m(\xi))\left\{\alpha\left(\frac{T^3}{6} - \frac{T{t_3}^2}{2} + \frac{t_3^3}{3}\right) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{T^4}{8} - \frac{T^2t_3^2}{4} + \frac{t_3^4}{8}\right) + \beta(\theta - m(\xi))\left(\frac{T^5}{10} - \frac{{t_3}^2T^3}{6} + \frac{t_3^5}{15}\right)\right\}\right\}\right]$

(24)

Carbon emission cost associated with the holding cost, $HC = h\left[(\tau - 1)\left\{\alpha\left(\frac{t_3^2}{2} - t_3t_2 + \frac{t_2^2}{2}\right) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{t_2^3}{3} - \frac{t_3t_2^2}{2} + \frac{t_3^3}{6}\right) + \beta(\theta - m(\xi))\left(\frac{t_2^4}{4} - \frac{t_3t_2^3}{3} + \frac{t_3^4}{12}\right) - (\theta - m(\xi))\left(\alpha\left(\frac{t_2^3}{6} - \frac{t_2{t_3}^2}{2} + \frac{t_3^3}{3}\right) + \left(\beta + \alpha(\theta - m(\xi))\right)\left(\frac{t_2^4}{8} - \frac{{t_3}^2t_2^2}{4} + \frac{t_3^4}{8}\right) + \beta(\theta - m(\xi))\left(\frac{t_2^5}{10} -\right.\right.\right.\right.$

$\frac{t_3{}^2 t_2^3}{6}+\frac{t_3^5}{15}\Big)\Big)\Big\}+\Big\{\alpha\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)+\big(\beta+\alpha(\theta-m(\xi))\big)\left(\frac{T^3}{3}-\frac{t_3T^2}{2}+\frac{t_3^3}{6}\right)+\beta\big(\theta-$
$m(\xi)\big)\left(\frac{T^4}{4}-\frac{t_3T^3}{3}+\frac{t_3^4}{12}\right)-\big(\theta-m(\xi)\big)\Big\{\alpha\left(\frac{T^3}{6}-\frac{Tt_3{}^2}{2}+\frac{t_3^3}{3}\right)\Big)+\big(\beta+\alpha(\theta-m(\xi))\big)\Big(\frac{T^4}{8}-$
$\frac{T^2t_3^2}{4}+\frac{t_3^4}{8}\Big)+\beta\big(\theta-m(\xi)\big)\left(\frac{T^5}{10}-\frac{t_3{}^2T^3}{6}+\frac{t_3^5}{15}\right)\Big\}\Big\}\Big]$

Deterioration Cost is

$$DC=C_d(\theta-m(\xi))[\int_{t_2}^{t_3}P(t)dt-\int_{t_2}^{T}D(t)dt]$$

$$DC=C_d\big(\theta-m(\xi)\big)\left[\tau\left(\alpha(t_3-t_2)+\beta\left(\frac{t_3{}^2}{2}-\frac{t_2{}^2}{2}\right)\right)-\left(\alpha(T-t_2)+\beta\left(\frac{T^2}{2}-\frac{t_2{}^2}{2}\right)\right)\right] \quad (25)$$

Shortage cost

$$SC=C_s\left\{-\int_0^{t_1}Q(t)dt-\int_{t_1}^{t_2}Q(t)dt\right\}$$

$$SC=C_s\left[\left\{\alpha\left(\frac{t_1^2}{2}\right)+(\beta-\alpha\rho)\left(\frac{t_1^3}{6}\right)-\beta\rho\left(\frac{t_1^4}{12}\right)\right\}+(\tau-1)\left\{\alpha\left(\frac{t_2^2}{2}-t_1t_2+\frac{t_1^2}{2}\right)+\beta\left(\frac{t_2^3}{3}-\frac{t_1t_2^2}{2}+\frac{t_1^3}{6}\right)\right\}\right] \quad \textbf{(26)}$$

Lost sales cost

$$LC=C_l[-\int_0^{t_1}(1-e^{-\delta t})D(t)dt]$$
$$=-\rho C_l\left[\alpha\left(\frac{t_1^2}{2}\right)+\beta\left(\frac{t_1^3}{3}\right)\right] \quad \textbf{(27)}$$

Therefore, the total cost would be

$$\mathrm{TC}(t_1,t_2,t_3,T,\xi)=[C_0+PC+HC+DC+SC+LC] \quad \textbf{(28)}$$

To minimize the objective function, we need to differentiate the total cost function TC w. r. t to the decision variable t_1 and T. The necessary conditions to get the extreme values are as follows

$$\frac{\partial TC(t_1,T)}{\partial t_1}=0 \text{ and } \frac{\partial TC(t_1,T)}{\partial T}=0$$

In the case of a hessian matrix, the determinant of the principal minor must be positive. This is the Hessian matrix of the total cost function:

$$\begin{bmatrix}\frac{\partial^2 TC}{\partial t_1{}^2} & \frac{\partial^2 TC}{\partial t_1\partial T}\\ \frac{\partial^2 TC}{\partial T\partial t_1} & \frac{\partial^2 TC}{\partial T^2}\end{bmatrix}$$

IV. Numerical example

We employ the following inventory system as an illustration of proposed model, which gives values of various parameters in proper units

Example 1 (Case 1): We have considered the values of parameters based on the previous studies, but resalable estimation has been done

$$\alpha = 500, \beta = 1.5, C_o = 250, C_p = 15, C_{pe} = 1.5, a = 0.05, C_d = 5, C_{de} = 0.45, C_s = 5, C_l = 3, h = 1.2, h_e = 0.5, \theta = 0.05, \rho = 0.1, \tau = 2$$

Solving the outlined problem provides following solutions

$t_1^* = 1.160988$, $t_2^* = 1.547984$, $t_3^* = 2.321976$, $T^* = 4.643952$, $\xi^* = 70.5918$, $TC^*(t_1, t_2, t_3, T, \xi) = 5531.35$

Example 2 (Case 2): We have considered the values of parameters based on the previous studies, but resalable estimation has been done

$$\alpha = 700, \beta = 1.2, C_0 = 250, C_p = 2, C_{pe} = 2, a = 0.05, C_d = 5, C_{de} = 5, C_s = 5, C_l = 3, h = 1.2, h_e = 0.25, \theta = 0.05, \rho = 0.1, \tau = 2.2$$

Solving the outlined problem provides following solutions

$t_1^* = 1.160988$, $t_2^* = 1.547984$, $t_3^* = 2.321976$, $T^* = 4.643952$,

$\xi^* = 70.5916$, $TC^*(t_1, t_2, t_3, T, \xi) = 5531.43$.

V. SENSITIVITY ANALYSIS

Changes in some parameters such as demand parameters, deterioration rate etc. are studied by performing a sensitivity analysis. Listed below in table 1 and table 2, are the results of the analysis.

TABLE 1: CASE-1

		ξ^*	t_1^*	t_2^*	t_3^*	T^*	TC^*
Change in a	110	71.14356	1.173216	1.56428	2.34642	4.69284	6315.972
	105	70.88604	1.167312	1.556408	2.334612	4.669224	5921.664
	95	70.25292	1.15422	1.538952	2.308428	4.616856	5145.660
	90	69.86184	1.146924	1.529236	2.293854	4.587708	4764.876
Change in b	4.4	69.90648	1.148508	1.531348	2.297022	4.594044	5303.244
	4.2	70.25496	1.154652	1.53954	2.30931	4.61862	5415.228
	3.8	70.91532	1.167516	1.556692	2.335038	4.670076	5652.012
	3.6	71.22564	1.17426	1.565676	2.348514	4.697028	5777.232
Change in θ	0.11	78.36828	1.160988	1.547984	2.321976	4.643952	5543.412
	0.105	74.2752	1.160988	1.547984	2.321976	4.643952	5532.612
	0.095	67.25856	1.160988	1.547984	2.321976	4.643952	5526.612
	0.09	64.22856	1.160988	1.547984	2.321976	4.643952	5525.412
change in β	0.11	70.59096	1.160988	1.54798	2.32197	4.64394	5531.16
	0.105	70.5912	1.160988	1.54798	2.32197	4.64394	5531.28
	0.095	70.5918	1.160988	1.547988	2.321982	4.643964	5531.544
	0.09	70.59204	1.160988	1.547988	2.321982	4.643964	5531.676
change in γ	0.11	70.59096	1.160988	1.54798	2.32197	4.64394	5521.8
	0.105	70.5912	1.160988	1.54798	2.32197	4.64394	5526.6
	0.095	70.5918	1.160988	1.547988	2.321982	4.643964	5536.236

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	0.09	70.59204	1.160988	1.547988	2.321982	4.643964	5541.06
change in δ	0.22	63.49176	1.138908	1.518536	2.277804	4.555608	4601.388
	0.21	67.11492	1.149576	1.532772	2.299158	4.598316	5045.58
	0.19	73.91892	1.173132	1.564176	2.346264	4.692528	6061.356
	0.18	77.09376	1.18602	1.58136	2.37204	4.74408	6638.088

TABLE II CASE-2

		ξ^*	t_1^*	t_2^*	t_3^*	T*	TC*
change in a	154	49.68252	0.406032	0.541376	0.812064	1.62408	2079.024
	147	48.84516	0.402982	0.537308	0.805968	1.61196	2079.084
	133	46.98528	0.397726	0.5303	0.795456	1.59096	1852.404
	126	45.94896	0.395446	0.52726	0.790896	1.58184	1776.588
change in b	4.4	47.94276	0.40012	0.533492	0.800244	1.60044	1926.18
	4.2	47.94552	0.400174	0.533564	0.800352	1.60068	1927.128
	3.8	47.95092	0.400276	0.5337	0.800556	1.60116	1929.024
	3.6	47.95356	0.400326	0.533768	0.800652	1.60128	1929.972
change in θ	0.11	53.2092	0.400224	0.533632	0.800448	1.60092	1940.112
	0.105	50.4402	0.400224	0.533632	0.800448	1.60092	1934.1
	0.095	45.6936	0.400224	0.533632	0.800448	1.60092	1926.888
	0.09	43.64388	0.400224	0.533632	0.800448	1.60092	1925.676
change in β	0.11	47.91312	0.400128	0.533504	0.800256	1.60056	1927.32
	0.105	47.93064	0.400176	0.533568	0.800352	1.60068	1927.704
	0.095	47.9658	0.400272	0.533696	0.800544	1.60104	1928.448
	0.09	47.98332	0.40032	0.53376	0.80064	1.60128	1928.832
change in γ	0.11	47.85156	0.399954	0.533272	0.799908	1.59984	1926.012
	0.105	47.89992	0.40009	0.533452	0.800184	1.60032	1927.044
	0.095	47.99664	0.40036	0.533812	0.800724	1.6014	1929.108
	0.09	48.04524	0.400492	0.533988	0.800988	1.602	1930.152
change in δ	0.44	47.9292	0.40017	0.53356	0.80034	1.60068	1927.848
	0.42	47.93868	0.400198	0.533596	0.8004	1.6008	1927.956
	0.38	47.95776	0.400252	0.533668	0.800508	1.60104	1928.196
	0.36	47.96736	0.400278	0.533704	0.800556	1.60116	1928.304

Observations

1. From the above table, it is noticed that the small increment in demand parameter 'a' resulted in increment in the total cost and cycle length.
2. Meanwhile, optimal value of ξ^* is also reported to be positive sensitive toward the demand rate.
3. It is revealed that the increment in demand parameter 'b' resulted in the reduction in cost and optimal cycle length.
4. The results shown that the partial backlogging parameter 'δ' is reported to be negative sensitive toward the optimal values of ξ^* and cycle length.
5. With Increment in constant deterioration rate θ, Optimal value of ξ^* is also increases.

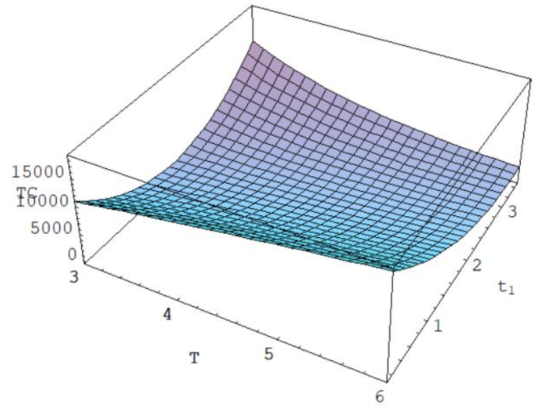

Figure 3: Convexity of TC^* (Case 1) w. r. t. T^* and t_1^*

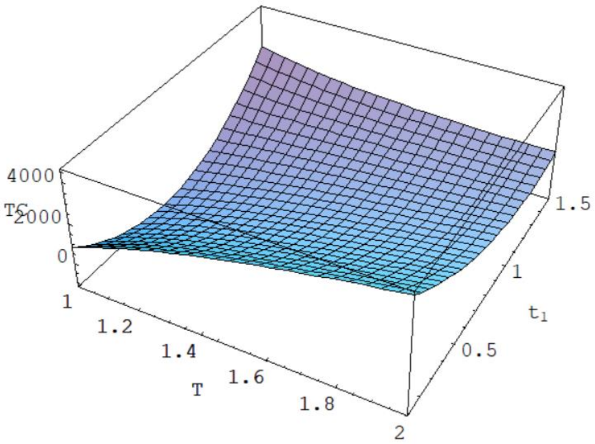

Figure 4: Convexity of TC^* (Case 2) w. r. t. T^* and t_1^*

VI. CONCLUSION

The deterioration rate for deteriorated items can be controlled by controlling production inventory and determining the time-dependent demand rate of the deteriorated item inventory. Several factors influence production cost, including demand and carbon emission. In this article we have examined two cases: the first where the shortage occurs at the end of the model and the second when it starts. Throughout the study, preservation is taken into consideration. The minimum value of the total relevant cost has been determined. The model is illustrated numerically and a sensitivity analysis is presented at the end. In addition to inflation and trade credit, this study can incorporate other inventory control system parameters such as income and sales tax.

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